A Low-Complexity Detector for BPPM Systems Under Additive Gaussian Mixture Noise

Vasileios G. Ataloglou, Georgia D. Ntouni, *Student Member, IEEE*, Vasileios M. Kapinas, *Member, IEEE*, and George K. Karagiannidis, *Fellow, IEEE*

***Abstract*—We analyze the performance of the single-threshold detector (STD) for binary pulse position modulation in the presence of symmetric Gaussian mixture noise (GMN). We first derive a general closed-form expression for the average error probability that is later used to investigate the optimality of the STD with zero threshold, which is the simplest form of STD. We also establish conditions sufficient to guarantee minimum error rates with the low-complexity STD for a wide range of signal-to-noise ratio. The theoretical results are verified through simulations of GMN with various noise parameters.**

***Index Terms*—Binary pulse position modulation (BPPM), single-threshold detector, Gaussian mixture noise.**

I. Introduction

ADDITIVE white Gaussian noise (AWGN) is undoubtedly among the most commonly used channel models, mainly because several independent and identically distributed (i.i.d.) sources of noise converge to a Gaussian distribution due to the central limit theorem. However, in many environments there is not an adequate number of i.i.d. sources to apply this principle. For these cases, Gaussian mixture is used, since any probability density function (pdf) can be sufficiently approximated by a finite sum of Gaussian densities [1]. So far, Gaussian mixture has been used to model various types of noise and interference in multicarrier [2], ultra-wideband [3], power-line [4], and underwater [5] communication systems.

The error performance of communication systems in the presence of Gaussian mixture noise (GMN) is strongly dependent on the selection of the detector. Normally, the maximum-likelihood detector (MLD) is the optimal one for equiprobable constellation symbols. However, the MLD criterion is not equivalent to the well-known minimum Euclidean distance metric as in the case of AWGN, thus leading to significant performance degradation when applied to non-Gaussian noise environments. For this reason, detectors based on the Huber's M-metric, Myriad and Meridian metrics, generalized Cauchy metric, and L_p-norm metric can be employed (see [6], [7] and references therein), which though are either too complex or require parameter optimization in an adaptive sense. In general, the optimal detector that minimizes the average error probability under non-Gaussian noise needs to be very robust, requiring the calculation of multiple threshold values [8].

To this end, a low-complexity single-threshold detector (STD) for binary pulse amplitude modulation systems under additive GMN was proposed in [7]. In this letter, we extend the work in [7] and investigate the STD performance of binary pulse-position modulation (BPPM) systems with additive symmetric GMN. Our primary aim is to set up the problem for the optimal threshold value that minimizes the symbol error rate for any given noise parameters and signal-to-noise ratio (SNR). Based on that, we establish sufficient conditions for the optimality of the zero-threshold STD, which is the simplest form of STD. Although more complicated decision rules can result in better error rate performance, the simplicity of the STD renders it suitable for applications requiring very low-complexity detection, which is in line with the naive nature of the binary communication systems studied here.

Notations: In the sequel, $\mathrm{Re}\{z\}$ and $\mathrm{Im}\{z\}$ denote the real and imaginary parts of a complex z, while $\mu_X = \mathrm{E}[X]$ and $\sigma_X^2 = \mathrm{var}[X] = \mathrm{E}\big[(X-\mu_X)^2\big]$ give the mean and variance of X. Also, $\mathbb{R}_0^+$ and $\mathbb{N}$ refer to the set of non-negative real and natural numbers, respectively, while $Q(x) = (1/\sqrt{2\pi})\int_x^{+\infty}\exp(-t^2/2)\,dt$ is the Gaussian Q-function.

II. System Model

We consider a communications system employing BPPM for the transmission of two symbols, namely $s_1 = \sqrt{\mathcal{E}}$ and $s_2 = j\sqrt{\mathcal{E}}$, where $\mathcal{E}$ represents the average symbol energy. The received signal is represented by, $r = s + n$, where $s \in \{s_1, s_2\}$ and $n = n_x + jn_y$ is a complex random variable (RV) standing for the symmetric GMN added to the transmitted signal. At the receiver, an STD is employed, which compares $r_x = \mathrm{Re}\{r\}$ and $r_y = \mathrm{Im}\{r\}$ with the considered threshold, τ, and decides in favor of s_1 if $r_y - r_x \le \tau$ or in favor of s_2 if $r_y - r_x > \tau$.

