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Filter-and-forward relaying in cognitive networks with blind channel estimation

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Saeideh Mohammadkhani[1] ✉, *George K. Karagiannidis*[2]

[1]*Electrical Engineering Department, Iran University of Science and Technology, Tehran, Iran*
[2]*Department of Electrical and Computer Engineering, Aristotle University of Thessaloniki, Thessaloniki 541 24, Greece*
✉ *E-mail: s_mohammadkhani@elec.iust.ac.ir*

Abstract: The authors study the design of cooperative beamforming for an underlay cognitive relay network over frequency selective fading channels. The network under consideration consists of one primary link, one secondary link and relay nodes that assist the communication. It is assumed imperfect blind channel estimation, i.e. only the second order statistics – with uncertainty – of the channel gain between the primary transmitter-relay and relay-primary receiver, is available. To mitigate the channel's frequency selectivity, each relay is equipped with a finite impulse response filter, which performs *filter and forward* relaying. The authors consider two different scenarios: in the first, the objective is to minimise the total transmit power of the relays subject to a constraint on the interference induce on the primary receiver. Furthermore, a constraint is also considered on the received signal-to-interference plus noise ratio (SINR) in the secondary receiver. In the second scenario, the objective is to maximise SINR in the secondary receiver, subject to constraints on the sum power of the relays and also on the noise and interference power in the primary receiver. Simulation results show how total power of relay varies with the uncertainty in the blind channel estimation in the first scenario. In addition, the achievable SINR deteriorates in the second scenario when the uncertainty increases.

1 Introduction

With the huge development of wireless communications and the resulting radio spectrum limitations, cognitive radio (CR), has attracted a considerable attention, as a dynamic new concept and technology to improve the spectrum utilisation. One of the common spectrum sharing strategies in CR networks (CRNs) is the *underlay* method. In this method, some secondary users (SUs) are allowed to use the allocated spectrum to the primary system, subject to keeping the resulting interference on the primary users (PUs) below a given threshold, which is known as the *interference temperature* [1]. Therefore, the transmit power of the SU is restricted by the interference constraint. Accordingly, the quality of service (QoS) of the SU is difficult to be guaranteed. Similar to other conventional wireless systems, beamforming was proposed to control the interference and to improve the QoS in CRNs [2–4]. However, applying beamforming in CRNs is more challenging than in conventional wireless networks, due to the strict interference constraints of the primary service.

1.1 Relevant literature

Scanning the relevant literature, cooperative or relaying beamforming was proposed for CRNs in [5–11], to improve SU QoS. Specifically, in [5] the rate maximisation problem and the design of the beamforming structure was investigated for relay-aided CRNs. An iterative algorithm was proposed to solve a non-convex maximisation problem, where in each iteration the corresponding subproblems are convex. In [6], considering the interference constraint at the PU in an underlay CRN, the beamforming weights were obtained via signal-to-interference plus noise ratio (SINR) maximisation. When the transmitted power of relays is small and the interference power is passive, then the beamforming weights were obtained in closed-form, while in the medium SINR region, they were evaluated by using convex optimisation. In [7], the authors analysed the outage probability of the CRNs, when it is assumed that the dependence between the received SINRs is due to the interference power constraint at the primary receiver. Furthermore, the outage probability of CRNs was investigated in [8–11] with different assumptions. Specifically, in [8], outage probability with multiple relays is derived in a closed-form. Considering two different types of power constraints at SU-Tx, the diversity effect on the outage probability was investigated. The first type of power constraint is the peak interference and the second type both peak interference and maximum transmit power of SU-Tx and relays. In [9, 10], closed-form expressions were derived, when the existence of a direct link for the SU is assumed. In these schemes, a selection combining receiver is used at the secondary receiver in order to combine the received signals from relay and transmitter. In [9], Nakagami-*m* fading was assumed, while in [10] Rayleigh fading for all links. Furthermore in [11], CRNs with multiple PUs is considered and an approximate formula for the ergodic capacity and a closed-form one for the outage probability, were obtained. However, in all the aforementioned works, fading is assumed to be frequency flat, which is not true in important practical wireless systems, as in orthogonal frequency division multiplexing, and single carrier transmission, where frequency selective fading is considered.

Unlike the relaying techniques, i.e. amplify and forward (AF) and decode and forward that have been proposed for flat fading channels [12], filter and forward (FF) has been introduced in [13] for frequency selective fading channels. According to this scheme, each relay is equipped with a finite impulse response (FIR) filter, which is used to compensate the channel's distortion effects. Specifically, in [14, 15], the authors studied the FF beamforming in CRNs with an underlay structure over frequency selective fading channels. The aim of these works was to minimise the sum of the total transmit power both of relays and secondary transmitter, while the interference constraint on the primary link is maintained. However, these FF relaying schemes perform continuous estimation of the channel state information (CSI) at the relays or destination. Furthermore, CSI is considered to be perfect, which is not true in practical applications due to quantisation, estimation error or Doppler spread. Therefore, it is necessary uncertainty to be introduced in the estimated CSI and optimisation to be performed regarding to this uncertainty in order to have a

Table 1 Summary of used notations in this paper

Symbols	Description
$(\cdot)^*$	conjugate
$(\cdot)^T$	transpose
$(\cdot)^H$	conjugate transpose
$\otimes$	Kronecker product
$\mathbb{E}(\cdot)$	statistical expectation
var(.)	variance of the distribution
tr(·)	trace of a matrix
diag($\boldsymbol{a}$)	diagonal matrix with the element of $\boldsymbol{a}$
diag{$\boldsymbol{A}$}	vector of the diagonal elements matrix $\boldsymbol{A}$
$\boldsymbol{I}_N$	identity matrix of size N
$\boldsymbol{0}_{N\times M}$	zero matrix of size $N\times M$
$\lambda(\boldsymbol{A})$	principle eigenvalue of the matrix $\boldsymbol{A}$
$\boldsymbol{A}\succeq 0$	positive semi-definite matrix

robust design, less sensitive to channel estimation errors. In the literature, there are two types of such robust design, which corresponds to different ways that the channel error is modelled: *stochastic* and *worst-case*. These schemes were studied for CRNs with and without relays, assuming flat fading channels in [16–22]. In [22], robust beamforming is assumed for CRNs over flat fading channel and the channel error is modelled by a Gaussian random variable (RV). For frequency selective fading channels, a robust FF beamforming was investigated in [23] for a relay network consisting of a transmitter, a receiver and R relay nodes, while the worst-case optimisation was investigated with spherical uncertainty. It should be noted that, in the aforementioned papers, the performance of beamforming schemes has been studied for training-based channel estimation. However, this method consumes extra bandwidth to accommodate the periodic known symbol and thus reduces the spectral efficiency. In CRNs, due to the limited collaboration between secondary and primary networks, training-based channel estimation cannot be used. Earlier techniques in blind channel estimation rely on some form of higher order statistics. However, recently another channel model was introduced in [24–26], where the *second order statistic* of the channel coefficients is assumed to be available. In addition, this blind estimation is not perfect and the uncertainty is considered for the statistical metrics under consideration, i.e. mean and covariance. However, this model was also used in relay networks with flat fading channels.

1.2 Contribution

In this paper, we investigate the use of FF beamforming in CRNs with blind channel estimation, i.e. only the second order statistics of the channel from the PU-Tx to relays and relays to the PU-Rx is assumed to be available. In addition, the blind estimation is not perfect and a parameter is introduced to denote the uncertainty in the estimated statistical metrics. We also assume that the cognitive relays have perfect CSI of the SU link. The CSI estimation process of the SUs can be carried out using several methods. For example, in [27, 28], a two-phase method was proposed, where in the first phase, the channel between a relay and a receiver is estimated while in the second phase, the channel between transmitter and relay is estimated. Based on these assumptions, we study the following two beamforming scenarios:

(i) In the first scenario, the aim is to minimise the total transmitted power of the relays. Two constraints are considered for primary and secondary links, which guarantee that the interference at the primary receiver remains below a given threshold and SINR at the secondary receiver is above a predefined value, respectively. We show that the design of the beamforming weights results to a semi-definite programming (SDP) problem, which can be efficiently solved by using the interior point method.
(ii) In the second scenario, SINR in the SU-Rx is maximised subject to a constraint on the interference and noise at the PU receiver (PU-Rx) and to another constraint on the total transmit power of the relays. We show that in this case the design of the beamforming weights is a quasi-convex optimisation problem.

1.3 Structure

The rest of paper is organised as follows: the system and channel model is introduced in Section 2. In Sections 3 and 4, power

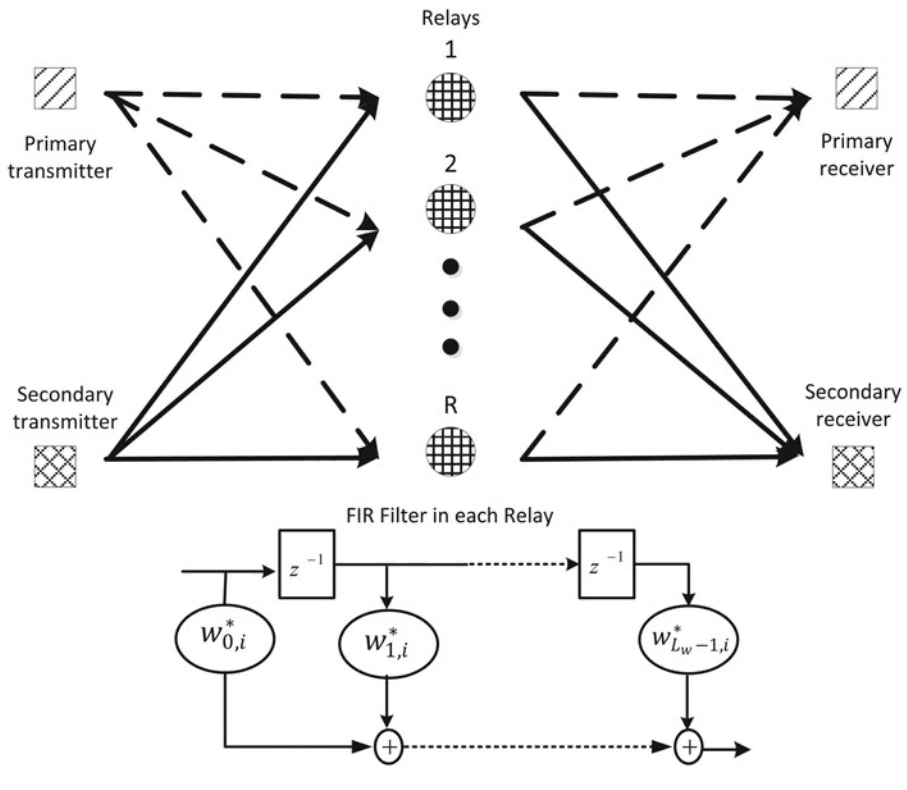

Fig. 1 *System model of the CR relay network under consideration*

minimisation and SINR maximisation problems are evaluated, respectively. Simulation results are presented in Section 4 and finally, some concluding remarks in Section 5. For convenience, the notations used in this paper are defined in Table 1.

