Maximizing Proportional Fairness in Wireless Powered Communications

Panagiotis D. Diamantoulakis, *Student Member, IEEE*, and George K. Karagiannidis, *Fellow, IEEE*

***Abstract*—This letter investigates the fundamental tradeoff between sum rate and fairness of a wireless-powered uplink communication system, consisting of one base station (BS) and multiple energy harvesting users, which are coordinated through the *harvest-then-transmit* protocol. To this end, the optimal rate and time allocation, when aiming to maximize the proportional fairness, is investigated. Two well known communication protocols are considered, namely, time division multiple access (TDMA) and nonorthogonal multiple access with time-sharing (NOMA-TS). It is shown that NOMA-TS outperforms the considered benchmark scheme, which is the NOMA with fixed decoding order and adaptive power allocation, while TDMA proves to be an appropriate choice, when all the users are located in similar distances from the BS.**

***Index Terms*—Energy harvesting, fairness, NOMA, resource allocation, SWIPT.**

I. INTRODUCTION

ENERGY harvesting is a promising solution for the energy sustainability of wireless communication networks [1], [2]. However, the main disadvantage of traditional energy harvesting methods is that they rely on natural resources, such as solar and wind energy, which are uncontrollable. For this reason, harvesting energy from radio frequency signals, which also transfer information, seems to be an interesting alternative. This technique, commonly known as simultaneous information and wireless power transfer (SWIPT), has been studied assuming various scenarios, such as orthogonal frequency division multiple access (OFDMA) [3], multiple-input multiple-output (MIMO) systems [4]–[7], and cooperative networks [8]–[11].

Among the proposed applications of SWIPT, wireless-powered uplink communication, is possibly the most interesting, since it enables distributed users to communicate with a base station (BS), even when they have no other energy sources. This application, initially proposed in [12], is based on the *harvest-then-transmit* protocol [5], [12], [13], according to which the users first harvest energy, and then they transmit their independent messages to the BS, by using the harvested energy. In [12] and [13], different communication protocols are considered, i.e., time division multiple access (TDMA) and non-orthogonal multiple access (NOMA), respectively, while either the sum or the equal rate is maximized, by adapting the time allocated to energy harvesting. However, when the sum rate is maximized, fairness is considerably reduced, due to the *double near-far* problem, which appears when some users far from the BS receive a smaller amount of wireless energy than the nearer users, while they need to transmit with more power. On the other hand, equal rate maximization considerably reduces the sum-rate, which, in case of NOMA, also happens because the achievable capacity of some users might exceed the maximum equal rate [13].

In order to balance the trade-off between performance and fairness, an alternative metric, such as the proportional fairness, can be used and maximized. This metric is widely applied in wireless networks in order to balance user fairness and network sum-rate. This scenario has been partially studied in [14], under the assumption of NOMA with fixed decoding order and adaptive power allocation, which is considered as benchmark in the present letter. The fixed decoding order implies that the BS decodes the users messages during the successive interference cancellation (SIC) process with a constant order. This assumption reduces the degrees of freedom and degrades the system's performance, since some users always experience more interference and their messages are always decoded last. Moreover, NOMA with adaptive power allocation induces higher complexity than NOMA with fixed power allocation and especially TDMA.

In this work, we aim to maximize the proportional fairness by proper time and rates allocation, considering two communication protocols, i.e., TDMA and NOMA with time-sharing (NOMA-TS), when asymmetrical (unequal) rates are allowed. Note that NOMA-TS is a generalization of NOMA with fixed decoding order, so that a user, whose message suffers from strong interference for a specific decoding order, can experience a better reception reliability for another decoding order, during the implementation of SIC. The corresponding optimization problems are efficiently solved in closed form, by using convex optimization, and more specifically *Lagrange dual decomposition*. The evaluation of the proposed strategies through extensive simulations reveals that NOMA-TS maximizes proportional fairness and outperforms the benchmark, while TDMA proves to be an appropriate choice, when all users are located in similar distances from the BS.

II. SYSTEM MODEL

We consider the uplink of a wireless network, consisting of N energy harvesting users and one BS. We assume that all nodes are equipped with a single antenna, while they share the same frequency band. The communication is divided into time frames of unitary duration. Let, $\mathcal{N} = \{1, \ldots, N\}$, denote the set of users. Assuming channel reciprocity, the channel power gain from BS to user $n \in \mathcal{N}$ and the corresponding reversed is denoted by h_n. Also, the harvest-then-transmit protocol is employed, according to which there are the following two distinct phases during a time frame.