Since the two noise coordinates n_x, n_y are Gaussian mixtures, their densities are given by [7] and [5]

$$f_{n_x}(x) = \sum_{i=1}^{N} \frac{w_{x_i}}{\sigma_{x_i}\sqrt{2\pi}} \exp\left(-\frac{(x-\mu_{x_i})^2}{2\sigma_{x_i}^2}\right), \quad (1)$$

$$f_{n_y}(y) = \sum_{j=1}^{M} \frac{w_{y_j}}{\sigma_{y_j}\sqrt{2\pi}} \exp\left(-\frac{(y-\mu_{y_j})^2}{2\sigma_{y_j}^2}\right), \quad (2)$$

where $N, M \in \mathbb{N}$ denote the number of Gaussian components in the mixture noise, while μ_{x_i}, μ_{y_j} and $\sigma_{x_i}^2$, $\sigma_{y_j}^2$, for $i = 1, \ldots, N$ and $j = 1, \ldots, M$, are their corresponding

Manuscript received October 14, 2016; accepted November 15, 2016. Date of publication November 18, 2016; date of current version February 16, 2017. The work of G. D. Ntouni and G. K. Karagiannidis was supported by the "Research Projects for Excellence IKY/Siemens." The work of V. M. Kapinas was supported by the Qatar National Research Fund under Grant NPRP 6-1326-2-532. The associate editor coordinating the review of this paper and approving it for publication was Z. Rezki.

The authors are with the Department of Electrical and Computer Engineering, Aristotle University of Thessaloniki, 54636 Thessaloniki, Greece (e-mail: vatalogg@auth.gr; gntouni@auth.gr; kapinas@auth.gr; geokarag@auth.gr).

Digital Object Identifier 10.1109/LWC.2016.2630693

means and variances, respectively. Also, $w_{x_i}, w_{y_j} \in [0,1]$ are appropriate weights satisfying $\sum_{i=1}^{N} w_{x_i} = \sum_{j=1}^{M} w_{y_j} = 1$.

In our system, we consider that the number of terms N, M and the rest noise parameters are known values. Besides, noise symmetry implies the following simple relations for all i, j

$$\left.\begin{array}{lll} \mu_{x_i} = -\mu_{x_{N+1-i}} & \sigma_{x_i} = \sigma_{x_{N+1-i}} & w_{x_i} = w_{x_{N+1-i}} \\ \mu_{y_j} = -\mu_{y_{M+1-j}} & \sigma_{y_j} = \sigma_{y_{M+1-j}} & w_{y_j} = w_{y_{M+1-j}} \end{array}\right\} \quad (3)$$

which render the mean and variance of the $N-$component Gaussian mixture n_x (for both even and odd N) to be equal to $\mu_x = \sum_{i=1}^{N} w_{x_i}\mu_{x_i} = 0$ and $\sigma_x^2 = \sum_{i=1}^{N} w_{x_i}(\mu_{x_i}^2 + \sigma_{x_i}^2)$, respectively, while the statistics of n_y are similar [9].[1]

III. Performance Analysis

A. Error Probability and Problem Formulation

The calculation of the average symbol error probability (ASEP) in terms of the threshold value τ involves the computation of the pdf of $n_y - n_x$. Given that n_x, n_y are statistically independent RVs, and after mathematical manipulations, it can be proved that the ASEP of the considered system is given by

$$P(\tau; \mathcal{E}, w_{ij}, \mu_{ij}, \sigma_{ij}) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{M} w_{ij} \times \left[Q\left(\frac{\sqrt{\mathcal{E}} + \tau - \mu_{ij}}{\sigma_{ij}}\right) + Q\left(\frac{\sqrt{\mathcal{E}} - \tau + \mu_{ij}}{\sigma_{ij}}\right)\right], \quad (4)$$

where $w_{ij} = w_{x_i}w_{y_j} \in [0,1]$ satisfying $\sum_{i=1}^{N}\sum_{j=1}^{M} w_{ij} = 1$, $\mu_{ij} = \mu_{y_j} - \mu_{x_i}$, and $\sigma_{ij}^2 = \sigma_{x_i}^2 + \sigma_{y_j}^2$, for all pairs (i, j). The average received SNR can be defined as $\gamma = \mathcal{E}/\sigma^2$, where $\sigma^2 = \sum_{i=1}^{N}\sum_{j=1}^{M} w_{ij}(\mu_{ij}^2 + \sigma_{ij}^2)$.