2 System and channel model

An underlay CRN with a pair of PU-Tx and PU-Rx, a pair of SU-Tx and SU-Rx and R relay nodes is assumed as in Fig. 1. Furthermore, it is assumed that the transmitters have limited coverage area and they need help of the relays to be able to communicate with their respective receivers. In other words, there are not direct links between the transmitters and receivers. The communication is performed over two time slots.

The vector of the received signals, $\boldsymbol{r}(n)$, at the relays can be written as

$$\boldsymbol{r}(n) = \sum_{l=0}^{L_{\mathrm{p},f}-1} \sqrt{P_{\mathrm{P}}}\boldsymbol{f}_{\mathrm{p},l}x^{(\mathrm{P})}(n-l) + \sum_{l=0}^{L_{\mathrm{s},f}-1} \sqrt{P_{\mathrm{S}}}\boldsymbol{f}_{\mathrm{s},l}x^{(\mathrm{S})}(n-l) + \boldsymbol{v}(n)$$
$$= \sqrt{P_{\mathrm{p}}}\boldsymbol{F}_{\mathrm{p}}\boldsymbol{x}^{(\mathrm{P})} + \sqrt{P_{\mathrm{s}}}\boldsymbol{F}_{\mathrm{s}}\boldsymbol{x}^{(\mathrm{S})} + \boldsymbol{v}(n), \tag{1}$$

where (see equation at bottom of the page)

The parameters $f_{\mathrm{p},l,i}$ and $f_{\mathrm{s},l,i}$ are the lth tap of the channels between the PU-Tx and the ith relay and the SU-Tx and the ith relay, respectively. In addition, $x^{(\mathrm{P})}(n)$, $x^{(\mathrm{S})}(n)$, P_{P} and P_{S} are the transmit symbols by PU-Tx and SU-Tx, the PU-Tx transmit power and the SU-Tx transmit power, respectively, and n denotes the time. Finally, $\boldsymbol{v}(n) = [v_1(n), \dots v_R(n)]^{\mathrm{T}}$ is an $R \times 1$ noise vector, whose components are zero mean complex Gaussian RVs with variance σ_v^2.

In the second time slot, after passing the signals through the FIR filters, the transmit signal vector, $\boldsymbol{x}(n) \in \mathbb{C}^N$, by the relays at the time n will be

$$\boldsymbol{x}(n) = \sum_{l=0}^{L_w-1} \boldsymbol{W}_l^{\mathrm{H}}\boldsymbol{r}(n)$$
$$= \sum_{l=0}^{L_w-1} \boldsymbol{W}_l^{\mathrm{H}}\left(\sqrt{P_{\mathrm{p}}}\boldsymbol{F}_{\mathrm{p},l}\tilde{\boldsymbol{x}}^{(\mathrm{P})}(n) + \sqrt{P_{\mathrm{s}}}\boldsymbol{F}_{\mathrm{s},l}\tilde{\boldsymbol{x}}^{(\mathrm{S})}(n) + \boldsymbol{v}(n-l)\right)$$
$$= \sqrt{P_{\mathrm{p}}}\boldsymbol{W}^{\mathrm{H}}\boldsymbol{\Xi}_{\mathrm{p}}\tilde{\boldsymbol{x}}^{(\mathrm{P})}(n) + \sqrt{P_{\mathrm{s}}}\boldsymbol{W}^{\mathrm{H}}\boldsymbol{\Xi}_{\mathrm{s}}\tilde{\boldsymbol{x}}^{(\mathrm{S})}(n) + \boldsymbol{W}^{\mathrm{H}}\tilde{\boldsymbol{v}}(n), \tag{2}$$

where

$$\boldsymbol{W}_l = \mathrm{diag}\{w_{l,1}, \dots, w_{l,R}\},$$

and $w_{l,i}$, $l = 0,\dots, L_w - 1$, $i = 1,\dots, R$ is the lth coefficient of the FIR filter at the ith relay. In addition

$$\boldsymbol{F}_{\mathrm{p},l} \triangleq [\boldsymbol{0}_{R\times l}, \boldsymbol{F}_{\mathrm{p}}, \boldsymbol{0}_{R\times(L_w-1-l)}],$$
$$\boldsymbol{F}_{\mathrm{s},l} \triangleq [\boldsymbol{0}_{R\times l}, \boldsymbol{F}_{\mathrm{s}}, \boldsymbol{0}_{R\times(L_w-1-l)}],$$
$$\tilde{\boldsymbol{x}}^{(\mathrm{P})}(n) \triangleq [x^{(\mathrm{P})}(n), \dots, x^{(\mathrm{P})}(n-(L_{\mathrm{p},f}+L_w-2)]^{\mathrm{T}},$$
$$\tilde{\boldsymbol{x}}^{(\mathrm{P})}(n) \triangleq [x^{(\mathrm{S})}(n), \dots, x^{(\mathrm{S})}(n-(L_{\mathrm{s},f}+L_w-2)]^{\mathrm{T}},$$
$$\boldsymbol{W} \triangleq [\boldsymbol{W}_0, \dots, \boldsymbol{W}_{L_w-1}]^{\mathrm{T}},$$
$$\boldsymbol{\Xi}_{\mathrm{p}} \triangleq [\boldsymbol{F}_{\mathrm{p},0}^{\mathrm{T}}, \dots, \boldsymbol{F}_{\mathrm{p},L_w-1}^{\mathrm{T}}]^{\mathrm{T}}, \quad \boldsymbol{\Xi}_{\mathrm{s}} \triangleq [\boldsymbol{F}_{\mathrm{s},0}^{\mathrm{T}}, \dots, \boldsymbol{F}_{\mathrm{s},L_w-1}^{\mathrm{T}}]^{\mathrm{T}},$$
$$\tilde{\boldsymbol{v}}(n) \triangleq [\boldsymbol{v}^{\mathrm{T}}(n), \dots, \boldsymbol{v}^{\mathrm{T}}(n-(L_w+1))]^{\mathrm{T}}. \tag{3}$$

The received signals in PU-Rx, $r^{(\mathrm{P})}(n)$, and SU-Rx, $r^{(\mathrm{S})}(n)$, are defined as

$$r^{(\mathrm{P})}(n) = \sum_{l=0}^{L_{\mathrm{p},g}-1} \boldsymbol{g}_{\mathrm{p},l}^{\mathrm{T}}\boldsymbol{x}(n-l) + v^{(\mathrm{P})}(n), \tag{4}$$

$$r^{(\mathrm{S})}(n) = \sum_{l=0}^{L_{\mathrm{s},g}-1} \boldsymbol{g}_{\mathrm{s},l}^{\mathrm{T}}\boldsymbol{x}(n-l) + v^{(\mathrm{S})}(n), \tag{5}$$

where $\boldsymbol{g}_{\mathrm{p},l} \triangleq [g_{\mathrm{p},l,1}, \dots, g_{\mathrm{p},l,R}]^{\mathrm{T}}$ and $\boldsymbol{g}_{\mathrm{s},l} \triangleq [g_{\mathrm{s},l,1}, \dots, g_{\mathrm{s},l,R}]^{\mathrm{T}}$ with $g_{\mathrm{p},l,i}$ and $g_{\mathrm{s},l,i}$ be the lth tap of the channels between the ith relay and PU-Rx, SU-Rx, respectively. Furthermore, $v^{(\mathrm{P})}(n)$ and $v^{(\mathrm{S})}(n)$ are zero mean complex Gaussian RVs, which represent noise, at PU-Rx and SU-Rx, with variance $\sigma^2_{v(\mathrm{P})}$ and $\sigma^2_{v(\mathrm{S})}$, respectively. By substituting (2) in (4), and using a similar approach as in [13] we obtain

$$r^{(\mathrm{P})}(n) = \sqrt{P_{\mathrm{P}}}\boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{p}}\breve{\boldsymbol{F}}_{\mathrm{p,p}}\breve{\boldsymbol{x}}^{(\mathrm{P})}(n) + \sqrt{P_{\mathrm{S}}}\boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{p}}\breve{\boldsymbol{F}}_{\mathrm{s,p}}\breve{\boldsymbol{x}}^{(\mathrm{S})}(n) + \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{p}}\tilde{\boldsymbol{I}}\,\breve{\boldsymbol{v}}(n) + v^{(\mathrm{P})}(n)$$
$$= \underbrace{\sqrt{P_{\mathrm{P}}}\boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{p}}\bar{\boldsymbol{f}}_{\mathrm{p,p}}x^{(\mathrm{P})}(n)}_{\text{desired signal}} + \underbrace{\sqrt{P_{\mathrm{P}}}\boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{p}}\bar{\boldsymbol{F}}_{\mathrm{p,p}}\bar{\boldsymbol{x}}^{(\mathrm{P})}(n)}_{\text{ISI}} + \underbrace{\sqrt{P_{\mathrm{S}}}\boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{p}}\breve{\boldsymbol{F}}_{\mathrm{s,p}}\breve{\boldsymbol{x}}^{(\mathrm{S})}(n)}_{\text{interference from other user}} + \underbrace{\boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{p}}\tilde{\boldsymbol{I}}\,\breve{\boldsymbol{v}}(n) + v^{(\mathrm{P})}(n)}_{\text{noise}}, \tag{6}$$