Phase 1: The BS transmits power, denoted by P, which is used by the users in order to charge their batteries. The duration of this phase is denoted by $0 \leq T \leq 1$. Assuming equal energy harvesting efficiency at each receiver, $0 \leq \eta \leq 1$, and that the harvested energy due to receiver noise is negligible compared with the sufficiently large P, the amount of harvested energy is given by [12]–[14]

$$E_n = \eta P h_n T. \tag{1}$$

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The authors are with the Department of Electrical and Computer Engineering, Aristotle University of Thessaloniki, 54124 Thessaloniki, Greece (e-mail: padiaman@auth.gr; geokarag@auth.gr).

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Phase 2: The remaining amount of time, i.e., $1 - T$ is assigned to the users, in order to transmit their messages. As mentioned above, two different communication protocols are considered, namely TDMA and NOMA-TS. The achievable rates that correspond to each of the considered protocols are described in the following subsections.

A. Capacity Region of TDMA

We assume that the users transmit their independent information to the BS by using TDMA, so that there is no interference among users. In this case,

$$\sum_{n=1}^{N} t_n \leq 1 - T, \tag{2}$$

where $t_n \geq 0$ denotes the amount of time that is allocated to each user. The average transmit power by each user is, $P_n = \frac{E_n}{t_n}$. Moreover, the capacity region is bounded by [12]

$$R_n \leq t_n \log_2\left(1 + \frac{P_n h_n}{N_0}\right), \tag{3}$$

and N_0 is the additive white gaussian noise (AWGN) power. The constraint in (3) can be rewritten as

$$R_n \leq t_n \log_2\left(1 + \frac{a_n T}{t_n}\right), \tag{4}$$

where, $a_n = \frac{\eta h_n^2 P}{N_0}$.

B. Capacity Region of NOMA-TS

When NOMA is utilized, all users can simultaneously use the whole duration of phase 2 in order to transmit their messages. Thus, the average transmit power by each user is, $P_n = \frac{E_n}{1-T}$.

In order to decode the users' messages SIC is employed at the BS. Therefore, for decoding the first message, interference is created due to all other messages, while on the second message, interference is created due to all other messages except the first and the second one, and so on. Without loss of generality, the users' indices are assigned in a way that the values $h_n \forall N$ are sorted in ascending order, i.e., $h_1 \leq h_2 \leq \cdots \leq h_N$. In order to increase fairness, time-sharing can be used, the basic principle of which is that the order of decoding for the users can change for specific fractions of the duration $1 - T$.

When NOMA-TS is used, the capacity region is bounded by [13]

$$\sum_{n=1}^{m} R_n \leq (1-T)\log_2\left(1 + \frac{\sum_{n=1}^{m} P_n h_n}{N_0}\right), \quad \forall m = 1, \ldots, N. \tag{5}$$

Eq. (5) can be rewritten as

$$\sum_{n=1}^{m} R_n \leq (1-T)\log_2\left(1 + \frac{g_m T}{1-T}\right), \quad \forall m = 1, \ldots, N, \tag{6}$$

where, $g_m = \frac{\eta P \sum_{n=1}^{m} h_n^2}{N_0}$.

III. Problem Formulation and Solution

The aim of the proposed analysis is to maximize the system's performance while achieving a balance between the sum-rate and fairness. To this end, the *proportional fairness* is used, which is defined as the sum of logarithms of the individual rates. Next, the proportional fairness maximization problem is defined and solved, for TDMA and NOMA-TS.

A. TDMA

When TDMA is used the proportional fairness maximization problem can be expressed as

$$\begin{aligned} \max_{R_n, t_n, \forall n \in \mathcal{N}, T} \quad & \sum_{n=1}^{N} \ln(R_n) \\ \textbf{s.t.} \quad & \mathrm{C}_1 : \text{Eq. (2)}, \mathrm{C}_2 : \text{Eq. (4)}, \mathrm{C}_3 : t_n, T \geq 0. \end{aligned} \tag{7}$$

The optimization problem in (7) is a concave one, which can be solved by standard numerical methods such as interior point or bisection method. However, we use the Lagrange dual decomposition, which proves to be extremely efficient, since, given the Lagrange multipliers (LMs), closed form expressions for $R_n, t_n, \forall n \in \mathcal{N}, T$ are derived. Thus, it is guaranteed that the optimal solution can be obtained in polynomial time [15].