It is easy to show that the symmetry relations are still valid for the new noise parameters, i.e.,[2]

$$\left.\begin{array}{l} \mu_{ij} = -\mu_{N+1-i,M+1-j} \\ \sigma_{ij} = \sigma_{N+1-i,M+1-j} \\ w_{ij} = w_{N+1-i,M+1-j} \end{array}\right\}. \quad (5)$$

The optimal value of the STD is the solution of the problem

$$\tau_{\text{opt}} = \arg\min_{\tau} P(\tau; \mathcal{E}, w_{ij}, \mu_{ij}, \sigma_{ij}). \quad (6)$$

It is useful to note here that, the optimization in (6) can be performed only for non-negative values of τ, since (4) is an even function of τ due to the symmetry of the GMN. Therefore, in the sequel, we consider that $\tau \in \mathbb{R}_0^+$.

B. Optimality Conditions

The optimization problem in (6) is not always convex and τ_{opt} can be obtained through exhaustive search, iterative algorithms, or approximations [7]. It comes out that, the task of finding the *optimal STD* (i.e., the STD with $\tau = \tau_{\text{opt}}$) generally involves high computational complexity. Instead, it would be very useful to investigate the conditions under which the very simple *standard STD* (i.e., the STD with $\tau = 0$) is optimal or not, without the need to solve the intractable problem in (6).

[1] The provided formulas for μ_x and σ_x^2 hold true for both even and odd N.

[2] Note that, in the right-hand side of (5), we have separated the two subscripts of the noise parameters with comma in order to avoid confusion.

Proposition 1 (Optimality Condition of the Standard STD): The standard STD is not the optimal STD if it holds that

$$\sum_{i=1}^{N}\sum_{j=1}^{M} \frac{w_{ij}}{\sigma_{ij}^3}(\sqrt{\mathcal{E}} + \mu_{ij}) \exp\left(-\frac{(\sqrt{\mathcal{E}} + \mu_{ij})^2}{2\sigma_{ij}^2}\right) < 0. \quad (7)$$

Proof: From $\partial P(\tau; \mathcal{E}, w_{ij}, \mu_{ij}, \sigma_{ij})/\partial\tau|_{\tau=0} = 0$, we get

$$\sum_{i=1}^{N}\sum_{j=1}^{M} \frac{w_{ij}}{\sigma_{ij}} \exp\left(-\frac{(\sqrt{\mathcal{E}} - \mu_{ij})^2}{2\sigma_{ij}^2}\right) = \sum_{i=1}^{N}\sum_{j=1}^{M} \frac{w_{ij}}{\sigma_{ij}} \exp\left(-\frac{(\sqrt{\mathcal{E}} + \mu_{ij})^2}{2\sigma_{ij}^2}\right), \quad (8)$$

which is true due to the symmetry relations in (5). As a result, $\tau = 0$ gives either a local maximum or a local minimum. The exact nature of the extrema can be determined by evaluating

$$\partial^2 P(\tau; \mathcal{E}, w_{ij}, \mu_{ij}, \sigma_{ij})/\partial\tau^2\Big|_{\tau=0} = \frac{1}{2\sqrt{2\pi}}\sum_{i=1}^{N}\sum_{j=1}^{M} \frac{w_{ij}}{\sigma_{ij}^3}\left[(\sqrt{\mathcal{E}} - \mu_{ij}) \exp\left(-\frac{(\sqrt{\mathcal{E}} - \mu_{ij})^2}{2\sigma_{ij}^2}\right) + (\sqrt{\mathcal{E}} + \mu_{ij}) \exp\left(-\frac{(\sqrt{\mathcal{E}} + \mu_{ij})^2}{2\sigma_{ij}^2}\right)\right]. \quad (9)$$

Obviously, if (7) is satisfied, then (9) becomes negative, due to the Gaussian mixture symmetry. Hence, $\tau = 0$ gives a local maximum and there exist non-trivial values of $\tau > 0$ such that $P(\tau; \mathcal{E}, w_{ij}, \mu_{ij}, \sigma_{ij}) < P(0; \mathcal{E}, w_{ij}, \mu_{ij}, \sigma_{ij})$. ■

On the contrary, if the left-hand side of (7) is positive, then $\tau = 0$ may give a local minimum, but no safe conclusion about the global optimality of the standard STD can be derived. To this end, a more strict condition is provided by the next proposition.

