Moments-Based Approach to the Performance Analysis of Equal Gain Diversity in Nakagami-m Fading

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***Abstract*—In this letter, an alternative moments-based approach for the performance analysis of an L-branch predetection equal gain combiner (EGC) over independent or correlated Nakagami-m fading channels is presented. Exact closed-form expressions are derived for the moments of the EGC output signal-to-noise ratio (SNR), while the corresponding moment-generating function (MGF) is accurately approximated with the aid of Padé approximants theory. Important performance criteria are studied; the average output SNR, which is expressed in closed form both for independent and correlative fading and for arbitrary system parameters, the average symbol-error probability for several coherent, noncoherent, and multilevel modulation schemes, and the outage probability, which are both accurately approximated using the well-known MGF approach. The proposed mathematical analysis is illustrated by various numerical results, and computer simulations have been performed to verify the validity and the accuracy of the theoretical approach.**

***Index Terms*—Bit-error rate (BER), correlated fading, equal gain combining (EGC), Nakagami-m fading, outage probability, Rayleigh fading.**

I. Introduction

DIVERSITY combining at the receiver has been extensively used in wireless communications systems for many years to reduce the effects of fading and to improve the received signal strength. Various techniques are known to combine the signals from multiple diversity branches. The most popular of them are selection combining (SC), equal gain combining (EGC), maximal ratio combining (MRC), or a combination of SC and MRC, called generalized selection combining (GSC). Among them, EGC provides an intermediate solution, as far as the performance and the implementation complexity are concerned. In EGC receivers, each signal branch is weighted with the same factor, irrespective of the signal amplitude. However, co-phasing of all signals is needed to avoid signal cancellation.

Previous work related to the performance of predetection EGC, assuming statistically independent Nakagami-m or Rayleigh channel fading, can be found in [1]–[6]. In particular, Beaulieu and Abu-Dayya used in [1] an approximate infinite series representation for the probability density function (pdf) of the sum of Nakagami-m random variables (RVs) to evaluate the EGC performance. Zhang in [2] studied the error performance of EGC for Rayleigh fading, using a special lemma (Gil–Palaez) and Hermite numerical integration. Alouini and Simon in [3] derived an expression for the average symbol-error probability (ASEP) in the form of a double finite-infinite integral and an integrand composed of special functions, while Gauss–Hermite numerical integration was used to increase the computational speed. Karagiannidis and Kotsopoulos proposed in [4] and [5] an alternative approach for the evaluation of the error performance for binary signaling schemes, which is efficient for low-order diversity, since in [6], by Annamallai *et al.*, the average error performance was analyzed and semianalytical expressions with infinite integrals were derived, transforming the error integral into the frequency domain. Moreover, some closed-forms for the ASEP of several modulation schemes were presented. In contrast to the independent fading case, there are few approaches for the performance evaluation of predetection EGC over correlated fading channels, and all of them are limited to dual diversity receivers [7]–[9].

Ascertaining the absence of a unified approach to the performance analysis of EGC receivers, both for independent and correlative fading, due to the difficulty in finding a useful expression for the pdf of the EGC output signal-to-noise ratio (SNR), this letter is an attempt to face this problem using an alternative moments-based approach. Deriving simple closed-form expressions for the moments and approximating the moment generating function (MGF) of the output SNR using Padé approximants, important performance criteria, such as the average output SNR, the ASEP, and the outage probability are studied. The average output SNR is extracted in a simple closed-form expression both for independent and correlative fading, and for an arbitrary number of input paths and system's parameters, while the ASEP for several coherent, noncoherent, and multilevel modulation schemes and the outage probability can be accurately approximated using the well-known MGF approach [10]. Numerical results and simulations were used to check the validity and the accuracy of the proposed analysis and to point out the effect of the unbalanced input SNRs, the fading severity, and the fading correlation on the system's performance.