and

$$r^{(S)}(n) = \sqrt{P_S}\boldsymbol{\omega}^H\mathbf{G}_s\breve{\mathbf{F}}_{s,s}\breve{\mathbf{x}}^{(S)}(n) + \sqrt{P_P}\boldsymbol{\omega}^H\mathbf{G}_s\breve{\mathbf{F}}_{p,s}\breve{\mathbf{x}}^{(P)}(n) + \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{s}}\tilde{\tilde{\boldsymbol{I}}}\breve{\breve{\boldsymbol{v}}}(n) + v^{(\mathrm{S})}(n)$$
$$= \underbrace{\sqrt{P_{\mathrm{S}}}\boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{s}}\bar{\boldsymbol{f}}_{\mathrm{s,s}}x^{(\mathrm{S})}(n)}_{\text{desired signal}} + \underbrace{\sqrt{P_{\mathrm{S}}}\boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{s}}\bar{\boldsymbol{F}}_{\mathrm{s,s}}\bar{\boldsymbol{x}}^{(\mathrm{S})}(n)}_{\text{ISI}} + \underbrace{\sqrt{P_P}\boldsymbol{\omega}^H\mathbf{G}_s\breve{\mathbf{F}}_{p,s}\breve{\breve{\mathbf{x}}}^{(P)}(n)}_{\text{interference from other user}} + \underbrace{\boldsymbol{\omega}^H\mathbf{G}_s\tilde{\tilde{\mathbf{I}}}\breve{\breve{\mathbf{v}}}(n) + v^{(S)}(n)}_{\text{noise}}, \tag{7}$$

where

$$\boldsymbol{\omega} \triangleq [\boldsymbol{\omega}_0, \dots, \boldsymbol{\omega}_{L_w-1}], \quad \boldsymbol{\omega}_l \triangleq \mathrm{diag}\{\boldsymbol{W}_l\},$$
$$\breve{\boldsymbol{F}}_{\mathrm{p,p}} \triangleq [\bar{\boldsymbol{f}}_{\mathrm{p,p}}, \bar{\boldsymbol{F}}_{\mathrm{p,p}}], \quad \breve{\boldsymbol{F}}_{\mathrm{s,s}} \triangleq [\bar{\boldsymbol{f}}_{\mathrm{s,s}}, \bar{\boldsymbol{F}}_{\mathrm{s,s}}],$$
$$\breve{\boldsymbol{F}}_{\mathrm{p,p}} \triangleq [\boldsymbol{\Xi}^{\mathrm{T}}_{\mathrm{p,p},0}, \dots, \boldsymbol{\Xi}^{\mathrm{T}}_{\mathrm{p,p},L_{\mathrm{p},g}-1}]^{\mathrm{T}},$$
$$\breve{\boldsymbol{F}}_{\mathrm{s,p}} \triangleq [\boldsymbol{\Xi}^{\mathrm{T}}_{\mathrm{s,p},0}, \dots, \boldsymbol{\Xi}^{\mathrm{T}}_{\mathrm{s,p},L_{\mathrm{p},g}-1}]^{\mathrm{T}},$$
$$\breve{\boldsymbol{F}}_{\mathrm{s,s}} \triangleq [\boldsymbol{\Xi}^{\mathrm{T}}_{\mathrm{s,s},0}, \dots, \boldsymbol{\Xi}^{\mathrm{T}}_{\mathrm{s,s},L_{\mathrm{s},g}-1}]^{\mathrm{T}},$$
$$\breve{\boldsymbol{F}}_{\mathrm{p,s}} \triangleq [\boldsymbol{\Xi}^{\mathrm{T}}_{\mathrm{p,s},0}, \dots, \boldsymbol{\Xi}^{\mathrm{T}}_{\mathrm{p,s},L_{\mathrm{s},g}-1}]^{\mathrm{T}},$$
$$\boldsymbol{G}_{\mathrm{p}} \triangleq [\boldsymbol{I}_{L_w} \otimes \boldsymbol{G}_{\mathrm{p},0}, \dots, \boldsymbol{L}_{L_w} \otimes \boldsymbol{G}_{\mathrm{p},L_{\mathrm{p},g}-1}],$$
$$\boldsymbol{G}_{\mathrm{s}} \triangleq [\boldsymbol{I}_{L_w} \otimes \boldsymbol{G}_{\mathrm{s},0}, \dots, \boldsymbol{L}_{L_w} \otimes \boldsymbol{G}_{\mathrm{s},L_{\mathrm{s},g}-1}],$$
$$\boldsymbol{G}_{\mathrm{p},l} \triangleq \mathrm{diag}\{\boldsymbol{g}_{\mathrm{p},l}\}, \quad \boldsymbol{G}_{\mathrm{s},l} \triangleq \mathrm{diag}\{\boldsymbol{g}_{\mathrm{s},l}\},$$
$$\tilde{\boldsymbol{I}} \triangleq [\breve{\boldsymbol{I}}_0^{\mathrm{T}}, \dots, \breve{\boldsymbol{I}}^{\mathrm{T}}_{L_{\mathrm{p},g}-1}]^{\mathrm{T}}, \quad \tilde{\tilde{\boldsymbol{I}}} \triangleq [\breve{\breve{\boldsymbol{I}}}_0^{\mathrm{T}}, \dots, \breve{\breve{\boldsymbol{I}}}^{\mathrm{T}}_{L_{\mathrm{s},g}-1}]^{\mathrm{T}},$$

$$\boldsymbol{F}_{\mathrm{p}} \triangleq [\boldsymbol{f}_{\mathrm{p},0}, \dots, \boldsymbol{f}_{\mathrm{p},L_{\mathrm{p},f}-1}] \quad \text{with} \quad \boldsymbol{f}_{\mathrm{p},l} \triangleq [f_{\mathrm{p},l,1}, \dots, f_{\mathrm{p},l,R}]^{\mathrm{T}}, \quad l = 0, \dots, L_{\mathrm{p},f}-1$$
$$\boldsymbol{F}_{\mathrm{s}} \triangleq [\boldsymbol{f}_{\mathrm{s},0}, \dots, \boldsymbol{f}_{\mathrm{s},L_{\mathrm{s},f}-1}] \quad \text{with} \quad \boldsymbol{f}_{\mathrm{s},l} \triangleq [f_{\mathrm{s},l,1}, \dots, f_{\mathrm{s},l,R}]^{\mathrm{T}}, \quad l = 0, \dots, L_{\mathrm{s},f}-1.$$

$$\boldsymbol{\Xi}_{\mathrm{p,p},l} \triangleq [\mathbf{0}_{RL_w \times l}, \boldsymbol{\Xi}_{\mathrm{p}}, \mathbf{0}_{RL_w \times (L_{\mathrm{p},g}-1-l)}],$$
$$\boldsymbol{\Xi}_{\mathrm{s,p},l} \triangleq [\mathbf{0}_{RL_w \times l}, \boldsymbol{\Xi}_{\mathrm{s}}, \mathbf{0}_{RL_w \times (L_{\mathrm{p},g}-1-l)}],$$
$$\boldsymbol{\Xi}_{\mathrm{p,s},l} \triangleq [\mathbf{0}_{RL_w \times l}, \boldsymbol{\Xi}_{\mathrm{p}}, \mathbf{0}_{RL_w \times (L_{\mathrm{s},g}-1-l)}],$$
$$\boldsymbol{\Xi}_{\mathrm{s,s},l} \triangleq [\mathbf{0}_{RL_w \times l}, \boldsymbol{\Xi}_{\mathrm{s}}, \mathbf{0}_{RL_w \times (L_{\mathrm{s},g}-1-l)}],$$
$$\breve{\boldsymbol{I}}_l \triangleq [\mathbf{0}_{RL_w \times l}, \boldsymbol{I}_{RL_w}, \mathbf{0}_{RL_w \times (L_{\mathrm{p},g}-1-l)}],$$
$$\breve{\breve{\boldsymbol{I}}}_l \triangleq [\mathbf{0}_{RL_w \times l}, \boldsymbol{I}_{RL_w}, \mathbf{0}_{RL_w \times (L_{\mathrm{s},g}-1-l)}],$$
$$\breve{\boldsymbol{v}}(n) \triangleq [\boldsymbol{v}^{\mathrm{T}}(n), \ldots, \boldsymbol{v}^{\mathrm{T}}(n-(L_w+L_{\mathrm{p},g}-2))]^{\mathrm{T}},$$
$$\breve{\breve{\boldsymbol{v}}}(n) \triangleq [\boldsymbol{v}^{\mathrm{T}}(n), \ldots, \boldsymbol{v}^{\mathrm{T}}(n-(L_w+L_{\mathrm{s},g}-2))]^{\mathrm{T}},$$
$$\breve{\boldsymbol{x}}^{(\mathrm{P})}(n) \triangleq [x^{(\mathrm{P})}(n), \bar{\boldsymbol{x}}^{(\mathrm{P})}(n)]^{\mathrm{T}}, \quad \breve{\breve{\boldsymbol{x}}}^{(\mathrm{S})}(n) \triangleq [x^{(\mathrm{S})}(n), \bar{\boldsymbol{x}}^{(\mathrm{S})}(n)]^{\mathrm{T}},$$
$$\breve{\boldsymbol{x}}^{(\mathrm{P})}(n) \triangleq [x^{(\mathrm{P})}, \ldots, x^{(\mathrm{P})}(n-L_{\mathrm{p},f}+L_w+L_{\mathrm{p},g}-3))]^{\mathrm{T}},$$
$$\breve{\breve{\boldsymbol{x}}}^{(\mathrm{P})}(n) \triangleq [x^{(\mathrm{P})}, \ldots, x^{(\mathrm{P})}(n-L_{\mathrm{p},f}+L_w+L_{\mathrm{s},g}-3))]^{\mathrm{T}},$$
$$\breve{\boldsymbol{x}}^{(\mathrm{S})}(n) \triangleq [x^{(\mathrm{S})}, \ldots, x^{(\mathrm{S})}(n-L_{\mathrm{s},f}+L_w+L_{\mathrm{p},g}-3))]^{\mathrm{T}},$$
$$\breve{\breve{\boldsymbol{x}}}^{(\mathrm{S})}(n) \triangleq [x^{(\mathrm{S})}, \ldots, x^{(\mathrm{S})}(n-L_{\mathrm{s},f}+L_w+L_{\mathrm{s},g}-3))]^{\mathrm{T}}.$$