Proposition 2 (Mean Values Condition): The standard STD is the optimal STD if the average symbol energy and the means of the GMN satisfy the following condition

$$\max_{i=1,\ldots,N} \mu_{x_i} + \max_{j=1,\ldots,M} \mu_{y_j} - \sqrt{\mathcal{E}} \leq 0. \quad (10)$$

Proof: Without loss of generality we assume even N, M. Then, due to the symmetry of the GMN, $\partial P(\tau; \mathcal{E}, w_{ij}, \mu_{ij}, \sigma_{ij})/\partial\tau = 0$, can be written in the form

$$\sum_{i=1}^{N/2}\sum_{j=1}^{M/2} \frac{w_{ij}}{\sigma_{ij}}\left[\exp\left(-\frac{a_1^2}{2\sigma_{ij}^2}\right) + \exp\left(-\frac{a_2^2}{2\sigma_{ij}^2}\right) + \exp\left(-\frac{a_3^2}{2\sigma_{ij}^2}\right) + \exp\left(-\frac{a_4^2}{2\sigma_{ij}^2}\right)\right] - \sum_{i=1}^{N/2}\sum_{j=1}^{M/2} \frac{w_{ij}}{\sigma_{ij}}\left[\exp\left(-\frac{b_1^2}{2\sigma_{ij}^2}\right) + \exp\left(-\frac{b_2^2}{2\sigma_{ij}^2}\right) + \exp\left(-\frac{b_3^2}{2\sigma_{ij}^2}\right) + \exp\left(-\frac{b_4^2}{2\sigma_{ij}^2}\right)\right] = 0, \quad (11)$$

where the auxiliary parameters a_k, b_k, for $k = 1, \ldots, 4$, read

$$a_1 = -\tau + \mu_{y_j} - \mu_{x_i} + \sqrt{\mathcal{E}},\ b_1 = \tau + \mu_{y_j} - \mu_{x_i} + \sqrt{\mathcal{E}},$$
$$a_2 = -\tau + \mu_{y_j} + \mu_{x_i} + \sqrt{\mathcal{E}},\ b_2 = \tau + \mu_{y_j} + \mu_{x_i} + \sqrt{\mathcal{E}},$$

TABLE I
GMN PARAMETERS FOR THE FIRST SCENARIO

(a) n_x parameters $(N = 4)$

i	1	2	3	4
w_{x_i}	0.34	0.16	0.16	0.34
μ_{x_i}	0.12	0.14	-0.14	-0.12
$\sigma^2_{x_i}$	0.10	0.03	0.03	0.10

(b) n_y parameters $(M = 5)$

j	1	2	3	4	5
w_{y_j}	0.380	0.090	0.060	0.090	0.380
μ_{y_j}	0.100	0.080	0.000	-0.080	-0.100
$\sigma^2_{y_j}$	0.100	0.001	0.024	0.001	0.100

TABLE II
GMN PARAMETERS FOR THE SECOND SCENARIO

(a) n_x parameters $(N = 4)$

i	1	2	3	4
w_{x_i}	0.42	0.08	0.08	0.42
μ_{x_i}	0.12	2.00	-2.00	-0.12
$\sigma^2_{x_i}$	0.10	0.06	0.06	0.10

(b) n_y parameters $(M = 5)$

j	1	2	3	4	5
w_{y_j}	0.400	0.060	0.080	0.060	0.400
μ_{y_j}	0.100	2.100	0.000	-2.100	-0.100
$\sigma^2_{y_j}$	0.100	0.080	0.024	0.080	0.100

$$a_3 = -\tau - \mu_{y_j} - \mu_{x_i} + \sqrt{\mathcal{E}},\ \ b_3 = \tau - \mu_{y_j} - \mu_{x_i} + \sqrt{\mathcal{E}},$$
$$a_4 = -\tau - \mu_{y_j} + \mu_{x_i} + \sqrt{\mathcal{E}},\ \ b_4 = \tau - \mu_{y_j} + \mu_{x_i} + \sqrt{\mathcal{E}}.$$