II. Statistics of the EGC Output SNR

A. System and Channel Model

We consider an L-branch predetection EGC diversity receiver, operating in a fading environment. The received signal at the ith antenna at timing instant t is

$$z_i(t) = r_i(t)\, s(t) \cos\left[2\pi f_c t + \theta_i(t)\right] + n_i(t), \quad i = 1, 2, \ldots, L \tag{1}$$

where f_c is the carrier frequency, $n_i(t)$ is the additive white Gaussian noise (AWGN) with a two-sided power spectral density $N_0/2$, $s(t)$ is the transmitted signal, $\theta_i(t)$ is the random phase due to Doppler shift and oscillators frequency mismatch,

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and $r_i(t)$ is the fading envelope. The phase $\theta_i(t)$ is uniformly distributed over the range and the noise components are assumed to be statistically independent of the signals and uncorrelated with each other. The receiver equally weighs all input signals and then sums them to produce the decision statistic. For equally likely transmitted symbols, the instantaneous output SNR per symbol is given by

$$\gamma_{\text{out}} = \frac{E_s}{L\,N_0}\left(r_1 + r_2 \cdots + r_L\right)^2 \tag{2}$$

with E_s being the energy per symbol. In a Nakagami-m fading environment, $r_1, r_2, \ldots, r_L$ are RVs with pdfs given by [11]

$$f_{r_i}\left(r_i\right) = \frac{2}{\Gamma\left(m\right)}\left(\frac{m}{\Omega_i}\right)^m r_i^{2m-1}\exp\left(-\frac{m}{\Omega_i}r_i^2\right), r_i \geq 0 \tag{3}$$

whereas $\Gamma(x)$ is the Gamma function, Ω_i represents the average signal power at the ith branch, and m is an arbitrary fading severity parameter which can take values from 0.5 through infinity. Here, it is assumed that the m parameter is the same for all branches, which is true in practical applications, and the power delay profile of the input paths could be uniform ($\Omega_i = \Omega$) or nonuniform, representing antenna diversity or multipath diversity over frequency-selective fading channels, respectively.

B. *Moments of the Output SNR*

Using (2), the nth moment of the EGC output SNR is, by definition

$$\begin{aligned}\mu_n = E\left\langle\gamma_{\text{out}}^n\right\rangle &= E\left\langle\left[\frac{E_s}{L\,N_0}\left(r_1 + r_2 \cdots + r_L\right)^2\right]^n\right\rangle \\ &= \left(\frac{E_s}{L\,N_0}\right)^n E\left\langle\left(r_1 + r_2 \cdots + r_L\right)^{2\,n}\right\rangle\end{aligned} \tag{4}$$

where $E\left\langle\bullet\right\rangle$ means expectation. Using the multinomial theorem [12] and $\gamma_i = E_s r_i^2/N_0$, $i = 1, \ldots, L$, with γ_i being the instantaneous SNR at the ith input path of the combiner, results in

$$\mu_n = \frac{(2n)!}{L^n}\sum_{\substack{n_1,\ldots,n_L=0\\ n_1+\cdots+n_L=2n}}^{2n}\left[\frac{E\left\langle\gamma_1^{n_1/2}\ldots\gamma_L^{n_L/2}\right\rangle}{\prod_{i=1}^{L} n_i!}\right]. \tag{5}$$

1) Independent Fading: For independent but not necessarily identically distributed (i.d.) branches, $E\left\langle\gamma_1^{n_1/2}\ldots\gamma_L^{n_L/2}\right\rangle$ is expressed as

$$E\left\langle\gamma_1^{n_1/2}\ldots\gamma_L^{n_L/2}\right\rangle = E\left\langle\gamma_1^{n_1/2}\right\rangle\cdots E\left\langle\gamma_L^{n_L/2}\right\rangle. \tag{6}$$

