Performance of free-space optical communications over a mixture composite irradiance channel

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The performance of a typical free-space optical link using a recently proposed composite model for turbulence and pointing error effects is studied. This model offers analytical tractability in the statistical description of the above phenomena and was derived using the mixture Gamma distribution for turbulence. New analytical derivations of the outage probability and bit-error rate for some binary modulation schemes are presented in closed form.

System and channel model: We assume a free-space optical (FSO) system where the laser beams propagate along a horizontal path through a channel with additive white Gaussian noise (AWGN), in the presence of scintillation and pointing error effects. The receiver integrates the photocurrent signal, which is related to the incident optical power, by the detector responsivity for each bit period. The received electrical signal is given by

$$y = \eta I x + n \tag{1}$$

where x is the binary transmitted intensity signal, I denotes the received irradiance which is constant over a large number of transmitted bits, η represents the effective photo-current conversion ratio of the receiver, and n is the AWGN with variance σ_n^2. Both intensity modulation/direct detection (IM/DD) and heterodyne detection are taken into account.

The irradiance is considered to be a product of two independent random factors, i.e. $I = I_a I_p$, where I_a is due to atmospheric turbulence and I_p due to geometric spread and pointing errors. The probability density function (PDF) of I_α is given by the mixture Gamma (MG) model of [1, eqn. (1)], i.e.

$$f_{I_\alpha}(I_\alpha) = \sum_{i=1}^{N} a_i I_a^{b_i-1}\, \mathrm{e}^{-\zeta_i I_a}, \quad I_a > 0 \tag{2}$$

which can efficiently approximate any PDF by selecting a proper number of summation terms N. Following the assumptions in [1, sec. III], the PDF of I_p is expressed as

$$f_{I_p}(I_p) = \frac{\gamma^2}{A_0^{\gamma^2}} I_p^{\gamma^2-1}, \quad 0 \le I_p \le A_0 \tag{3}$$

The parameters included in the above PDFs are explained in detail in [1].

In this context, a novel composite model for the PDF of I was recently launched in [1] as

$$f_I(I) = \frac{\gamma^2}{A_0^{\gamma^2}} I^{\gamma^2-1} \sum_{i=1}^{N} a_i \zeta_i^{\gamma^2-b_i}\Gamma\left(b_i-\gamma^2, \frac{\zeta_i}{A_0} I\right), \quad I > 0 \tag{4}$$

where $\Gamma(\cdot,\cdot)$ is the upper incomplete Gamma function defined in [2, eqn. (8.350.2)]. Furthermore, the cumulative distribution function (CDF) of I was found to be

$$F_I(I) = \sum_{i=1}^{N} a_i \zeta_i^{-b_i}\left[\left(\frac{\zeta_i I}{A_0}\right)^{\gamma^2}\Gamma\left(b_i-\gamma^2, \frac{\zeta_i I}{A_0}\right) + \gamma\left(b_i, \frac{\zeta_i I}{A_0}\right)\right] \tag{5}$$

where $\gamma(\cdot,\cdot)$ is the lower incomplete Gamma function defined in [2, eqn. (8.350.1)].

In the following sections, we derive the electrical signal-to-noise ratio (SNR), the outage probability, and the average bit-error rate (BER) performance for some binary modulation schemes.

Electrical SNR statistics: The instantaneous electrical SNR and the average electrical SNR can be defined as $\mu = (\eta I)^r/\sigma_n^2$ and $\bar{\mu} = (\eta \bar{I})^r/\sigma_n^2$, respectively [3]. The parameter r is equal to 1 for heterodyne detection and 2 for IM/DD. That is, $\mu = \mu_{\mathrm{het}}$, $\bar{\mu} = \bar{\mu}_{\mathrm{het}}$ for $r = 1$ and $\mu = \mu_{\mathrm{IM/DD}}$, $\bar{\mu} = \bar{\mu}_{\mathrm{IM/DD}}$ for $r = 2$. The mean value of irradiance, $\bar{I}$, is given as [1]

$$\bar{I} = \frac{\gamma^2 A_0}{1+\gamma^2}\sum_{i=1}^{N} a_i \zeta_i^{-(1+b_i)}\Gamma(1+b_i) \tag{6}$$