If (10) is satisfied, then $a_k + \tau = b_k - \tau \geq 0$, for all k. Then, for $\tau > 0$, it holds that $|b_k| > |a_k|$, $k = 1, \ldots, 4$, which renders the expression on the left-hand side of (11) positive, since $x \mapsto \exp(-x^2)$ is strictly decreasing on $[0, \infty)$. Similarly, for $\tau < 0$, this expression becomes negative. Thus, the equality in (11) holds only for $\tau = 0$, which renders it the unique minimum, and hence the proposition is proved. ■

Corollary 1 (Average SNR Condition): The standard STD is the optimal STD if the average SNR satisfies the inequality

$$\gamma \geq \frac{\max_{i=1,\ldots,N, j=1,\ldots,M} \mu^2_{ij}}{\sum_{i=1}^{N} \sum_{j=1}^{M} w_{ij}\left(\mu^2_{ij} + \sigma^2_{ij}\right)}. \quad (12)$$

Proof: The proof follows directly from Proposition 2, the definition of the SNR, and the symmetry relations in (5). ■

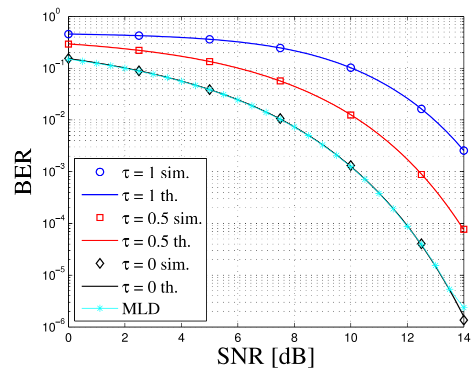
The practical importance of Proposition 2 and Corollary 1 is evident in the case of impulsive noise, where $\mu_{x_i} = \mu_{y_j} = 0$, for all i, j. In view of (10) and (12), the standard STD becomes the optimal STD and the probability of error can be computed via the simplified form

$$P\big(0; \mathcal{E}, w_{ij}, 0, \sigma_{ij}\big) = \sum_{i=1}^{N} \sum_{j=1}^{M} w_{ij} Q\left(\frac{\sqrt{\mathcal{E}}}{\sigma_{ij}}\right). \quad (13)$$

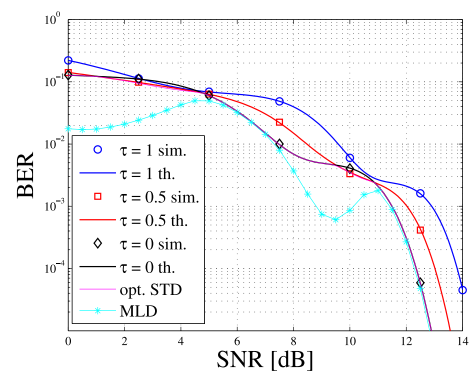
Proposition 3 (Range of Optimal Threshold τ_{opt}): The optimal threshold value, τ_{opt}, that minimizes $P(\tau; \mathcal{E}, w_{ij}, \mu_{ij}, \sigma_{ij})$ always lies within the interval $[0, \max_{i=1,\ldots,N, j=1,\ldots,M} \mu_{ij}]$.

Proof: In the case where $\tau > \mu_{ij}$ for all pairs (i, j), it is $(\tau - \mu_{ij} + \sqrt{\mathcal{E}})^2 - (-\tau + \mu_{ij} + \sqrt{\mathcal{E}})^2 = 4\sqrt{\mathcal{E}}(\tau - \mu_{ij}) > 0$, which implies that $\partial P(\tau; \mathcal{E}, w_{ij}, \mu_{ij}, \sigma_{ij})/\partial \tau$ is always positive or equivalently that $\tau \mapsto P(\tau; \mathcal{E}, w_{ij}, \mu_{ij}, \sigma_{ij})$ is strictly increasing. Hence, the maximum value of μ_{ij} over all pairs sets an upper bound in the interval where τ_{opt} belongs. ■

We highlight here that, Proposition 1 provides only a sufficient but not necessary condition for the standard STD to be not the optimal STD. This implies that if, for a given value of $\mathcal{E}$, (7) is not satisfied, then the optimization in (6) still needs to be performed for possible non-trivial values of τ_{opt}. Also, from Proposition 2, it can be inferred that $\tau = 0$ is always the optimal threshold for a sufficiently high value of $\mathcal{E}$ and beyond. However, τ_{opt} may vary for values of $\mathcal{E}$ that do not satisfy (10). For these values of $\mathcal{E}$, Proposition 3 defines the search space for (6).



