On the Optimal Tone Spacing for Interference Mitigation in OFDM-IM Systems

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***Abstract*— Orthogonal frequency division multiplexing with index modulation (OFDM-IM) has been recently proposed as an efficient technique to improve the error performance and enhance the spectral efficiency achieved by the classical OFDM. In this letter, we minimize the presence of intercarrier and intersymbol interference, experienced by OFDM-IM systems under mobility conditions and multipath scenarios, by selecting the appropriate tone spacing between adjacent subcarriers. Finally, we prove that the optimal value of tone spacing increases the system capacity, occupying only the necessary amount of bandwidth, and provide closed-form expressions for the interference power per active subcarrier.**

***Index Terms*— Orthogonal frequency division multiplexing with index modulation (OFDM-IM), doubly-selective fading channels, tone spacing.**

I. INTRODUCTION

IN VIEW of the continuously increasing traffic demand, the use of enhanced orthogonal frequency division multiplexing (OFDM) techniques, capable of bringing essential capacity and error performance gains, has become indispensable in the design of next generation wireless communication systems [1], [2]. To this end, a new multicarrier scheme, namely OFDM with index modulation (OFDM-IM) has been recently proposed in [3]. However, like in conventional OFDM systems, the frequency selectivity of the multipath channel results in intersymbol interference (ISI) and, hence, a cyclic prefix (CP) of greater duration than the maximum delay spread has to be inserted in order to combat the detrimental effects on signal transmission. Nevertheless, the consequential increase in the transmitted symbol duration affects the presence of time selectivity in the system causing intercarrier interference (ICI), especially in the case of high-mobility environments [4]. To this end, several schemes have been proposed so far to minimize the induced interference [5]. Indicatively, in [6], the selection of optimal tone spacing was suggested as a means to minimize the combined effect of ICI and ISI in multicarrier systems employing carrier aggregation.

In this letter, taking into account various mobility conditions and multipath scenarios, we study the combined effect of ICI and ISI on the performance of OFDM-IM by selecting the appropriate value of tone spacing. Particularly, the interference anticipated by the active subcarriers of an OFDM-IM system is evaluated in a closed-form expression and minimized with the selection of the optimal tone spacing. This optimization increases the signal-to-interference-plus-noise ratio (SINR) and thus the system capacity. Besides, the bandwidth utilization is enhanced as well, since a lower value of tone spacing requires less bandwidth for the signal to be transmitted.

To the best of our knowledge, the optimal selection of tone spacing has never been studied before in the context of OFDM-IM. This concept is proved here to be very promising, while, if combined with a wise selection of other physical layer parameters, it can significantly improve the overall system performance. Finally, when applied to more robust multicarrier systems, such as interleaving-based, power allocation-aided, and multiantenna OFDM-IM, then versatile and multifold advantages are expected to be envisaged [7].

Notation: In the sequel, for each $x \in \mathbb{R}$, $\lceil x \rceil$ and $\lfloor x \rfloor$ denote the integer ceiling and floor functions, respectively. Furthermore, $\mathrm{E}\{\cdot\}$ stands for the statistical expectation, while the colon-equals symbol, $:=$, defines the assignment operator.

II. SYSTEM MODEL

Consider an OFDM-IM system with N subcarriers splitted in $g = N/\nu$ groups, where ν is the number of subcarriers per group which are considered adjacent to each other. Unlike conventional OFDM, here, information is conveyed not only by data symbols but also through spatial indexing of the active subcarriers [3]. Particularly, in each group, only m out of the ν subcarriers are active carrying an M-ary symbol, while the rest are set equal to zero. As a result, the number of transmitted bits per OFDM-IM symbol is

$$p = mg\log_2 M + g\lfloor \log_2 C(\nu, m)\rfloor, \quad (1)$$

where $C(\nu, m) = \frac{\nu!}{m!(\nu-m)!}$ denotes the binomial coefficient.

Assuming independent and identically distributed (i.i.d.) time-varying multipath Rayleigh fading, the ith channel gain can be modeled as $h_i(n; l) \sim \mathcal{CN}(0, \sigma_l^2)$, which is a circularly-symmetric complex Gaussian random variable with zero mean and variance σ_l^2, representing the lth channel tap in the nth time sample of the ith OFDM-IM symbol.