It is assumed that only the second order statistic of CSI for primary link is available and estimated in a processing centre placed in one of the relays or very close to the relays group. Therefore, the channel coefficients for the primary link can be modelled as [24]

$$\begin{aligned} f_{\mathrm{p},l,i} &= \bar{f}_{\mathrm{p},l,i} + \tilde{f}_{\mathrm{p},l,i}, \\ g_{\mathrm{p},l,i} &= \bar{g}_{\mathrm{p},l,i} + \tilde{g}_{\mathrm{p},l,i}, \end{aligned} \tag{8}$$

where $\bar{f}_{\mathrm{p},l,i}$ and $\bar{g}_{\mathrm{p},l,i}$ are the means of $f_{\mathrm{p},l,i}$ and $g_{\mathrm{p},l,i}$, respectively, and $\tilde{f}_{\mathrm{p},l,i}$ and $\tilde{g}_{\mathrm{p},l,i}$ are zero-mean RVs. For the channel coefficients with exponential power delay profile, we consider

$$\begin{aligned} \bar{f}_{\mathrm{p},l,i} &= \frac{\mathrm{e}^{\mathrm{j}\theta_{f,l,i}}}{\sqrt{1+\alpha_f}}\mathrm{e}^{-lT_{\mathrm{s}}/\sigma_t}, \quad \mathrm{var}(\tilde{f}_{\mathrm{p},l,i}) = \frac{\alpha_f}{1+\alpha_f}\mathrm{e}^{-lT_{\mathrm{s}}/\sigma_t}, \\ \bar{g}_{\mathrm{p},l,i} &= \frac{\mathrm{e}^{\mathrm{j}\theta_{g,l,i}}}{\sqrt{1+\alpha_g}}\mathrm{e}^{-lT_{\mathrm{s}}/\sigma_t}, \quad \mathrm{var}(\tilde{g}_{\mathrm{p},l,i}) = \frac{\alpha_g}{1+\alpha_g}\mathrm{e}^{-lT_{\mathrm{s}}/\sigma_t}, \end{aligned} \tag{9}$$

where $\theta_{f,l,i}$ and $\theta_{g,l,i}$ are uniform RVs randomly chosen from $[0, 2\pi]$ and α_f, α_g are the parameters that denote the uncertainty in channel gains. Next, we consider two performance metrics: the SINR and the transmit power, which are often used to characterise the reliability and transmission efficiency of a communication system, respectively.

Using (2) and a similar approach as in [13], the transmit power of the ith relay, P_i, can be derived as

$$P_i = \boldsymbol{\omega}^{\mathrm{H}}(\boldsymbol{D}_i + \boldsymbol{D}_i')\boldsymbol{\omega}, \tag{10}$$

where

$$\begin{aligned} \boldsymbol{D}_i &= P_{\mathrm{P}}(\boldsymbol{I}_{L_w} \otimes \boldsymbol{A}_i)\mathbb{E}(\boldsymbol{\Xi}_{\mathrm{p}}\boldsymbol{\Xi}_{\mathrm{p}}^{\mathrm{H}})(\boldsymbol{I}_{L_w} \otimes \boldsymbol{A}_i)^{\mathrm{H}} + \sigma_v^2(\boldsymbol{I}_{L_w} \otimes \boldsymbol{A}_i)(\boldsymbol{I}_{L_w} \otimes \boldsymbol{A}_i)^{\mathrm{H}}, \\ \boldsymbol{D}_i' &= P_{\mathrm{S}}(\boldsymbol{I}_{L_w} \otimes \boldsymbol{A}_i)\boldsymbol{\Xi}_{\mathrm{s}}\boldsymbol{\Xi}_{\mathrm{s}}^{\mathrm{H}}(\boldsymbol{I}_{L_w} \otimes \boldsymbol{A}_i)^{\mathrm{H}}, \\ \boldsymbol{A}_i &= \mathrm{diag}\{\boldsymbol{a}_i\}, \end{aligned} \tag{11}$$

and $\boldsymbol{a}_i$ is also the ith column in the identity matrix. Then, the total transmit power of the relays is equal to

$$P_{\mathrm{R}} = \sum_{i=0}^{R} P_i = \boldsymbol{\omega}^{\mathrm{H}}(\boldsymbol{D} + \boldsymbol{D}')\boldsymbol{\omega}, \tag{12}$$

where $\boldsymbol{D} = \sum_{i=1}^{R}\boldsymbol{D}_i$ and $\boldsymbol{D}' = \sum_{i=0}^{R}\boldsymbol{D}_i'$.

Using (7) and denoting $P_{\mathrm{D}}^{(\mathrm{S})}$, $P_{\mathrm{I}}^{(\mathrm{S})}$, $P_{\mathrm{n}}^{(\mathrm{S})}$ as the desired signal power, the received interference and noise powers at the SU-Rx, respectively

$$P_{\mathrm{D}}^{(\mathrm{S})} = P_{\mathrm{S}}\boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{s}}\bar{\boldsymbol{f}}_{\mathrm{s,s}}\bar{\boldsymbol{f}}_{\mathrm{s,s}}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{s}}^{\mathrm{H}}\boldsymbol{\omega} = \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{D}}^{(\mathrm{S})}\boldsymbol{\omega}. \tag{13}$$

Similarly, $P_{\mathrm{I}}^{(\mathrm{S})}$ and $P_{\mathrm{n}}^{(\mathrm{S})}$ can be obtained as

$$P_{\mathrm{I}}^{(\mathrm{S})} = \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{I(S)}}^{(\mathrm{S})}\boldsymbol{\omega} + \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{I(P)}}^{(\mathrm{S})}\boldsymbol{\omega}, \tag{14}$$

$$P_{\mathrm{n}}^{(\mathrm{S})} = \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{n}}^{(\mathrm{S})}\boldsymbol{\omega} + \sigma_{v(\mathrm{S})}^2, \tag{15}$$

where $\boldsymbol{Q}_{\mathrm{I(S)}}^{(\mathrm{S})}$, $\boldsymbol{Q}_{\mathrm{I(P)}}^{(\mathrm{S})}$ and $\boldsymbol{Q}_{\mathrm{n}}^{(\mathrm{S})}$ indicate the interference caused by the PU, the ISI effect and the relays noise effect on the SU-Rx and they can be expressed as

$$\boldsymbol{Q}_{\mathrm{I(S)}}^{(\mathrm{S})} = P_{\mathrm{S}}\boldsymbol{G}_{\mathrm{s}}\bar{\boldsymbol{F}}_{\mathrm{s,s}}\bar{\boldsymbol{F}}_{\mathrm{s,s}}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{s}}^{\mathrm{H}}, \tag{16}$$

$$\boldsymbol{Q}_{\mathrm{I(P)}}^{(\mathrm{S})} = P_{\mathrm{P}}\boldsymbol{G}_{\mathrm{s}}\mathbb{E}(\breve{\boldsymbol{F}}_{\mathrm{p,s}}\breve{\boldsymbol{F}}_{\mathrm{p,s}}^{\mathrm{H}})\boldsymbol{G}_{\mathrm{s}}^{\mathrm{H}}, \tag{17}$$

$$\boldsymbol{Q}_{\mathrm{n}}^{(\mathrm{S})} = \sigma_v^2\boldsymbol{G}_{\mathrm{s}}\tilde{\tilde{\boldsymbol{I}}}\tilde{\tilde{\boldsymbol{I}}}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{s}}^{\mathrm{H}}. \tag{18}$$

Using (16)–(18), SINR at the SU-Rx can be written as

$$\mathrm{SINR}^{(\mathrm{S})} = \frac{\boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{D}}^{(\mathrm{S})}\boldsymbol{\omega}}{\boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{I(S)}}^{(\mathrm{S})}\boldsymbol{\omega} + \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{I(P)}}^{(\mathrm{S})}\boldsymbol{\omega} + \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{n}}^{(\mathrm{S})}\boldsymbol{\omega} + \sigma_{v(\mathrm{S})}^2}. \tag{19}$$

Following the same way for the primary network and using (6), the interference power on the primary receiver is denoted η can be obtained as

$$\eta = \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{I(P)}}^{(\mathrm{P})}\boldsymbol{\omega} + \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{I(S)}}^{(\mathrm{P})}\boldsymbol{\omega} + \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{n}}^{(\mathrm{P})}\boldsymbol{\omega} + \sigma_{v(\mathrm{P})}^2, \tag{20}$$

where $\boldsymbol{Q}_{\mathrm{I(P)}}^{(\mathrm{P})}$, $\boldsymbol{Q}_{\mathrm{I(S)}}^{(\mathrm{P})}$ and $\boldsymbol{Q}_{\mathrm{n}}^{(\mathrm{P})}$ are the effect of the ISI and the interference from SU-Tx on PU-Rx, the effect of the relay noise on PU-Rx, respectively. These parameters can be written as

$$\boldsymbol{Q}_{\mathrm{I(P)}}^{(\mathrm{P})} = P_{\mathrm{P}}\mathbb{E}(\boldsymbol{G}_{\mathrm{p}}\bar{\boldsymbol{F}}_{\mathrm{p,p}}\bar{\boldsymbol{F}}_{\mathrm{p,p}}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{p}}^{\mathrm{H}}), \tag{21}$$

$$\boldsymbol{Q}_{\mathrm{I(S)}}^{(\mathrm{P})} = P_{\mathrm{S}}\mathbb{E}(\boldsymbol{G}_{\mathrm{p}}\breve{\boldsymbol{F}}_{\mathrm{s,p}}\breve{\boldsymbol{F}}_{\mathrm{s,p}}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{p}}^{\mathrm{H}}), \tag{22}$$

$$\boldsymbol{Q}_{\mathrm{n}}^{(\mathrm{P})} = \sigma_v^2\mathbb{E}(\boldsymbol{G}_{\mathrm{p}}\tilde{\boldsymbol{I}}\tilde{\boldsymbol{I}}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{p}}^{\mathrm{H}}). \tag{23}$$

Using (8), (11) resulted to

$$\mathbb{E}(\boldsymbol{\Xi}_{\mathrm{p}}\boldsymbol{\Xi}_{\mathrm{p}}^{\mathrm{H}}) = \mathbb{E}\left(\begin{bmatrix} \boldsymbol{F}_{\mathrm{p},0}\boldsymbol{F}_{\mathrm{p},0}^{\mathrm{H}} & \cdots & \boldsymbol{F}_{\mathrm{p},0}\boldsymbol{F}_{\mathrm{p},L_w-1}^{\mathrm{H}} \\ \vdots & \ddots & \vdots \\ \boldsymbol{F}_{\mathrm{p},L_w-1}\boldsymbol{F}_{\mathrm{p},0}^{\mathrm{H}} & \cdots & \boldsymbol{F}_{\mathrm{p},L_w-1}\boldsymbol{F}_{\mathrm{p},L_w-1}^{\mathrm{H}} \end{bmatrix}\right). \tag{24}$$