(a) GMN model of Table I.



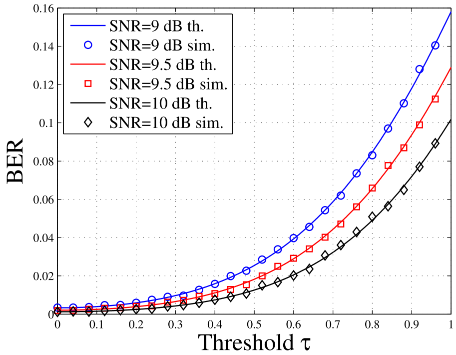
(b) GMN model of Table II.

Fig. 1. BER with respect to SNR for $\tau = 0$, 0.5, and 1.

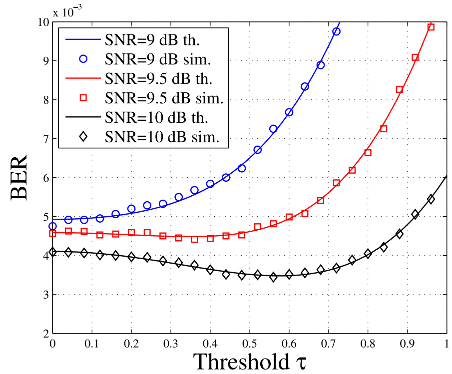
IV. SIMULATION RESULTS AND DISCUSSION

In this section, the validity of the theoretical results derived in Section III is confirmed by Monte Carlo simulations. Two different GMN models are considered, whose parameters are shown in Tables I and II. Their main difference is that the variation in the means of the 2nd model is much higher.

In Fig. 1, the bit error rate (BER) is plotted with respect to the SNR for $\tau = 0$, 0.5, and 1, while the optimal detector, namely the MLD, is also included for comparison. In all cases, the accuracy of the theoretical expression in (4) is obvious. For the 1st GMN model in Fig. 1a, the standard STD is the optimal STD and almost identical to the optimal detector in general for the whole SNR regime. However, this is not true for the 2nd GMN model in Fig. 1b. Particularly, in the second case, the three curves intersect each other, which implies that τ_{opt} generally depends on the SNR. To make it more clear, the



(a) Noise model of Table I.



(b) Noise model of Table II.

Fig. 2. BER with respect to τ for $\gamma = 9$, 9.5, and 10 dB.

BER curve for the optimal STD is also depicted. Interestingly, the standard STD is the optimal STD over most of the SNR region apart from a few SNR values (around 2 and 10 dB) for which $\tau = 0.5$ gives lower BER than $\tau = 0$. In the same way, there are a few SNR values (around 3.5 dB) for which $\tau = 1$ gives again lower BER than $\tau = 0$. Regarding the performance of the standard STD with respect to the MLD, we observe its general optimality within the interval [5, 7] dB and for SNR values larger than 11 dB.

In Fig. 2, the BER is plotted with respect to τ for $\gamma = 9$, 9.5, and 10 dB. Once again, we observe that, the standard STD is the optimal STD for the 1st GMN model, as shown in Fig. 2a, since $\tau = 0$ gives the lowest BER for all SNR values. In Fig. 2b, it is clearly illustrated that, for SNR values a bit over 9 dB, $\tau = 0$ is not optimal and, therefore, τ_{opt} should be searched elsewhere. Indicatively, for $\gamma = 10$ dB, the optimal threshold value is $\tau_{\text{opt}} \simeq 0.57$, which provides an improvement of 15.3% over the standard STD.

The above findings can be interpreted in a more intuitive way with the aid of the propositions established in Section III-B. Firstly, from Table I, we compute the critical parameters for the 1st GMN model, namely $\max_{i=1,\dots,N} \mu_{x_i} = 0.14$, $\max_{j=1,\dots,M} \mu_{y_j} = 0.10$, and $\sigma^2 = 0.157$. Then, we see that 0.24 is smaller than 0.396, which is the smallest value of $\sqrt{\mathcal{E}}$, corresponding to $\gamma = 0$ dB. Therefore, Proposition 2 is always satisfied, which implies that the standard STD is the optimal STD. Concerning now the 2nd GMN model, from Table II, we compute $\max_{i=1,\dots,N} \mu_{x_i} = 2.0$, $\max_{j=1,\dots,M} \mu_{y_j} = 2.1$, and $\sigma^2 = 1.374$. Then, it is easy to see that (10) and (12) are satisfied for $\mathcal{E} \geq 16.81$ and equivalently $\gamma \geq 10.88$ dB, respectively. Hence, for these SNR values, Corollary 1 is fulfilled, validating the results depicted