We also adopt the wide sense stationary uncorrelated scattering model, considering an invariant over time channel correlation function and isotropic scattering. Consequently, the autocorrelation of the lth tap can be written as

$$\mathrm{E}\left\{h_i(n_1; l)h_i^*(n_2; l)\right\} = \sigma_l^2 J_0\left(2\pi f_d(n_1 - n_2)T_s\right), \quad (2)$$

with T_s being the time sample duration and $J_0(\cdot)$ standing for the zero-order Bessel function of the first kind. Additionally, $f_d = (\upsilon/c)f_c$ is the maximum Doppler frequency, where υ, f_c, and c denote the velocity of the receiver, the carrier frequency, and the light speed, respectively.

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As already discussed, a CP of length G is added to the OFDM-IM symbol, so as to mitigate the ISI. If the maximum delay spread is T_{ds}, the number of channel taps that should be considered are $L = \lceil T_{ds}/T_s \rceil$. Finally, we assume an exponentially decaying power delay profile (PDP), according to

$$\sigma_l^2 = \frac{e^{-al}}{\sum_{l_1=0}^{L-1} e^{-al_1}} = \frac{1-e^{-a}}{1-e^{-aL}} e^{-al}, \quad 0 \le l \le L-1, \quad (3)$$

where $a = T_s/T_{ds}$ is a normalized parameter characterizing the level of frequency selectivity of the fading channel.

III. Interference Analysis

The received signal at the kth subcarrier, for $0 \le k \le N-1$, can be expressed in the frequency domain as

$$\begin{aligned} y_i(k) &= H_i(k,k)x_i(k) + \sum_{r=0, r\neq k}^{N-1} H_i(k,r)x_i(r) + w_i(k) \\ &= y_i^{\text{use}}(k) + y_i^{\text{ici}}(k) + w_i(k), \end{aligned} \quad (4)$$

where $x_i(\cdot)$ stands for the ith transmitted OFDM-IM symbol, while $w_i(k)$ is the additive white Gaussian noise (AWGN) added to the kth subcarrier. The channel component, $H_i(k,r)$, indicating the effect of the rth on the kth subcarrier, reads [8]

$$H_i(k,r) = \frac{1}{N} \sum_{l=0}^{L-1} \sum_{n=0}^{N-1} h_i(n;l) e^{\frac{j2\pi r(n-l)}{N}} e^{-\frac{j2\pi nk}{N}}. \quad (5)$$

With the aid of (2), and after calculations, the variance of the zero-mean channel quantity in (5), for all i, can be given by

$$\mathrm{E}\left\{|H_i(k,r)|^2\right\} = V(r-k), \quad (6)$$

where the auxiliary function $\beta \mapsto V(\beta)$ is given by

$$V(\beta) = \frac{1}{N} + 2\sum_{q=1}^{N-1} \frac{N-q}{N^2} J_0(2\pi f_d T_s q) \cos\left(\frac{2\pi\beta q}{N}\right). \quad (7)$$

If the CP length is not sufficient (i.e., $L > G$), then ISI exists and the received symbol, corresponding to the ith transmitted symbol, can be expressed in the frequency domain as

$$z_i(k) = y_i(k) + y_i^{\text{isi}}(k) = y_i(k) + \sum_{r=0}^{N-1} H_i^{\text{isi}}(k,r) x_{i-1}(r), \quad (8)$$

where only the ISI from the previous symbol is considered.

A. ICI Per Active Subcarrier Due to Time Variation

The channel terms associated with the ICI at the kth subcarrier from the rth subcarrier due to time variation are shown in (4), while their variance in (6) depends only on the absolute value of $r-k$. Assuming now that the total power of the OFDM-IM symbol is P_t, the average transmitted power per active subcarrier is $P_s = (\nu P_t)/(mN)$. Thus, the mean ICI power per active subcarrier is evaluated as (see Appendix)

$$\begin{aligned} P_{\text{ici}} = \frac{2P_t}{N^2}\Bigg[&\sum_{\beta_1=1}^{\nu-1} \left(\frac{N(m-1)(\nu-\beta_1)}{m(\nu-1)} + \frac{(N-\nu)\beta_1}{\nu} \right) \\ &\times V(\beta_1) + \sum_{\beta_2=\nu}^{N-1} (N-\beta_2) V(\beta_2) \Bigg]. \end{aligned} \quad (9)$$