Since $\mathbb{E}(\boldsymbol{\Xi}_{\rm p}\boldsymbol{\Xi}_{\rm p}^{\rm H})$ is a Hermitian matrix, then for $i \le j$ holds that

$$\mathbb{E}(\boldsymbol{F}_{{\rm p},i}\boldsymbol{F}_{{\rm p},j}^{\rm H}) = \begin{cases} \mathbb{E}(\boldsymbol{f}_{{\rm p},j-i}\boldsymbol{f}_{{\rm p},0}^{\rm H} + \ldots + \boldsymbol{f}_{{\rm p},L_{{\rm p},f}-1}\boldsymbol{f}_{{\rm p},L_{{\rm p},f}-1-(j-i)}^{\rm H}), & j-i \le L_{{\rm p},f}-1 \\ 0, & j-i > L_{{\rm p},f}-1 \end{cases}$$
$$i, j = 0, \ldots, L_w - 1, \tag{25}$$

where

$$\mathbb{E}(\boldsymbol{f}_{{\rm p},l}\boldsymbol{f}_{{\rm p},l'})_{m,n} = \left(\bar{f}_{{\rm p},l,m}\bar{f}_{{\rm p},l',n}^{*} + \frac{\alpha_f}{1+\alpha_f}{\rm e}^{-lT_{\rm s}/\sigma_t}\delta_{ll'}\delta_{mn}\right). \tag{26}$$
$$m, n = 1, \ldots, R.$$

In (26), $\delta_{ll'}$ is the Kronecker delta (i.e. $\delta_{ll'} = 1$ if $l = l'$ and 0 otherwise).

For the $\boldsymbol{Q}_{\rm I(P)}^{\rm (S)}$ in (17), we have

$$\boldsymbol{Q}_{\rm p,s} = \mathbb{E}(\breve{\boldsymbol{F}}_{\rm p,s}\breve{\boldsymbol{F}}_{\rm p,s}^{\rm H}) = \mathbb{E}\left(\begin{bmatrix} \boldsymbol{\Xi}_{{\rm p,s},0}\boldsymbol{\Xi}_{{\rm p,s},0}^{\rm H} & \cdots & \boldsymbol{\Xi}_{{\rm p,s},0}\boldsymbol{\Xi}_{{\rm p,s},L_{{\rm s},g}-1}^{\rm H} \\ \vdots & \ddots & \vdots \\ \boldsymbol{\Xi}_{{\rm p,s},L_{{\rm s},g}-1}\boldsymbol{\Xi}_{{\rm p,s},0}^{\rm H} & \cdots & \boldsymbol{\Xi}_{{\rm p,s},L_{{\rm s},g}-1}\boldsymbol{\Xi}_{{\rm p,s},L_{{\rm s},g}-1}^{\rm H} \end{bmatrix}\right), \tag{27}$$

where $\boldsymbol{Q}_{\rm p,s}$ is a matrix whose entries are themselves matrices. In addition, the (i, j)th entry of this matrix is a matrix whose elements for $i \le j$ is defined as in (28) (see (28))

Similarly, for $\boldsymbol{Q}_{\rm I(S)}^{\rm (P)}$ in (22)

$$\boldsymbol{Q}_{\rm I(S)}^{\rm (P)} = P_{\rm s}\mathbb{E}(\boldsymbol{G}_{\rm p}\check{\boldsymbol{F}}_{\rm s,p}\check{\boldsymbol{F}}_{\rm s,p}^{\rm H}\boldsymbol{G}_{\rm p}^{\rm H}) = P_{\rm s}\sum_{i=0}^{L_{{\rm p},g}-1}\sum_{j=0}^{L_{{\rm p},g}-1}\mathbb{E}((\boldsymbol{I}_{L_w}\otimes\boldsymbol{G}_{{\rm p},i})\boldsymbol{\Xi}_{{\rm s,p},i}\boldsymbol{\Xi}_{{\rm s,p},j}^{\rm H}(\boldsymbol{I}_{L_w}\otimes\boldsymbol{G}_{{\rm p},j})^{\rm H}),$$

where its entry elements are defined in (29)–(31) (see (29))

where (see (30) and (31))

For $\boldsymbol{Q}_{\rm I(P)}^{\rm (P)}$, we have

$$\mathbb{E}(\boldsymbol{G}_{\rm p}\bar{\boldsymbol{F}}_{\rm p,p}\bar{\boldsymbol{F}}_{\rm p,p}^{\rm H}\boldsymbol{G}_{\rm p}^{\rm H}) = \sum_{i=0, i\ne j}^{L_{{\rm p},g}-1}\sum_{j=0}^{L_{{\rm p},g}-1}\mathcal{M}_{i,j} + \sum_{i=j=1}^{L_{{\rm p},g}-1}\mathcal{M}_{i,j} + \mathbb{E}((\boldsymbol{I}_{L_w}\otimes\boldsymbol{G}_{{\rm p},0})\bar{\boldsymbol{\Xi}}_{{\rm p,p},0}\bar{\boldsymbol{\Xi}}_{{\rm p,p},0}^{\rm H}(\boldsymbol{I}_{L_w}\otimes\boldsymbol{G}_{{\rm p},0})^{\rm H}), \tag{32}$$

where

$$\mathcal{M}_{i,j} = \mathbb{E}((\boldsymbol{I}_{L_w}\otimes\boldsymbol{G}_{{\rm p},i})\boldsymbol{\Xi}_{{\rm p,p},i}\boldsymbol{\Xi}_{{\rm p,p},j}^{\rm H}(\boldsymbol{I}_{L_w}\otimes\boldsymbol{G}_{{\rm p},j})^{\rm H})$$

and $\bar{\boldsymbol{\Xi}}_{{\rm p,p},0}$ denotes a submatrix of $\boldsymbol{\Xi}_{{\rm p,p},0}$ by deleting its first column. Therefore, for $m = n = 0$, holds that

$$(\bar{\boldsymbol{\Xi}}_{{\rm p,p},0}\bar{\boldsymbol{\Xi}}_{{\rm p,p},0}^{\rm H})_{m,n} = \sum_{l=1}^{L_{{\rm p},f}-1}\boldsymbol{f}_l\boldsymbol{f}_l^{\rm H}.$$

Finally, for $i \le j$, $\boldsymbol{Q}_{\rm n}^{\rm (P)}$ is obtained as in (33) and (34) (see equation (33) at the bottom of the next page)

$$\mathbb{E}(\boldsymbol{\Xi}_{{\rm p,s},i}\boldsymbol{\Xi}_{{\rm p,s},j}^{\rm H})_{m,n} = \begin{cases} \mathbb{E}(\boldsymbol{f}_{{\rm p},j-i+(n-m)}\boldsymbol{f}_{{\rm p},0}^{\rm H} + \ldots + \boldsymbol{f}_{{\rm p},L_{{\rm p},f}-1}\boldsymbol{f}_{{\rm p},L_{{\rm p},f}-1-(j-i+(n-m))}^{\rm H}), & 0 \le j-i+(n-m) \le L_{{\rm p},f}-1 \\ \mathbb{E}(\boldsymbol{f}_{{\rm p},0}\boldsymbol{f}_{{\rm p},-(j-i+(n-m))}^{\rm H} + \ldots + \boldsymbol{f}_{{\rm p},L_{{\rm p},f}-1+(j-i+(n-m))}\boldsymbol{f}_{{\rm p},L_{{\rm p},f}-1}^{\rm H}), & -(L_{{\rm p},f}-1) \le j-i+(n-m) \le 0 \end{cases} \tag{28}$$
$$i, j = 0, \ldots, L_{{\rm s},g}-1$$
$$m, n = 0, \ldots, L_w - 1.$$

$$\mathbb{E}((\boldsymbol{I}_{L_w}\otimes\boldsymbol{G}_{{\rm p},i})\boldsymbol{\Xi}_{{\rm s,p},i}\boldsymbol{\Xi}_{{\rm s,p},j}^{\rm H}(\boldsymbol{I}_{L_w}\otimes\boldsymbol{G}_{{\rm p},j})^{\rm H}) = \mathbb{E}\left(\begin{bmatrix} \boldsymbol{G}_{{\rm p},i}(\boldsymbol{\Xi}_{{\rm s,p},i}\boldsymbol{\Xi}_{{\rm s,p},j}^{\rm H})_{0,0}\boldsymbol{G}_{{\rm p},j}^{\rm H} & \cdots & \boldsymbol{G}_{{\rm p},i}(\boldsymbol{\Xi}_{{\rm s,p},i}\boldsymbol{\Xi}_{{\rm s,p},j}^{\rm H})_{0,L_w-1}\boldsymbol{G}_{{\rm p},j}^{\rm H} \\ \vdots & \ddots & \vdots \\ \boldsymbol{G}_{{\rm p},i}(\boldsymbol{\Xi}_{{\rm s,p},i}\boldsymbol{\Xi}_{{\rm s,p},j}^{\rm H})_{L_w-1,0}\boldsymbol{G}_{{\rm p},j}^{\rm H} & \cdots & \boldsymbol{G}_{{\rm p},i}(\boldsymbol{\Xi}_{{\rm s,p},i}\boldsymbol{\Xi}_{{\rm s,p},j}^{\rm H})_{L_w-1,L_w-1}\boldsymbol{G}_{{\rm p},j}^{\rm H} \end{bmatrix}\right), \tag{29}$$

$$(\boldsymbol{\Xi}_{{\rm s,p},i}\boldsymbol{\Xi}_{{\rm s,p},j}^{\rm H})_{m,n} = \begin{cases} \boldsymbol{f}_{{\rm s},j-i+(n-m)}\boldsymbol{f}_{{\rm s},0}^{\rm H} + \ldots + \boldsymbol{f}_{{\rm s},L_{{\rm s},f}-1}\boldsymbol{f}_{{\rm s},L_{{\rm s},f}-1-(j-i+(n-m))}^{\rm H}, & 0 \le j-i+(n-m) \le L_{{\rm s},f}-1 \\ \boldsymbol{f}_{{\rm s},0}\boldsymbol{f}_{{\rm s},-(j-i+(n-m))}^{\rm H} + \ldots + \boldsymbol{f}_{{\rm s},L_{{\rm s},f}-1+(j-i+(n-m))}\boldsymbol{f}_{{\rm s},L_{{\rm s},f}-1}^{\rm H}, & -(L_{{\rm s},f}-1) \le j-i+(n-m) \le 0. \end{cases} \tag{30}$$

$$i, j = 0, \ldots, L_{{\rm p},g}-1$$
$$m, n = 0, \ldots, L_w - 1,$$
$$\mathbb{E}(\boldsymbol{G}_{{\rm p},i}(\boldsymbol{\Xi}_{{\rm s,p},i}\boldsymbol{\Xi}_{{\rm s,p},j}^{\rm H})_{m,n}\boldsymbol{G}_{{\rm p},j}^{\rm H})_{l,l'} = (\bar{g}_{{\rm p},i,l}\bar{g}_{{\rm p},j,l'}^{*}(\boldsymbol{\Xi}_{{\rm s,p},i}\boldsymbol{\Xi}_{{\rm s,p},j}^{\rm H})_{(m,n)l,l'} + \frac{\alpha_g}{1+\alpha_g}(\boldsymbol{\Xi}_{{\rm s,p},i}\boldsymbol{\Xi}_{{\rm s,p},i}^{\rm H})_{(m,n)l,l'}{\rm e}^{-iT_{\rm s}/\sigma_t}\delta_{ij}\delta_{ll'}). \tag{31}$$