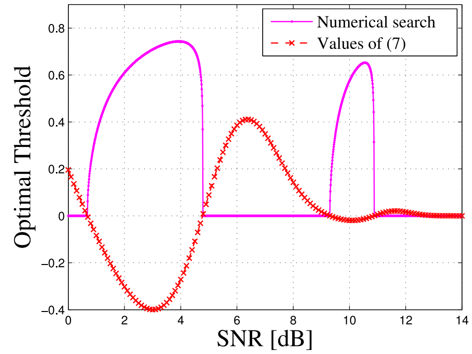


Fig. 3. Optimal threshold values of STD with respect to SNR and optimality condition of the standard STD according to Proposition 1 for the noise model of Table II.

in Fig. 1b, which clearly show that the standard STD is the optimal STD in this region. However, for $\gamma < 10.88$, the optimal threshold values shall be searched within the interval $[0, 4.1]$, as suggested by Proposition 3. After numerical search, we can find that $\tau_{\text{opt}} \neq 0$ for two intervals only, namely $[0.69, 4.77]$ dB and $[9.31, 10.87]$ dB, which can be verified by Proposition 1. This is clearly illustrated in Fig. 3, where it is shown that the non-trivial threshold values correspond almost one-to-one to the negative values of (7). This also explains why the red and black curves in Fig. 2b, associated with $\gamma = 9.5$ dB and $\gamma = 10$ dB, respectively, do not have a minimum at $\tau = 0$.

V. Conclusion

In this letter, we derived a closed-form expression for the ASEP of BPPM in the presence of symmetric GMN considering an STD at the receiver. The proper selection of the threshold value has been proved to be a key parameter in the performance of the system. We have shown that, the very low-complexity standard STD is optimal for a wide range of SNR.

References

[1] K. N. Plataniotis and D. Hatzinakos, "Gaussian mixtures and their applications to signal processing," in *Advanced Signal Processing Handbook*, S. Stergiopoulos, Ed. Boca Raton, FL, USA: CRC Press, 2001, ch. 3.

[2] Y. Gong, Z. Ding, and C. F. N. Cowan, "MMSE turbo equalizer for channels with cochannel interference," in *Proc. IEEE ICASSP*, Toulouse, France, 2006, pp. 1–4.

[3] T. Erseghe, V. Cellini, and G. Dona, "On UWB impulse radio receivers derived by modeling MAI as a Gaussian mixture process," *IEEE Trans. Wireless Commun.*, vol. 7, no. 6, pp. 2388–2396, Jun. 2008.

[4] J. Lin, M. Nassar, and B. L. Evans, "Impulsive noise mitigation in powerline communications using sparse Bayesian learning," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 7, pp. 1172–1183, Jul. 2013.

[5] S. Banerjee and M. Agrawal, "On the performance of underwater communication system in noise with Gaussian mixture statistics," in *Proc. NCC*, Kanpur, India, Mar. 2014, pp. 1–6.

[6] A. Nasri and R. Schober, "Adaptive L_p-norm metric for secondary BICM-OFDM systems," in *Proc. IEEE GLOBECOM*, Honolulu, HI, USA, Nov./Dec. 2009, pp. 1–6.

[7] S. Bayram and S. Gezici, "On the performance of single-threshold detectors for binary communications in the presence of Gaussian mixture noise," *IEEE Trans. Commun.*, vol. 58, no. 11, pp. 3047–3053, Nov. 2010.

[8] J. Mitra and L. Lampe, "Design and analysis of robust detectors for TH IR-UWB systems with multiuser interference," *IEEE Trans. Commun.*, vol. 57, no. 8, pp. 2210–2214, Aug. 2009.

[9] L. Trailovic and L. Y. Pao, "Variance estimation and ranking of Gaussian mixture distributions in target tracking applications," in *Proc. IEEE CDC*, Las Vegas, NV, USA, Dec. 2002, pp. 2195–2201.