B. ISI Per Subcarrier Due to Insufficient Cyclic Prefix

When the CP length is shorter than the number of channel taps, then ISI due to long delay spread exists. In this case, the channel term, corresponding to the ISI from the previous symbol at the kth subcarrier, can be expressed as [8, eq. (15)]

$$H_i^{\text{isi}}(k,r) = \frac{1}{N} \sum_{l=G}^{L-1} \sum_{n=0}^{l-G} h_i(n;l) e^{-\frac{j2\pi r(l-G)}{N}} e^{\frac{j2\pi n(r-k)}{N}}. \quad (10)$$

The associated mean ISI power per subcarrier for the case of an exponential PDP, can be successively calculated as

$$P_{\text{isi}} = \frac{P_s}{N} \mathrm{E}\left\{ \sum_{k=0}^{N-1} \frac{m}{\nu} \sum_{r=0}^{N-1} |H_i^{\text{isi}}(k,r)|^2 \right\} = \frac{P_t}{N^2} \sum_{l=G}^{L-1} \sum_{n=0}^{l-G} \sigma_l^2. \quad (11)$$

By substituting (3) in (11) and after utilizing [9, eq. (0.113)], P_{isi}, given that $L > G$, can yield the closed-form expression

$$P_{\text{isi}} = \left(\frac{P_t}{N^2}\right) \frac{L - G + b\left[1 - e^{a(L-G)}\right]}{1 - e^{aL}}, \quad (12)$$

where $b = (1-e^{-a})^{-1}$ is a dimensionless parameter. It should be noted here that the induced ISI is the same whether the subcarrier is active or not, given that it depends only on the previous transmission for which no knowledge is available.

C. Total Interference Per Active Subcarrier

If Δf is the tone spacing between two adjacent subcarriers, then the sample duration is $T_s = 1/(N\Delta f)$. Assuming a certain value for T_{ds}, the number of channel taps that should be considered depends on the selected Δf through the expression

$$L = \lceil T_{ds} N \Delta f \rceil. \quad (13)$$

If $L > G$, the mean value of the total interference per active subcarrier for the exponential PDP can be obtained as in (14).

IV. Total Interference Minimization

From (13), it comes out that Δf is somehow directly proportional to L. Therefore, from (14), it becomes obvious that a tradeoff exists between P_{ici} and P_{isi} in terms of Δf. To be more precise, as Δf increases, P_{ici} decreases, while P_{isi}, if it exists, increases. Thus, when ISI is dominant, a small value of Δf would be preferable, whereas a larger Δf is better for environments where the ICI due to time variation prevails. It should be noted here that, when the value of Δf changes, the time sample duration and the occupied bandwidth change accordingly. In order to guarantee the fairness of the comparison between systems with variable Δf, we assume that the spectral efficiency, which can be calculated from

$$\eta = \frac{N\left(m \log_2 M + \lfloor \log_2 C(\nu, m) \rfloor\right)}{\nu(N+G)}, \quad (15)$$

the total power of the OFDM-IM symbol, and the signal-to-noise ratio (SNR), do not depend on the selected value of Δf.

Algorithm 1 Optimal Tone Spacing Selection in OFDM-IM

1: Compute $\partial \tilde{P}_{\text{int}}/\partial \Delta f$ from (19)
2: Solve (numerically) $\partial \tilde{P}_{\text{int}}/\partial \Delta f = 0$ to determine Δf_0
3: **if** $\Delta f_0 \geq B/N$ **then**
4: Set $\Delta f_1 = B/N$ and compute $L_1 = \lceil T_{ds} N \Delta f_1 \rceil$
5: **else** Compute $L_1 = \lceil T_{ds} N \Delta f_0 \rceil$ and $\Delta f_1 = L_1/(N T_{ds})$
6: **if** $\Delta f_1 > B/N$ **then**
7: Set $\Delta f_1 = B/N$
8: **end if**
9: **end if**
10: Evaluate $P_1 := P_{\text{int}}$ from (14); $\Delta f = \Delta f_1$, $L = L_1$
11: Compute $L_2 = L_1 - 1$ and $\Delta f_2 = L_2/(N T_{ds})$
12: Evaluate $P_2 := P_{\text{int}}$ from (14); $\Delta f = \Delta f_2$, $L = L_2$
13: **if** $P_1 < P_2$ **then**
14: Select $\Delta f_{\text{opt}} = \Delta f_1$
15: **else** Select $\Delta f_{\text{opt}} = \Delta f_2$
16: **end if**