After a simple power transformation the composite PDF of μ is expressed as

$$f_\mu(\mu) = \frac{\gamma^2}{rA_0^{\gamma^2}}\frac{\bar{I}^{\gamma^2}}{\bar{\mu}^{(\gamma^2/r)}}\mu^{(\gamma^2/r)-1} \times \sum_{i=1}^{N} a_i \zeta_i^{\gamma^2-b_i}\Gamma\left(b_i-\gamma^2, \frac{\zeta_i \bar{I}}{A_0}\left(\frac{\mu}{\bar{\mu}}\right)^{1/r}\right), \quad \mu > 0 \tag{7}$$

Finally, a closed-form expression of the SNR CDF is deduced by properly using, $F_\mu(\mu) \triangleq \int_0^\mu f_\mu(\mu)\,\mathrm{d}\mu$, as

$$F_\mu(\mu) = \sum_{i=1}^{N} a_i \zeta_i^{-b_i}\left[\left(\frac{\zeta_i \bar{I}}{A_0}\left(\frac{\mu}{\bar{\mu}}\right)^{1/r}\right)^{\gamma^2} \times \Gamma\left(b_i-\gamma^2, \frac{\zeta_i \bar{I}}{A_0}\left(\frac{\mu}{\bar{\mu}}\right)^{1/r}\right) + \gamma\left(b_i, \frac{\zeta_i \bar{I}}{A_0}\left(\frac{\mu}{\bar{\mu}}\right)^{1/r}\right)\right] \tag{8}$$

Outage probability: The outage probability denotes the probability that the instantaneous SNR falls below a specified threshold, μ_{th}, i.e.

$$P_{\mathrm{out}} \triangleq \Pr(\mu \le \mu_{\mathrm{th}}) = F_\mu(\mu_{\mathrm{th}}) \tag{9}$$

Average BER: The average BER, P_b, for several binary modulation schemes can be easily derived if we substitute (8) into the following equation [3, eqn. 23]

$$P_b = \frac{q^p}{2\Gamma(p)}\int_0^\infty \mathrm{e}^{-q\mu}\mu^{p-1}F_\mu(\mu)\,\mathrm{d}\mu \tag{10}$$

where $\Gamma(\cdot)$ is the Gamma function [2, eq. (8.310.1)] and p, q refer to different modulation formats. For example, the values of $p = 1$, $q = 1$ correspond to differential binary phase-shift keying (DBPSK) scheme.

For heterodyne detection, the substitution leads to the following integral:

$$P_{b,\mathrm{het}} = \frac{q^p}{2\Gamma(p)}\sum_{i=1}^{N} a_i \zeta_i^{-b_i}\left(\frac{\zeta_i \bar{I}}{A_0\bar{\mu}_{\mathrm{het}}}\right)^{\gamma^2} \times \int_0^\infty \mathrm{e}^{-q\mu}\mu^{p+\gamma^2-1}\Gamma\left(b_i-\gamma^2, \frac{\zeta_i \bar{I}}{A_0\bar{\mu}_{\mathrm{het}}}\mu\right)\mathrm{d}\mu + \frac{q^p}{2\Gamma(p)}\sum_{i=1}^{N} a_i \zeta_i^{-b_i}\int_0^\infty \mathrm{e}^{-q\mu}\mu^{p-1}\gamma\left(b_i, \frac{\zeta_i \bar{I}}{A_0\bar{\mu}_{\mathrm{het}}}\mu\right)\mathrm{d}\mu \tag{11}$$