Theorem 1: Considering the optimization problem in (7), let $\mu \geq 0$ and $\lambda_n \geq 0$ denote the LMs, which corresponds to the constraints C_1 and C_2, respectively. Then, the optimal rate allocation policy is given by

$$R_n^* = \frac{1}{\lambda_n}. \tag{8}$$

The optimal t_n is given by

$$t_n^* = \frac{a_n T^*}{A_n^* - 1}, \tag{9}$$

where

$$A_n^* = -\frac{1}{W\left(-\exp\left(-\frac{1+\mu \ln(2)}{\lambda_n}\right)\right)}, \tag{10}$$

with $W(\cdot)$ being the principal branch of the Lambert W function [2], [16] and μ can be found after solving (11)

$$\sum_{n=1}^{N} \frac{a_n \lambda_n}{A_n^* \ln(2)} - \mu = 0. \tag{11}$$

Finally, the optimal time allocation between the two phases is

$$T^* = \frac{1}{\sum_{n=1}^{N} \frac{a_n}{A_n^* - 1} + 1}. \tag{12}$$

Proof: Since (7) is a concave optimization problem, it can be optimally solved by *Lagrange dual decomposition* [15]. To this end, the Lagrangian is needed, which is given by

$$\begin{aligned} \mathcal{L} = & \sum_{n=1}^{N} \ln(R_n) - \mu\left(\sum_{n=1}^{N} t_n + T - 1\right) \\ & - \lambda_n\left(R_n - t_n \log_2\left(1 + \frac{a_n T}{t_n}\right)\right). \end{aligned} \tag{13}$$

According to the Karush-Kuhn-Tucker (KKT) conditions, it must hold that $\frac{\partial \mathcal{L}}{\partial R_n} = 0$, $\frac{\partial \mathcal{L}}{\partial T} = 0$, and $\frac{\partial \mathcal{L}}{\partial t_n} = 0, \forall n \in \mathcal{N}$, which yield (8), (11), and

$$\frac{(A_n - 1)\lambda_n}{A_n} + \mu \ln(2) - \lambda_n \ln(A_n) = 0, \tag{14}$$

respectively, where $A_n = \frac{a_n T}{t_n} + 1$. Using (14) and after some manipulations we derive (9). Finally, it is easy to prove that the constraint C_1 should hold with equality, thus the optimal time allocation between the two phases is given by (12). Note that in (8) and (10), λ_n is calculated iteratively by using the subgradient method. ■

B. NOMA-TS

When NOMA-TS is used the proportional fairness maximization problem can be expressed as

$$\max_{R_n \forall n \in \mathcal{N}, T} \sum_{n=1}^{N} \ln(R_n)$$
$$\textbf{s.t.} \ \text{Eq. (6)}, 0 \leq T \leq 1. \tag{15}$$

Regarding (15), it is hard to directly solve for T, since it appears in all the capacity equations in (6). Thus, in order to avoid the utilization of Newton-Raphson method [13], and decrease the corresponding complexity, we introduce the auxiliary variables T_m. Thus, (15) can be reformulated as

$$\begin{aligned} &\max_{R_n \forall n \in \mathcal{N}, T_m} \sum_{n=1}^{N} \ln(R_n) \\ &\textbf{s.t.} \ C_1: \sum_{n=1}^{m} R_n \leq (1 - T_m) \log_2\left(1 + \frac{g_m T_m}{1 - T_m}\right), \\ &\qquad\qquad\qquad\qquad\qquad\qquad \forall m = 1, \ldots, N, \\ &\qquad C_2: T_m = T_N, \qquad\qquad\qquad \forall m = 1, \ldots, N-1, \\ &\qquad C_3: 0 \leq T_m \leq 1, \qquad\qquad\quad \forall m = 1, \ldots, N. \end{aligned} \tag{16}$$