Now, using the well-known expression for the nth moment of the SNR of a single Nakagami-m channel [10, eq. (2.23)]

$$E\left\langle\gamma_i^n\right\rangle = \frac{\Gamma\left(m+n\right)}{\Gamma\left(m\right)\,m^n}\overline{\gamma}_i^n \tag{7}$$

which holds also for noninteger values of n, (5) can be expressed as

$$\mu_n = \frac{(2n)!}{L^n\Gamma^L\left(m\right)}\sum_{\substack{n_1,\,\ldots,\,n_L=0\\ n_1+\cdots+n_L=2n}}^{2n}\prod_{i=1}^{L}\frac{\Gamma\left(m+\frac{n_i}{2}\right)\overline{\gamma}_i^{n_i/2}}{n_i!\,m^{n_i/2}}. \tag{8}$$

2) Correlative Fading: For correlative input paths, (6) does not hold, and $E\left\langle\gamma_1^{n_1/2}\ldots\gamma_L^{n_L/2}\right\rangle$ has to be evaluated taking into account the correlation among $\gamma_1, \gamma_2, \ldots, \gamma_L$. Recently, Karagiannidis *et al.* in [13] and [14] presented an efficient approximation to the multivariate Nakagami-m distribution with arbitrary correlation.[1] According to this approach, the joint pdf of $r_1, r_2, \ldots, r_L$ can be expressed as

$$\begin{aligned}f_{r_1,\ldots,r_L}\left(r_1,\ldots,r_L\right) &= \frac{\left|\mathbf{W}\right|^m r_1^{m-1}\,r_L^m \mathrm{e}^{-\sum_{n=1}^{L} w_{n,n}\,r_n^2/2}}{2^{m-1}\Gamma\left(m\right)} \\ &\quad\times\prod_{n=1}^{L-1}\frac{r_n I_{m-1}\left(\left|w_{n,n+1}\right|r_n\,r_{n+1}\right)}{\left|w_{n,n+1}\right|^{(m-1)}}\end{aligned} \tag{9}$$

with $\mathbf{W}$ being the inverse of the correlation matrix $\mathbf{\Sigma}$, i.e., $\mathbf{W} = \mathbf{\Sigma}^{-1}$ with elements $w_{i,j}, 1 \leq i, j \leq L$. For the special case of linear antenna arrays [10, p. 324], i.d. branches with exponential correlation is assumed, i.e., $\mathbf{\Sigma}_{i,j} \equiv \rho^{|i-j|}$, and (9) can be directly used to evaluate $f_{r_1,\ldots,r_L}\left(r_1,\ldots,r_L\right)$. In the general case of $\mathbf{\Sigma}$ being an arbitrary correlation matrix, its entries must be approximated with the elements of a Green's matrix, $\mathbf{C}$, in order for $\mathbf{W} = \mathbf{C}^{-1}$ to be tridiagonal. Following the same procedure as in the extraction of the cumulative distribution function (cdf) in [13], it can be easily found that $E\left\langle r_1^{n_1}\ldots r_L^{n_L}\right\rangle$ can be expressed as in (12), shown at the bottom of the page, where $|\mathbf{W}|$ denotes the determinant of $\mathbf{W}$ and $(z)_k$ is the Pochhammer symbol defined as $(z)_k = \Gamma(z+k)/\Gamma(z)$. Note that (12) is a form of the generalized Lauricella function [15, p. 64]. For the important practical case of dual diversity, (12) can be expressed after some manipulations as

$$\begin{aligned}E\left\langle\gamma_1^{n_1}\gamma_2^{n_2}\right\rangle &= \frac{\overline{\gamma}_1^{n_1}\,\overline{\gamma}_2^{n_2}\Gamma\left(m+n_1\right)\Gamma\left(m+n_2\right)}{m^{n_1+n_2}\,\Gamma^2\left(m\right)} \\ &\quad\times{}_2F_1\left(-n_1,-n_2;m;\rho\right)\end{aligned} \tag{10}$$

which also follows from the related previously published result in [11], with ρ being the power correlation coefficient among