By properly using [4, eqn. 2.10.3.2], an analytical solution is shown as follows:

$$\begin{aligned} P_{b,\mathrm{het}} = \frac{q^p}{2\Gamma(p)}\sum_{i=1}^{N} a_i \zeta_i^{-b_i}\Bigg[&\left(\frac{\bar{I}\zeta_i}{A_0\bar{\mu}_{\mathrm{het}}}\right)^{\gamma^2}\frac{\Gamma(b_i-\gamma^2)\Gamma(p+\gamma^2)}{q^{p+\gamma^2}} \\ &-\left(\frac{\bar{I}\zeta_i}{A_0\bar{\mu}_{\mathrm{het}}}\right)^{b_i}\frac{q^{-(p+b_i)}\Gamma(p+b_i)}{(b_i-\gamma^2)} \\ &\times {}_2F_1\left(b_i-\gamma^2, p+b_i; \frac{b_i-\gamma^2}{2}; -\frac{\bar{I}\zeta_i}{qA_0\bar{\mu}_{\mathrm{het}}}\right) \\ &+\left(\frac{\bar{I}\zeta_i}{A_0\bar{\mu}_{\mathrm{het}}}\right)^{b_i}\frac{q^{-(p+b_i)}\Gamma(p+b_i)}{b_i} \\ &\times {}_2F_1\left(b_i, p+b_i; b_i+1; -\frac{\bar{I}\zeta_i}{qA_0\bar{\mu}_{\mathrm{het}}}\right)\Bigg] \end{aligned} \tag{12}$$

where ${}_2F_1(\cdot, \cdot; \cdot; \cdot)$ is the hypergeometric series [2, eqn. (9.100)].

Similarly, for the IM/DD detection, we get after some trivial algebra

$$P_{b,\mathrm{IM/DD}} = \frac{q^p}{\Gamma(p)} \sum_{i=1}^{N} a_i \zeta_i^{-b_i} \left(\frac{\zeta_i \bar{I}}{A_0 \sqrt{\bar{\mu}_{\mathrm{IM/DD}}}} \right)^{\gamma^2} \times \int_0^\infty \mathrm{e}^{-q\mu^2} \mu^{2p+\gamma^2-1} \Gamma\left(b_i - \gamma^2, \frac{\zeta_i \bar{I}}{A_0 \sqrt{\bar{\mu}_{\mathrm{IM/DD}}}} \mu \right) \mathrm{d}\mu + \frac{q^p}{\Gamma(p)} \sum_{i=1}^{N} a_i \zeta^{-b_i} \times \int_0^\infty \mathrm{e}^{-q\mu^2} \mu^{2p-1} \gamma\left(b_i, \frac{\zeta_i \bar{I}}{A_0 \sqrt{\bar{\mu}_{\mathrm{IM/DD}}}} \mu \right) \mathrm{d}\mu \quad (13)$$

The two integrals in (13) can be analytically solved with the aid of [4, eqn. (2.10.3.9)]. Then, the following formula occurs:

$$P_{b,\mathrm{IM/DD}} = \frac{q^p}{\Gamma(p)} \sum_{i=1}^{N} a_i \zeta_i^{-b_i} \times \Bigg[\left(\frac{\bar{I}\zeta_i}{A_0 \sqrt{\bar{\mu}_{\mathrm{IM/DD}}}} \right)^{\gamma^2} \frac{\Gamma(b_i-\gamma^2)\Gamma(2p+\gamma^2/2)}{2q^{(2p+\gamma^2/2)}} + \left(\frac{\bar{I}\zeta_i}{A_0 \sqrt{\bar{\mu}_{\mathrm{IM/DD}}}} \right)^{b_i+1} \frac{q^{-(2p+b_i+1/2)}\Gamma(2p+b_i+1/2)}{2(b_i-\gamma^2+1)} \times {}_2F_2\left(\frac{b_i-\gamma^2+1}{2}, \frac{2p+b_i+1}{2}; \frac{3}{2}, \frac{b_i-\gamma^2+3}{2}; \frac{\bar{I}^2\zeta_i^2}{4qA_0^2\bar{\mu}_{\mathrm{IM/DD}}} \right) - \left(\frac{\bar{I}\zeta_i}{A_0 \sqrt{\bar{\mu}_{\mathrm{IM/DD}}}} \right)^{b_i} \frac{q^{-(2p+b_i/2)}\Gamma(2p+b_i/2)}{2(b_i-\gamma^2)} \times {}_2F_2\left(\frac{b_i-\gamma^2}{2}, \frac{2p+b_i}{2}; \frac{1}{2}, \frac{b_i-\gamma^2}{2}; \frac{\bar{I}^2\zeta_i^2}{4qA_0^2\bar{\mu}_{\mathrm{IM/DD}}} \right) + \left(\frac{\bar{I}\zeta_i}{A_0 \sqrt{\bar{\mu}_{\mathrm{IM/DD}}}} \right)^{b_i} \frac{q^{-(2p+b_i/2)}\Gamma(2p+b_i/2)}{2b_i} \times {}_2F_2\left(\frac{b_i}{2}, \frac{2p+b_i}{2}; \frac{1}{2}, \frac{b_i}{2}+1; \frac{\bar{I}^2\zeta_i^2}{4qA_0^2\bar{\mu}_{\mathrm{IM/DD}}} \right) - \left(\frac{\bar{I}\zeta_i}{A_0 \sqrt{\bar{\mu}_{\mathrm{IM/DD}}}} \right)^{b_i+1} \frac{q^{-(2p+b_i+1/2)}\Gamma(2p+b_i+1/2)}{2(b_i+1)} \times {}_2F_2\left(\frac{b_i+1}{2}, \frac{2p+b_i+1}{2}; \frac{3}{2}, \frac{b_i+3}{2}; \frac{\bar{I}^2\zeta_i^2}{4qA_0^2\bar{\mu}_{\mathrm{IM/DD}}} \right) \Bigg] \quad (14)$$