Theorem 2: Let $\lambda_m \geq 0$ and μ_m denote LMs, which correspond to the constraints C_1 and C_2, respectively. Then, the optimal rate allocation policy is given by

$$R_n^* = \frac{1}{\sum_{m=n}^{N} \lambda_m}, \tag{17}$$

and the optimal time allocation policy, T^*, is

$$T^* = T_m^* = \frac{B_m^* - 1}{g_m + B_m^*}, \tag{18}$$

where

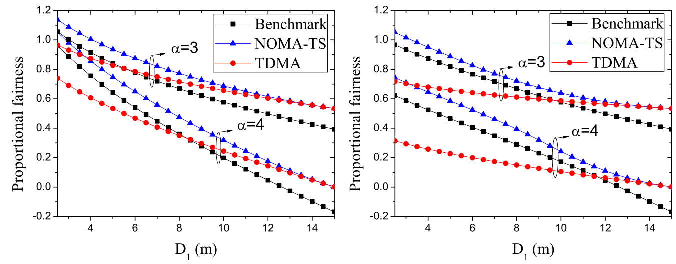
$$B_m^* = \frac{g_m - 1}{W\left((g_m - 1)\exp\left(\frac{c_m}{\lambda_m} - 1\right)\right)}, \tag{19}$$

and

$$c_m = \begin{cases} \mu_m \ln(2), & m = 1, \ldots, N-1, \\ -\sum_{i=1}^{N-1} \mu_i \ln(2), & m = N. \end{cases} \tag{20}$$

Proof: The optimization problem in (16) is concave and it can be also solved by using Lagrange dual decomposition [15]. The Lagrangian is given by

$$\begin{aligned} \mathcal{L} = &\sum_{n=1}^{N} \ln(R_n) - \sum_{m=1}^{N-1} \mu_m (T_m - T_N) \\ &- \sum_{m=1}^{N} \lambda_m \left(\sum_{n=1}^{m} R_n - (1 - T_m) \log_2\left(1 + \frac{g_m T_m}{1 - T_m}\right)\right). \end{aligned} \tag{21}$$



(a) $N_1 = 7$ and $N_2 = 3$ (b) $N_1 = 3$ and $N_2 = 7$

Fig. 1. Proportional fairness versus D_1.

According to KKT conditions, $\frac{\partial \mathcal{L}}{\partial R_n} = 0$, $\forall n$, from which (17) is derived, and $\frac{\partial \mathcal{L}}{\partial T_m} = 0$, $\forall m = 1, \ldots, N$, which yields

$$\lambda_m \frac{g_m + B_m - 1}{B_m} - \lambda_m \ln(B_m) - c_m = 0, \quad \forall m, \tag{22}$$

where

$$B_m = 1 + \frac{g_m T_m}{1 - T_m}. \tag{23}$$

Solving (22) and (23) for T_m, T^*, is given by (18). Also, λ_m is calculated iteratively by the subgradient method. ■

IV. Simulations and Discussions

Two users' groups are considered, each of which is located in different distance, $D_{i \in \{1,2\}}$, from the BS. Let N_i denote the number of users of the i-th group. It is assumed that the 1-st group is located closer to the BS. We further assume that $\eta = 0.5$, $h_n = 10^{-3} d_n^{\alpha}$, where d_n is the distance of the n-th user from the BS, α is the pathloss exponent, $N_0 = -160$ dBm, and $P = 1$ W [14]. The provided results focus on the impact of the number of users, pathloss value, and distances D_1 and D_2 on performance, while the proposed schemes are compared to the benchmark one in [14], i.e., NOMA with fixed decoding order and adaptive power allocation.

Figs. 1, 2, and 3 focus on the effect of D_1 on performance, while assuming $D_2 = 15$ m and groups with unequal number of users, i.e., $N_1 \neq N_2$. More specifically, In Fig. 1 the maximum proportional fairness achieved by each scheme is illustrated, for two values of α, i.e., $\alpha = 3$ and $\alpha = 4$. When $N_1 > N_2$, higher proportional fairness is achieved for all the considered protocols, especially when D_1 is small. Another notable observation from Fig. 1 is that NOMA-TS outperforms the other two schemes for the whole range of D_1 despite the values of α, N_1, N_2, while it performs much better than TDMA, when $N_2 > N_1$ and α increases, indicating the resilience of NOMA to higher pathloss values. Also, TDMA outperforms the benchmark NOMA scheme for the higher values of D_1 and especially when $N_1 > N_2$. Finally, when $D_1 = D_2$ the two proposed schemes achieve exactly the same performance. In this case, TDMA should be preferred, since in general, NOMA increases the decoding complexity at the BS.