Our aim is to determine the optimal value of Δf, namely the one that minimizes the total interference as expressed in (14). The optimization problem can be simply formulated as

$$\Delta f_{\text{opt}} = \arg\min_{\Delta f} P_{\text{int}} \quad \text{s.t. } N\Delta f \leq B \text{ and } L = \lceil T_{ds} N \Delta f \rceil, \tag{16}$$

where B is the maximum bandwidth that can be occupied.

This problem can be solved by examining two distinct cases separately; (a) $G > L$ and (b) $G \leq L$.

In case (a), the total interference becomes equal to P_{ici}, since $P_{\text{isi}} = 0$. Obviously, in order to minimize P_{int}, the maximum permitted value of Δf should be selected. Therefore, from

$$G > \lceil T_{ds} N \Delta f \rceil \geq T_{ds} N \Delta f \tag{17}$$

and the bandwidth constraint in (16), it is easy to infer that

$$\Delta f_{\text{opt}} = \max\left\{\frac{G}{N T_{ds}}, \frac{B}{N}\right\}. \tag{18}$$

In case (b), the ceiling function involved in (13) renders the solution of (16) intractable, since different values of Δf yielding the same L may result in the same P_{isi}. For this reason, we first get an approximation of P_{int} by substituting in (14) the number of channel taps by the quantity $T_{ds} N \Delta f$ as

$$\tilde{P}_{\text{int}} = P_{\text{ici}} + \left(\frac{P_t}{N^2}\right) \frac{T_{ds} N \Delta f - G + b\left[1 - e^{a(T_{ds} N \Delta f - G)}\right]}{1 - e^{a T_{ds} N \Delta f}} \tag{19}$$

and then we proceed with the steps of Algorithm 1. We note that, in Step 2, $\partial \tilde{P}_{\text{int}}/\partial \Delta f = 0$ is solved numerically, given that $\partial J_0(1/x)/\partial x = J_1(1/x)/x^2$. If Δf takes only certain values, the optimal tone spacing is the closest value to Δf_{opt} given by Algorithm 1, that is permitted.

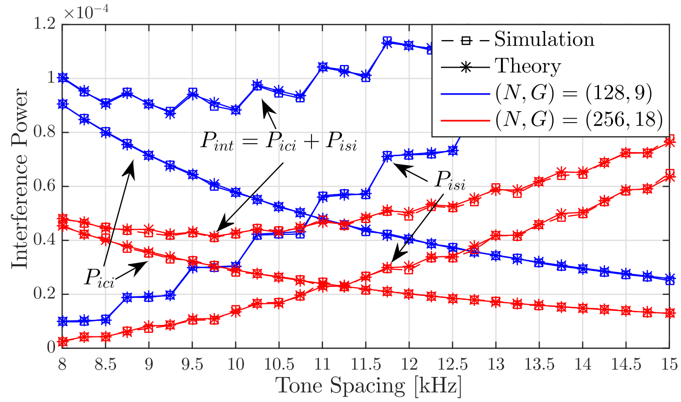

Fig. 1. P_{ici}, P_{isi}, P_{int} versus Δf in OFDM-IM with $(\nu, m) = (4, 3)$.

V. Simulations and Discussion

In this section, we validate the derived theoretical results with the aid of Monte Carlo simulations and we illustrate the effect of optimal tone spacing selection on the performance of OFDM-IM. In all cases, we assume binary phase-shift keying (BPSK) modulation, unit transmitted power per OFDM-IM symbol, $T_{ds} = 10$ µsec, $v = 300$ km/h, and $f_c = 2.5$ GHz, unless otherwise stated. Additionally, the maximum occupied bandwidth satisfies the constraint in (16).