where ${}_2F_2(\cdot, \cdot; \cdot, \cdot; \cdot)$ is the generalised hypergeometric series [2, eqn. (9.14.1)].

Numerical results: In what follows, we assume that the MG model approximates the Málaga PDF of [1, eqn. 6] according to the equations provided in [1, eqn. 9]. Then, proper numerical results are presented for the following parameter values $N = 15$, $\alpha' = 2$, $\beta' = 5$, $\varphi_A - \varphi_B = (\pi/2)$, $w_z/r = 5$, and $\sigma_s/r = 3$ which are kept constant, unless specified otherwise. Moreover, the transmitted power is normalised as $\Omega + 2b_0 = 1$ [1].

Fig. 1 demonstrates the outage probability in terms of the normalised threshold for several values of the parameter ρ. This parameter is used in the Málaga distribution and represents the amount of scattering power coupled to the line-of-sight component [1]. Heterodyne detection provides better performance than IM/DD, as expected [3]. Furthermore, the performance improves as ρ increases. This is valid since an increase of ρ leads to a decrease in turbulence intensity.

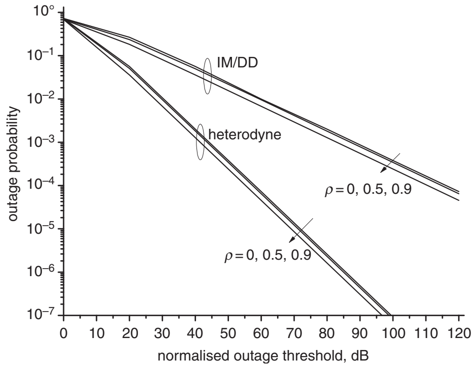

Fig. 1 *Outage probability against* $\bar{\mu}/\mu_{th}$

Fig. 2 visualises the BER performance of DBPSK against the average electrical SNR for several values of ρ. Again, the performance improves when ρ increases.

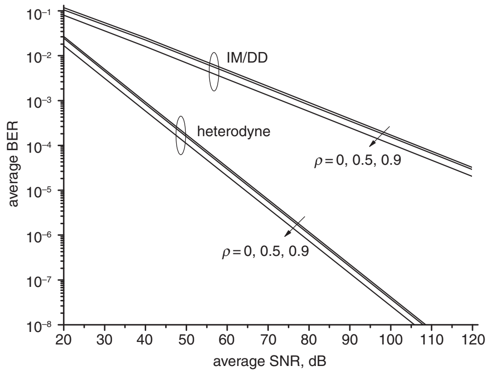

Fig. 2 *Average BER for DBPSK against average electrical SNR*

Conclusions: A versatile mixture composite model for turbulence and pointing error effects was employed for the performance evaluation of a typical FSO link. The outage probability and formulas for BER were extracted in closed-form and numerical results were demonstrated after assuming an appropriate set of parameters.

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