[1] [14, eq. (2)] should be written as $\mathbf{\Sigma}_{i,j} \equiv \rho^{|i-j|} = \operatorname{cov}\left(r_i^2, r_j^2\right)/\sqrt{\operatorname{var}\left(r_i^2\right)\operatorname{var}\left(r_j^2\right)}, 0 \leq \rho < 1$ and g_2 in the definition of G_n [after (7)], should be multiplied in the numerator instead of denominator.

$$\begin{aligned}E\left\langle r_1^{n_1}\ldots r_L^{n_L}\right\rangle = \frac{|\mathbf{W}|^m}{\left[\Gamma\left(m\right)\right]^L}\prod_{j=1}^{L}\frac{2^{n_j/2}\Gamma\left(m+\frac{n_j}{2}\right)}{w_{j,j}^{m+n_j/2}}\times\sum_{i_1,i_2,\ldots,i_{L-1}=0}^{\infty}&\left\{\frac{\left(m+\frac{n_1}{2}\right)_{i_1}\left(m+\frac{n_L}{2}\right)_{i_{L-1}}\left(m+\frac{n_2}{2}\right)_{i_1+i_2}\cdots\left(m+\frac{n_{L-1}}{2}\right)_{i_{L-2}+i_{L-1}}}{i_1!\ldots i_{L-1}!\left(m\right)_{i_1}\ldots\left(m\right)_{i_{L-1}}}\right. \\ &\left.\times\left[\frac{w_{1,2}^2}{w_{1,1}w_{2,2}}\right]^{i_1}\left[\frac{w_{2,3}^2}{w_{2,2}w_{3,3}}\right]^{i_2}\cdots\left[\frac{w_{L-1,L}^2}{w_{L-1,L-1}w_{L,L}}\right]^{i_{L-1}}\right\}\end{aligned} \tag{12}$$

the input paths. For $L=3$, (12) can be written in closed form as

$$E\left\langle r_1^{n_1}\, r_2^{n_2} r_3^{n_3}\right\rangle = \frac{|\mathbf{W}|^m}{[\Gamma(m)]^3}\prod_{j=1}^{3}\frac{2^{n_j/2}\Gamma\left(m+\frac{n_j}{2}\right)}{w_{j,j}^{m+n_j/2}} \times F_2\left[m+\frac{n_2}{2};\, m+\frac{n_1}{2},\, m+\frac{n_3}{2};\, m,\, m;\, x_1, x_2\right] \quad (11)$$

where $x_i = w_{i,i+1}^2/w_{i,i}w_{i+1,i+1}$ and $F_2[\alpha;\, \beta,\beta';\zeta,\zeta';\, x,\, y]$ is the hypergeometric function of two variables defined in [12, eq. (9.180)]. For the case of i.d. paths with exponential correlation [10, p. 324], $E\left\langle \gamma_1^{n_1}\, \gamma_2^{n_2}\, \gamma_3^{n_3}\right\rangle$ can be written as shown in (13) at the bottom of the page. It must be noted here that for the important practical case of dual EGC, all the moments can be expressed in closed form using (5) and (10).

C. Padé Approximants to the MGF of the Output SNR

The MGF of the EGC output SNR is, by definition

$$\mathcal{M}_{\gamma_{\rm out}}(s) = E\left\langle e^{s\gamma_{\rm out}}\right\rangle \quad (14)$$

and can be represented as a formal power series (e.g., Taylor) as

$$\mathcal{M}_{\gamma_{\rm out}}(s) = \sum_{n=0}^{\infty}\frac{1}{n!}E\left\langle \gamma_{\rm out}^n\right\rangle s^n = \sum_{n=0}^{\infty}\frac{\mu_n}{n!}s^n. \quad (15)$$