In Figs. 2 and 3 the average rate per user, i.e., the normalized sum rate, the minimum user rate, and the time allocated to energy harvesting, are illustrated, versus D_1, assuming $\alpha = 3.5$. It is seen that NOMA-TS outperforms the other two protocols, both in terms of normalized sum rate and minimum rate. This is because time-sharing increases the degrees of freedom and any point of the capacity region can be achieved. Moreover, Figs. 2 and 3 illustrate that, when $N_1 > N_2$, the minimum rate achieved by NOMA-TS is equal to the average

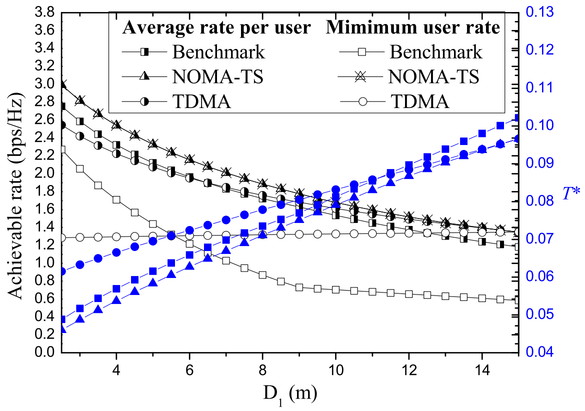


Fig. 2. Achievable rate and T^* for $N1 = 7$ and $N_2 = 3$.

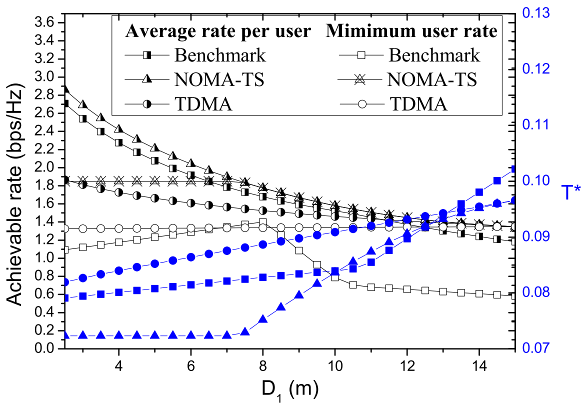


Fig. 3. Achievable rate and T^* for $N1 = 3$ and $N_2 = 7$.

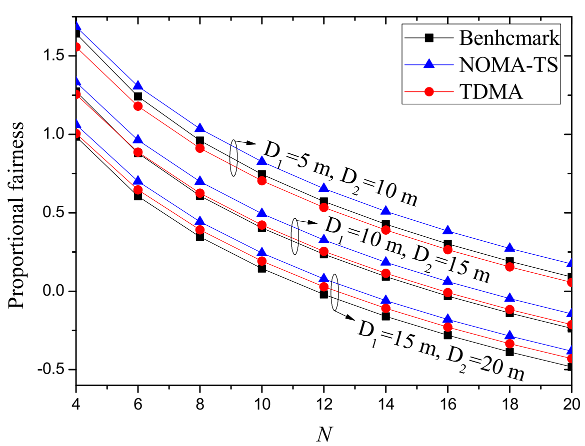


Fig. 4. Proportional fairness versus N.

rate per user, which however does not happen when $N_2 > N_1$ and D_1 is small. In contrast to NOMA, the minimum rate achieved by TDMA is not influenced by D_1. Furthermore, Figs. 2 and 3 reveal that the number of users in the two groups also affects the optimal value of T. More specifically, when $N_2 > N_1$ and the 1-st user group is located close to the BS, the time allocated to the energy harvesting phase increases. Finally, it is remarkable that NOMA-TS allocates less time to energy harvesting compared to TDMA, for the whole range of D_1, increasing the energy efficiency.

In Fig. 4 the considered schemes are evaluated with respect to the number of users, for three different distances realizations, i.e., i) $D_1 = 5$ m and $D_2 = 10$ m, ii) $D_1 = 10$ m and $D_2 = 15$ m, and iii) $D_1 = 15$ m and $D_2 = 20$ m, assuming that $N_1 = N_2$. It is observed that NOMA-TS achieves the highest proportional fairness, which implies that when it is used, instead of the other two protocols, more users can enter to the system. When both groups of user are closely located to the BS (e.g., $D_1 = 5$ m and $D_2 = 10$ m) NOMA with fixed decoding order and adaptive power allocation outperforms TDMA. The opposite happens when both groups are located far from the BS, e.g., when $D_1 = 10$ m and $D_2 = 15$ m and especially when $D_1 = 15$ m and $D_2 = 20$ m, in which case the benchmark scheme becomes more prone to interference.

V. Conclusion

In conclusion, the proposed techniques offer an efficient solution to time and rates allocation problem in wireless-powered communication systems with asymmetrical rates.

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