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that

$$\mathbb{E}(\boldsymbol{G}_{\mathrm{p},i}\boldsymbol{G}_{\mathrm{p},j}^{\mathrm{H}}) = \mathrm{diag}\left\{\bar{g}_{\mathrm{p},i,1}\bar{g}_{\mathrm{p},j,1}^{*} + \frac{\alpha_g}{1+\alpha_g}\mathrm{e}^{-iT_{\mathrm{s}}/\sigma_t}\delta_{ij},\ \ldots,\ \bar{g}_{\mathrm{p},i,R}\bar{g}_{\mathrm{p},j,R}^{*} + \frac{\alpha_g}{1+\alpha_g}\mathrm{e}^{-iT_{\mathrm{s}}/\sigma_t}\delta_{ij}\right\}. \tag{34}$$

3 Sum relaying power minimisation

This section deals with the design of beamforming coefficients in the relay nodes. The objective is to minimise the total transmit power of all relays, P_{R}, subject to QoS requirements at the secondary network, which is expressed as a minimum SINR of the secondary receiver. In addition, other constraints are on the received interference and noise at the primary receiver, which must be below a given threshold. The resulting optimisation problem can be expressed as

$$\begin{aligned} \min_{\boldsymbol{w}} \quad & P_{\mathrm{R}} \\ \text{s.t.} \quad & \mathrm{SINR}^{(\mathrm{S})} \geq \gamma \\ & \eta \leq \zeta, \end{aligned} \tag{35}$$

where γ, ζ are the minimum of the SINR at the SU-Rx and the maximum tolerable interference and noise for the PU-Rx, respectively.

Using (12)–(19), (35) can be reformulated as

$$\begin{aligned} \min_{\boldsymbol{w}} \quad & \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{D}'\boldsymbol{\omega} + \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{D}\boldsymbol{\omega} \\ \text{s.t.} \quad & \frac{\boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{D}}^{(\mathrm{S})}\boldsymbol{\omega}}{\boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{I(S)}}^{(\mathrm{S})}\boldsymbol{\omega} + \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{I(P)}}^{(\mathrm{S})}\boldsymbol{\omega} + \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{n}}^{(\mathrm{S})}\boldsymbol{\omega} + \sigma_{\nu(\mathrm{S})}^{2}} \geq \gamma \\ & \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{I(P)}}^{(\mathrm{P})}\boldsymbol{\omega} + \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{I(S)}}^{(\mathrm{P})}\boldsymbol{\omega} + \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{n}}^{(\mathrm{P})}\boldsymbol{\omega} + \sigma_{\nu(\mathrm{P})}^{2} \leq \zeta. \end{aligned} \tag{36}$$

The minimisation problem in (36) is non-convex and – to the best of our knowledge – does not have a solution with an acceptable computational complexity. However, a suboptimum solution can be achieved using the semi-definite relaxation method [29]. In order to utilise this method, we define $\boldsymbol{X} \triangleq \boldsymbol{w}\boldsymbol{w}^{\mathrm{H}}$ and rewrite (36) as

$$\begin{aligned} \min_{\boldsymbol{X}} \quad & \mathrm{tr}(\boldsymbol{X}(\boldsymbol{D}+\boldsymbol{D}')) \\ \text{s.t.} \quad & \mathrm{tr}(\boldsymbol{X}(\boldsymbol{Q}_{\mathrm{D}}^{(\mathrm{S})} - \gamma(\boldsymbol{Q}_{\mathrm{I(S)}}^{(\mathrm{S})} + \boldsymbol{Q}_{\mathrm{I(P)}}^{(\mathrm{S})} + \boldsymbol{Q}_{\mathrm{n}}^{(\mathrm{S})}))) \geq \gamma\sigma_{\nu(\mathrm{S})}^{2} \\ & \mathrm{tr}(\boldsymbol{X}(\boldsymbol{Q}_{\mathrm{I(P)}}^{(\mathrm{P})} + \boldsymbol{Q}_{\mathrm{I(S)}}^{(\mathrm{P})}) + \boldsymbol{Q}_{\mathrm{n}}^{(\mathrm{P})})) \leq \zeta - \sigma_{n_2^{(\mathrm{P})}}^{2} \\ & \mathrm{rank}(\boldsymbol{X}) = 1, \boldsymbol{X} \geq 0, \end{aligned} \tag{37}$$

where tr(.) denotes the trace of a matrix. Since $\mathrm{rank}(\boldsymbol{X})$ is not a convex function, the optimisation problem is still non-convex. If the fourth constraint, $\mathrm{rank}(\boldsymbol{X}) = 1$, is omitted, then, it is relaxed to a SDP, which can be solved by using the interior point method. Note that this is a reliable and efficient technique included in well-known software tools such as CVX [30]. Although $\boldsymbol{X}$ is obtained from (37) regardless of the constraint $\mathrm{rank}(\boldsymbol{X}) = 1$, it is proved in [31] that the obtained $\boldsymbol{X}$ is a matrix with rank one.

The computational complexity of this algorithm is mainly from the computation of the SDP. The computational complexity of SDP within tolerance e is $O(m_{\mathrm{sdp}}n_{\mathrm{sdp}}^{3.5} + m_{\mathrm{sdp}}^{2}n_{\mathrm{sdp}}^{2.5} + m_{\mathrm{sdp}}^{3}n_{\mathrm{sdp}}^{0.5})\cdot \log(1/e)$ that n_{sdp} is the dimension of the semi-definite cone and m_{sdp} is the number of linear constraints [20]. Therefore, the computational complexity is obtained by considering $m_{\mathrm{sdp}} = 3$ and $n_{\mathrm{sdp}} = (RL_w)^2$.

4 SU-Rx SINR maximisation

This section aims to design the beamforming weights in the relay nodes with the objective of maximising the SINR at the SU-Rx, subject to a constraint on the transmit power at the relay nodes. In addition, there is another constraint on the received interference and noise power at PU-Rx. The resulting optimisation problem can be expressed as

$$\begin{aligned} \max_{\boldsymbol{w}} \quad & \mathrm{SINR}^{(\mathrm{S})} \\ \text{s.t.} \quad & P_{\mathrm{R}} \leq P_{\mathrm{tmax}} \\ & \eta \leq \zeta, \end{aligned} \tag{38}$$

where P_{tmax} is the maximum of transmit power of the relays.

Using (12)–(19), (38) is reformulated as

$$\begin{aligned} \max_{\boldsymbol{w}} \quad & \frac{\boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{D}}^{(\mathrm{S})}\boldsymbol{\omega}}{\boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{I(S)}}^{(\mathrm{S})}\boldsymbol{\omega} + \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{I(P)}}^{(\mathrm{S})}\boldsymbol{\omega} + \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{n}}^{(\mathrm{S})}\boldsymbol{\omega} + \sigma_{\nu(\mathrm{S})}^{2}} \\ \text{s.t.} \quad & \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{D}'\boldsymbol{\omega} + \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{D}\boldsymbol{\omega} \leq P_{\mathrm{tmax}} \\ & \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{I(P)}}^{(\mathrm{P})}\boldsymbol{\omega} + \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{I(S)}}^{(\mathrm{P})}\boldsymbol{\omega} + \boldsymbol{\omega}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{n}}^{(\mathrm{P})}\boldsymbol{\omega} + \sigma_{\nu(\mathrm{P})}^{2} \leq \zeta. \end{aligned} \tag{39}$$

Using an auxiliary variable, τ, and defining $\boldsymbol{X} = \boldsymbol{\omega}\boldsymbol{\omega}^{\mathrm{H}}$, the optimisation problem is expressed as

$$\begin{aligned} \max_{\boldsymbol{X},\tau} \quad & \tau \\ \text{s.t.} \quad & \mathrm{tr}(\boldsymbol{X}(\boldsymbol{Q}_{\mathrm{D}}^{(\mathrm{S})} - \tau(\boldsymbol{Q}_{\mathrm{I(S)}}^{(\mathrm{S})} + \boldsymbol{Q}_{\mathrm{I(P)}}^{(\mathrm{S})} + \boldsymbol{Q}_{\mathrm{n}}^{(\mathrm{S})}))) \geq \tau c \\ & \mathrm{tr}(\boldsymbol{X}(\boldsymbol{D}+\boldsymbol{D}')) \leq P_{\mathrm{tmax}} \\ & \mathrm{tr}(\boldsymbol{X}(\boldsymbol{Q}_{\mathrm{I(P)}}^{(\mathrm{P})} + \boldsymbol{Q}_{\mathrm{I(S)}}^{(\mathrm{P})} + \boldsymbol{Q}_{\mathrm{n}}^{(\mathrm{P})})) \leq \zeta - \sigma_{\nu(\mathrm{P})}^{2} \\ & \tau \geq 0, \boldsymbol{X} \geq 0, \mathrm{rank}(\boldsymbol{X}) = 1. \end{aligned} \tag{40}$$