In Fig. 1, we depict the power of interference coming from ICI and/or ISI versus Δf for an OFDM-IM configuration with $(\nu, m) = (4, 3)$. Firstly, it is observed that the simulations match perfectly with the theoretical results, while there exists a global minimum, namely an optimal tone spacing that minimizes the total interference. This value equals 9.25 kHz for $(N, G) = (128, 9)$ and 9.75 kHz for $(N, G) = (256, 18)$. Obviously, Δf_{opt} corresponds to an L value that is slightly larger than G, because at this point the ISI power becomes high enough. Secondly, the effect of the ceiling function in (13) is evident, since for those Δf yielding the same L, the ISI power remains constant, whereas the ICI power decreases with increasing Δf, which explains the rationale of Algorithm 1.

In Figs. 2 and 3, the optimal tone spacing values are plotted with respect to f_c and v, respectively, for two different values of T_{ds}, namely 5 µsec and 10 µsec. In all cases, we assume $(\nu, m) = (16, 11)$, which provides a spectral efficiency of 1.343 bps/Hz. As expected, lower Δf values are optimal for the long delay spread scenario, which corresponds to a multipath environment with more severe ISI. Moreover, the optimal tone spacing increases with increasing f_c and v, as a result of the ICI escalation. Interestingly, using the largest possible number of subcarriers, N, is advantageous as it leads to a wider range of L, which consequently allows for the selection of more refined Δf values. In the same context, the monotonicity of P_{int} with respect to Δf is less affected by the ceiling function in (13) as N increases. Moreover, our extensive simulations revealed that the optimal tone spacing

$$P_{\text{int}} = \frac{2P_t}{N^2}\left[\frac{L - G + b\left[1 - e^{a(L-G)}\right]}{2(1 - e^{aL})} + \sum_{\beta_1=1}^{\nu-1}\left(\frac{N(m-1)(\nu-\beta_1)}{m(\nu-1)} + \frac{(N-\nu)\beta_1}{\nu}\right)V(\beta_1) + \sum_{\beta_2=\nu}^{N-1}(N-\beta_2)V(\beta_2)\right] \tag{14}$$

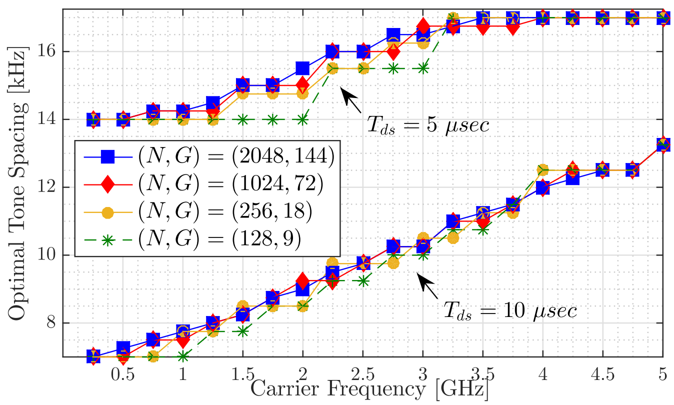

Fig. 2. Δf_{opt} versus f_c in OFDM-IM with $(\nu, m) = (16, 11)$.

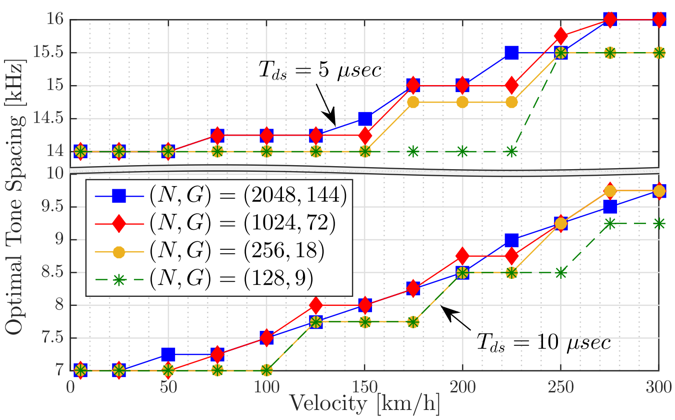

Fig. 3. Δf_{opt} versus v in OFDM-IM with $(\nu, m) = (16, 11)$.