In (40), $\mathrm{rank}(\boldsymbol{X}) = 1$ is a non-convex constraint. Hence, by relaxing this constraint, the optimisation problem is transformed to a quasi-convex problem and the feasible set is convex for each value of τ [29]. If we assume τ^* to be the optimum value of τ and solve (40), then if $\tau \leq \tau^*$, the problem is feasible, otherwise, it is infeasible. Using the bisection method to solve this problem we assume an interval $[l, u]$ which contains τ^*. Then, by defining τ as the midpoint of this interval, i.e. $\tau = (l+u)/2$, the optimisation problem can be solved to find $\boldsymbol{X}$. Therefore, a new form of the optimisation problem is

$$\begin{aligned} \text{find} \quad & \boldsymbol{X} \\ \text{s.t.} \quad & \mathrm{tr}(\boldsymbol{X}(\boldsymbol{Q}_{\mathrm{D}}^{(\mathrm{S})} - \tau(\boldsymbol{Q}_{\mathrm{I(S)}}^{(\mathrm{S})} + \boldsymbol{Q}_{\mathrm{I(P)}}^{(\mathrm{S})} + \boldsymbol{Q}_{\mathrm{n}}^{(\mathrm{S})}))) \geq \tau\sigma_{\nu(\mathrm{S})}^{2} \\ & \mathrm{tr}(\boldsymbol{X}(\boldsymbol{D}+\boldsymbol{D}')) \leq P_{\mathrm{tmax}} \\ & \mathrm{tr}(\boldsymbol{X}(\boldsymbol{Q}_{\mathrm{I(P)}}^{(\mathrm{P})} + \boldsymbol{Q}_{\mathrm{I(S)}}^{(\mathrm{P})} + \boldsymbol{Q}_{\mathrm{n}}^{(\mathrm{P})})) \leq \zeta - \sigma_{\nu(\mathrm{P})}^{2} \\ & \boldsymbol{X} \geq 0. \end{aligned} \tag{41}$$

If the constraints in (41) are satisfied for the selected value of τ, i.e.

$$\mathbb{E}(\boldsymbol{G}_{\mathrm{p}}\tilde{\boldsymbol{I}}\tilde{\boldsymbol{I}}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{p}}^{\mathrm{H}})_{i,j} = \begin{cases} \mathbb{E}(\boldsymbol{G}_{\mathrm{p},j-i}\boldsymbol{G}_{\mathrm{p},0}^{\mathrm{H}} + \ldots + \boldsymbol{G}_{\mathrm{p},L_{\mathrm{p},g}-1}\boldsymbol{G}_{\mathrm{p},L_{\mathrm{p},g}-1-(j-i)}^{\mathrm{H}}), & j-i \leq L_{\mathrm{p},g}-1 \\ 0, & j-i \geq L_{\mathrm{p},g}-1 \end{cases} \quad i,j = 0,\ \ldots,\ L_w - 1, \tag{33}$$

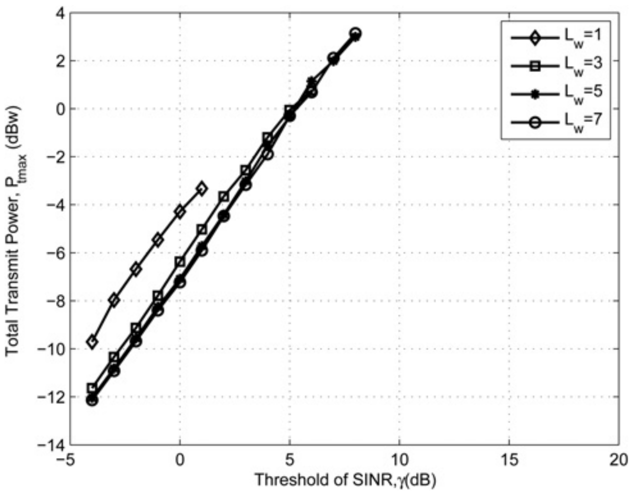

Fig. 2 *Total transmit power of the relays against SINR threshold in secondary receiver for different values of L_w*

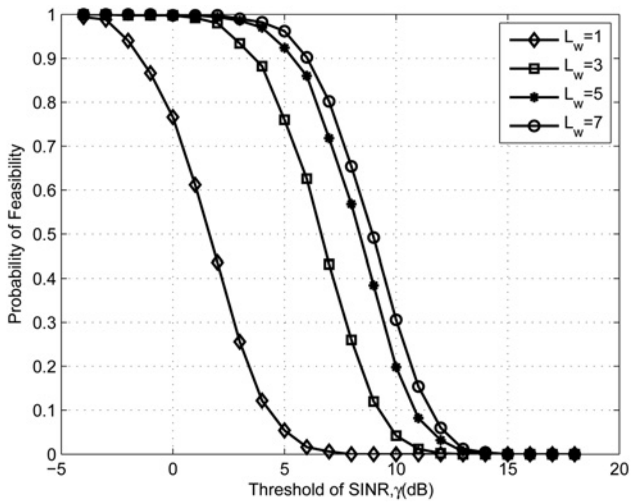

Fig. 3 *Probability of feasibility against SINR threshold in the secondary receiver (γ) for different values of L_w*

the optimisation problem is feasible, we set $l = \tau$. Otherwise, $u = \tau$ and for the new values of l and u, we assume $\tau = (l + u)/2$. These iterations are repeated until the convergence criterion for bisection search, i.e. $u - l \leq \varepsilon$, is fulfilled [29]. The selection of the appropriate values for l and u is critical as it affects the convergence of the algorithm to a global maximum. Since SINR is always positive, at the first iteration, it is assumed $l = 0$. To compute the initial value of u, we solve the objective function in (39) for each of the two constraints, and we find two values for (u_1, u_2) as (see (42))

where $\boldsymbol{A} \triangleq \boldsymbol{D} + \boldsymbol{D}'$ and $\boldsymbol{A}' \triangleq \boldsymbol{Q}_{\mathrm{I(P)}}^{(\mathrm{P})} + \boldsymbol{Q}_{\mathrm{I(S)}}^{(\mathrm{P})} + \boldsymbol{Q}_{\mathrm{n}}^{(\mathrm{P})}$. In order to derive the expression in (42), we use the approach in [32] and select the value $u = \min(u_1, u_2)$.

In each iteration of bisection method, the feasibility check problem is solved such as its computational complexity is $O(4(RL_w)^7 + 16(RL_w)^5 + 64(RL_w)^1)\log(1/e))$. The number of iterations for the bisection search is given by $\log_2(u - l)/\varepsilon$. Therefore the overall complexity is in order of $O(\log_2(u - l)/\varepsilon * (4(RL_w)^7 + 16(RL_w)^5 + 64(RL_w)^1)\log(1/\varepsilon)))$.

5 Simulations and discussion

In this section, we evaluate via simulations the performance of the proposed schemes. We consider two scenarios for the simulations. According to the first one, all channel coefficients are perfectly known. In this case, the channels between transmitter-relay-receiver in Fig. 1 are modelled as complex Gaussian RVs with zero mean and exponential power delay profile [33]

$$p_x(t) = \frac{1}{\sigma_t}\sum_{l=0}^{L_x-1} \mathrm{e}^{-t/\sigma_t}\delta(t - lT_\mathrm{s}), \tag{43}$$

where $L_x \in \{L_{\mathrm{p},f}, L_{\mathrm{s},g}, L_{\mathrm{s},f}, L_{\mathrm{s},g}\}$, $L_{\mathrm{p},f} = L_{\mathrm{s},f} = L_{\mathrm{p},g} = L_{\mathrm{s},g} = 5$, T_s is the symbol length and $\sigma_t = 2T_\mathrm{s}$ represents the delay spread. The noise variances in the relays and receivers are assumed to be 0.1 and the number of relays, $R = 10$. In the second scenario, we assume imperfect blind channel estimation for the primary channel coefficients and investigate the effect of uncertainty in the statistic properties, i.e. α_f and α_g.

Figs. 2 and 3 depict the minimum total transmit power of the relays and the probability of feasibility against minimum required SINR, when the first scenario is assumed with $\zeta = 0$ dBw. All simulation results are averaged over 1000 channel realisations. Since the channel and noise are RVs, the feasibility in the optimisation problem is also a RV. Indeed, it is not easy to always guarantee the desired SINR for the secondary receiver and interference limit for primary receiver when the desired SINR value is high or interference limit is low. Therefore, a problem is called feasible, if it is solvable for more than a half of simulation runs. Otherwise, it is infeasible and the corresponding points are discarded. Based on this definition, the curve of probability of feasibility is obtained. It is evident from these figures that the transmitted power is lower for the FF relaying, compared with AF that is equivalent to one-tap filtering. Moreover, by increasing the SINR threshold in the secondary receiver, the total transmitted power is increased. In addition, from Fig. 3 we conclude that by increasing the filter length, the probability of feasibility increases and a higher SINR threshold is achieved.

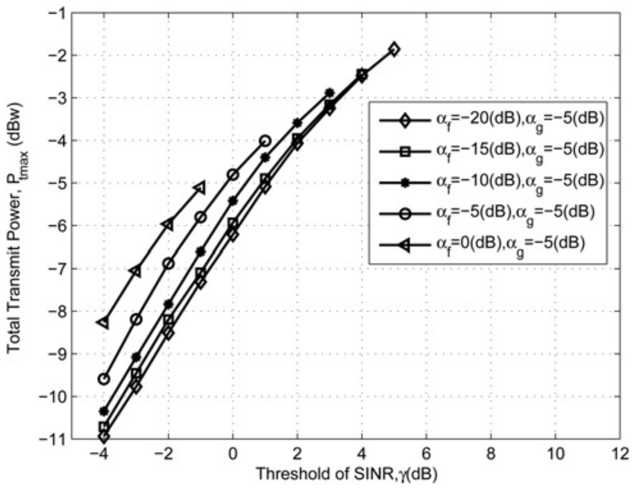

Fig. 4 *Total transmit power of the relays against SINR threshold in secondary receiver for different values of α_f*

$$\begin{aligned} u_1 &= (P_{\mathrm{tmax}})\lambda\{((P_{\mathrm{tmax}})\boldsymbol{A}^{-1/2}(\boldsymbol{Q}_{\mathrm{I(S)}}^{(\mathrm{S})} + \boldsymbol{Q}_{\mathrm{I(P)}}^{(\mathrm{S})} + \boldsymbol{Q}_{\mathrm{n}}^{(\mathrm{S})})\boldsymbol{A}^{-1/2} + \sigma_{\nu(\mathrm{S})}^2\boldsymbol{I})^{-1}\boldsymbol{A}^{-1/2}\boldsymbol{Q}_{\mathrm{D}}^{(\mathrm{S})}\boldsymbol{A}^{-1/2}\}, \\ u_2 &= (\zeta - \sigma_{\nu(\mathrm{P})}^2)\lambda\{((\zeta - \sigma_{\nu(\mathrm{P})}^2)\boldsymbol{A}'^{-1/2}(\boldsymbol{Q}_{\mathrm{I(S)}}^{(\mathrm{S})} + \boldsymbol{Q}_{\mathrm{I(P)}}^{(\mathrm{S})} + \boldsymbol{Q}_{\mathrm{n}}^{(\mathrm{S})})\boldsymbol{A}'^{-1/2} + \sigma_{\nu(\mathrm{S})}^2\boldsymbol{I})^{-1}\boldsymbol{A}'^{-1/2}\boldsymbol{Q}_{\mathrm{D}}^{(\mathrm{S})}\boldsymbol{A}'^{-1/2}\}, \end{aligned} \tag{42}$$