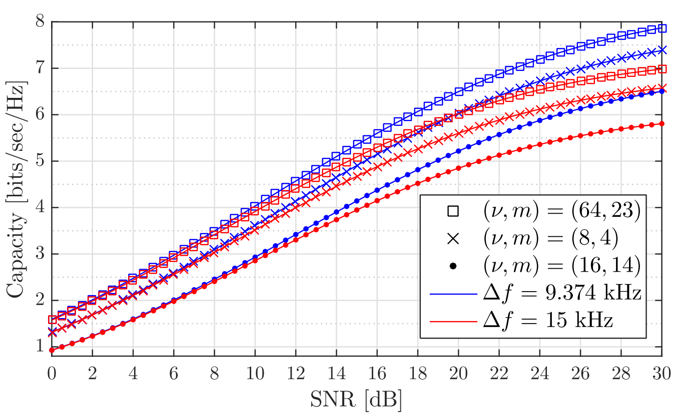

Fig. 4. Capacity versus SNR in OFDM-IM with $(N, G) = (128, 9)$, at $\eta = 1.168$ bps/Hz. The SNR is defined here as $P_t/(N\sigma_s^2)$.

does not depend on the (ν, m) pair but rather on the (N, G) one, in the case where m is not much smaller than ν.

In Fig. 4, three OFDM-IM configurations, with different (ν, m) but the same spectral efficiency, namely 1.168 bps/Hz, are compared in terms of capacity, $\mathcal{C}$, for two values of Δf; the typical value adopted in the LTE-Advanced standard (i.e., 15 kHz) and the optimal value derived from Algorithm 1 (i.e., 9.374 kHz). Under the assumption that the information carried by the subcarrier indices is not taken into account, a lower bound on the average capacity per active subcarrier, given by

$$\mathcal{C} = \frac{\nu}{mN} \mathrm{E}\left\{ \sum_{k=0}^{N-1} \log_2\left(1 + \gamma_i(k)\right) \right\}, \tag{20}$$

has been used in the simulations, where

$$\gamma_i(k) = \frac{|y_i^{\text{use}}(k)|^2}{|y_i^{\text{ici}}(k) + y_i^{\text{isi}}(k)|^2 + \sigma_s^2} \tag{21}$$

is the instantaneous SINR per subcarrier and σ_s^2 is the noise variance per subcarrier. The capacity gain of OFDM-IM with optimal Δf over the one with typical Δf is evident, which becomes more profound in the high-SNR regime. At the same time, the occupied bandwidth is reduced by more than 38% compared to the scheme with typical Δf. Finally, OFDM-IM systems with close values for m/ν perform similarly for the same Δf and better as m/ν gets smaller.

Appendix

Proof of the Mean ICI Power Per Active Subcarrier in OFDM-IM, Equation (9)

The mean ICI per active subcarrier comes from the ICI induced by the subcarriers of the same group (i.e, intra-group ICI) and the mean ICI induced by the subcarriers of all the other groups (i.e., inter-group ICI). For the kth active subcarrier that belongs to the S_λ group, $\lambda = 1, 2, .., g$, we get that

$$P_{\text{ici},k}^{\text{intra}} = P_s \left(\frac{m-1}{\nu-1}\right) \sum_{r=0, r \in S_\lambda}^{N-1} \mathrm{E}\left\{ |H_i(k,r)|^2 \right\} \quad \text{and} \tag{22}$$

$$P_{\text{ici},k}^{\text{inter}} = P_s \left(\frac{m}{\nu}\right) \sum_{r=0, r \notin S_\lambda}^{N-1} \mathrm{E}\left\{ |H_i(k,r)|^2 \right\}. \tag{23}$$

Therefore, the mean ICI per active subcarrier equals

$$P_{\text{ici}} = \frac{1}{N} \sum_{k=0}^{N-1} \left(P_{\text{ici},k}^{\text{intra}} + P_{\text{ici},k}^{\text{inter}} \right). \tag{24}$$

Taking into account (6) and after some manipulations, P_{ici} in (24) can finally obtain the closed-form expression in (9).

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