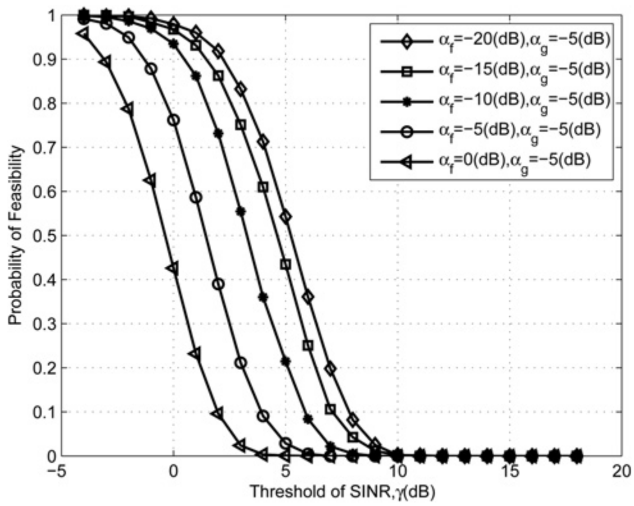

Fig. 5 *Probability of feasibility against SINR threshold in the secondary receiver (γ) for different values of α_f*

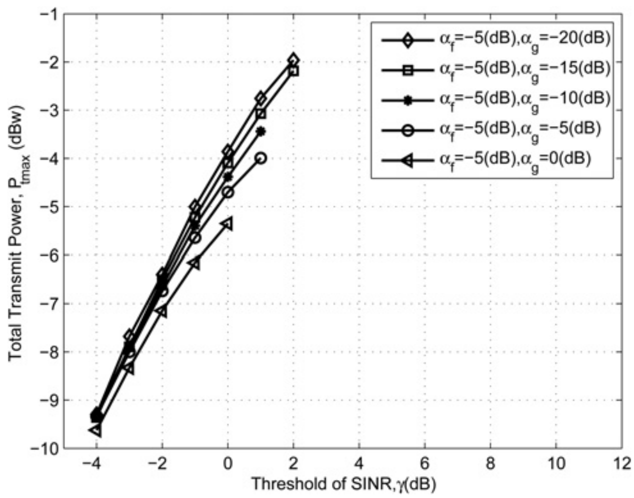

Fig. 6 *Total transmit power of the relays against SINR threshold in secondary receiver for different values of α_g*

Figs. 4–7 present the effect of channel's uncertainty, when the second scenario is assumed. In these figures, filter length is assumed $L_w = 5$. Because, we see in Figs. 2 and 3, when the filter length increases more than 5, the change in power and feasibility is very little. It can be observed that when the CSI error of transmitter-relay link (α_f) increases, the uncertainty in channel coefficients estimation also increases. Therefore, the total transmit power is increased while the probability of feasibility is decreased. The results also show that the total relay power decreases with increasing α_g. This is because the channel uncertainty causes the beamformers to become more conservative and the power reduced to control interference at the primary receiver.

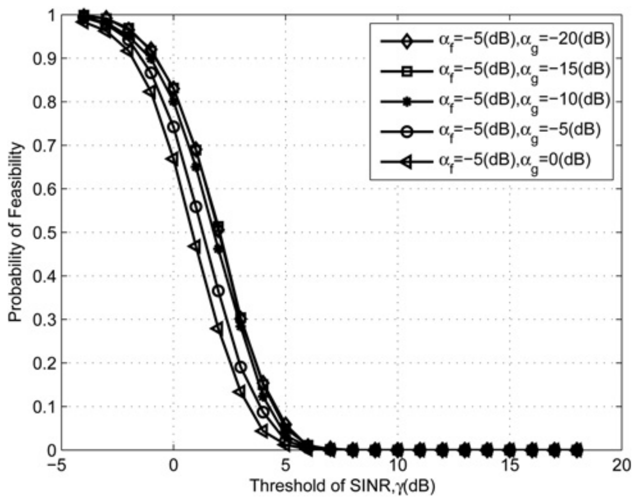

Fig. 7 *Probability of feasibility against SINR threshold in the secondary receiver (γ) for different values of α_g*

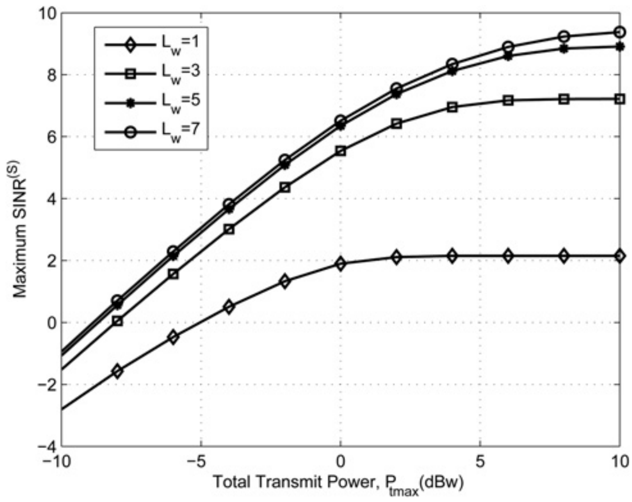

Fig. 8 *Maximum $SINR^{(S)}$ against P_{tmax} for different values of L_w*

Fig. 8 shows SINR in the SU-Rx against the maximum transmit power of the relays for different filter lengths, assuming perfect blind channel estimation. It is observed that by increasing the transmit power limit, SINR in the SU-Rx is also increased. Moreover, it is seen that the SINR for the same the constraint on the relay transmit power monotonically increases with respect to L_w in comparison with the AF, i.e. $L_w = 1$, as expected, and the FF relay achieves most of the gain with only a few FF filter taps.

In Figs. 9 and 10, the maximum SINR in secondary receiver is plotted for different values of α_f and α_g. As it can be seen from these figures, the maximum SINR is decreased as the α_f (or α_g) is increased.

In Fig. 10, we see that before a specified amount for P_{tmax}, the power constraint is more important and since the amount of power is independent of α_g, variation in α_g do not have effect on SINR. When P_{tmax} increases, the amount of interference limit has more effect and with increasing α_g, the maximum SINR is decreased.

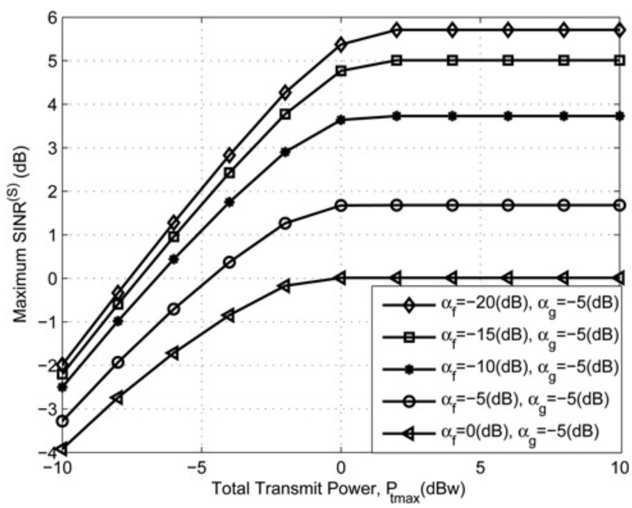

Fig. 9 *Maximum $SINR^{(S)}$ against P_{tmax} for different values of α_f*

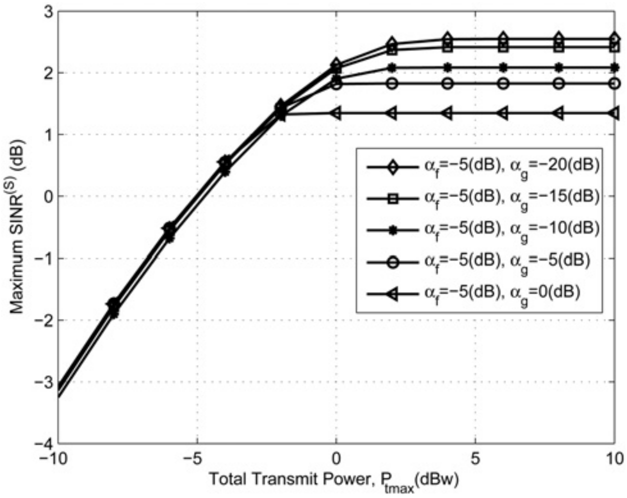

Fig. 10 *Maximum $SINR^{(S)}$ against P_{tmax} for different values of α_g*

6 Conclusions

In this paper, we investigated the design of cooperative beamforming for CR relay networks over frequency selective channels. It was assumed that only the second order statistics with uncertainty between primary transmitter-relays-primary receiver is available. Using FF relaying, we considered two different schemes for the beamforming design. The first scheme tries to minimise the total transmit power of the relays subject to QoS constraints at the receivers. In this approach, beamforming coefficients were obtained by convex optimisation via SDP. The second scheme maximises the SINR in the secondary receiver, subject to a power constraint at the relays and an interference constraint at the primary receiver. We show that in this case the design of the beamforming weights is a quasi-convex optimisation problem. Extensive Monte-Carlo simulations show that the proposed methods which are based on FF relaying technique improved the objectives. Furthermore, by increasing the uncertainty in the channel gains, the different QoS criteria behave in different ways. The proposed beamforming design can be efficiently used in cognitive relaying networks operated over frequency selective fading channels.

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