Although the moments of all orders, μ_n, are finite and can be evaluated in closed form using the analysis of Section II, in practice, only a finite number N can be used, truncating the series (15) as

$$\mathcal{M}_{\gamma_{\rm out}}(s) \cong \sum_{n=0}^{N}\frac{\mu_n}{n!}s^n + O\left(s^{N+1}\right) \quad (16)$$

with $O\left(s^{N+1}\right)$ being the remainder after the truncation. In many cases, we cannot conclude that the power series in (15) has a positive radius of convergence and where or whether it is convergent. Hence, we have to obtain the best approximation to the unknown underlying function $\mathcal{M}_{\gamma_{\rm out}}(s)$, evaluating only a finite number of the moments. This can be efficiently achieved using the Padé approximation method, which is already used in several scientific fields to approximate series as that in (15), where practically only few coefficients are known, and the series converges too slowly or diverges [16], [17]. Padé approximants method was also proposed in the past to approximate unknown pdfs and cdfs in radar analysis [17]–[19].

A Padé approximant is that rational function approximation of a specified order B for the denominator and A for the nominator, whose power series expansion agrees with the order power expansion of $\mathcal{M}_{\gamma_{\rm out}}(s)$ [16]. The rational function

$$R_{[A/B]}(s) \equiv \frac{\sum_{i=0}^{A} a_i s^i}{1+\sum_{i=1}^{B} b_i s^i} \quad (17)$$

is said to be a Padé approximant to the series (15), if

$$R_{[A/B]}(s) = \sum_{n=0}^{A+B}\frac{\mu_n}{n!}s^n + O\left(s^{N+1}\right). \quad (18)$$

Hence, the moments $\mu_n, n=1,\ldots,A+B$ need to be evaluated in order to construct the approximant $R_{[A/B]}(s)$. Next, $\mathcal{M}_{\gamma_{\rm out}}(s)$ will be approximated using subdiagonal ($R_{[A-1/A]}$) Padé approximants, since it is only for such approximants that the convergence rate and the uniqueness can be assured [19]. Several issues concerning approximants, such as methods to determine the two sets of coefficients $\{a_i\}$ and $\{b_i\}$, the so-called "order determination," i.e., the knowledge of the optimum upper value of A, a formula for the remainder $O\left(s^{N+1}\right)$, convergence analysis, etc., are included in [16], [17], and [19]. Padé approximants are available in most of the well-known mathematical software packets, such as MATHEMATICA, MATLAB, and MAPLE.

III. Performance Analysis

A. Average Output SNR

The first moment, μ_1, of the EGC output SNR, represents an important performance criterion of wireless communication systems operating over fading channels [10]. This criterion is the average output SNR, which is often measured at the output of the receiver. It is the easiest to evaluate and serves as an excellent indicator of the system's fidelity.

1) Independent Fading: The average combined SNR at the EGC output, for independent Nakagami-m fading, arbitrary number of branches L, arbitrary values for the fading severity parameter m, and unequal input branch SNRs can be obtained in a closed-form expression by setting $n=1$ in (8), yielding

$$\overline{\gamma}_{\rm out} = \frac{\Gamma^2\left(m+\frac{1}{2}\right)}{\Gamma^2(m)\,mL}\sum_{i,j=1}^{L}\sqrt{\overline{\gamma}_i\overline{\gamma}_j}. \quad (19)$$

Note that assuming a uniform power delay profile (i.e., $\overline{\gamma}_i = \overline{\gamma}$), it is easily verified that (8) reduces to an expression previously published in [3]. Setting $m=1$ in (19), the average output SNR for the important case of independent Rayleigh fading is derived as

$$\overline{\gamma}_{\rm out} = \frac{\pi}{4L}\sum_{i,j=1}^{L}\sqrt{\overline{\gamma}_i\,\overline{\gamma}_j}. \quad (20)$$

$$E\left\langle \gamma_1^{n_1}\gamma_2^{n_2}\gamma_3^{n_3}\right\rangle = \left(\frac{\overline{\gamma}\left(1-\rho^2\right)^m}{m}\right)^{n_1+n_2+n_3} \times \frac{\prod_{i=1}^{3}\Gamma(m+n_i)}{(1+\rho^2)^{m+n_2}\,\Gamma^3(m)} F_2\left[m+n_2; m+n_1, m+n_3; m, m; \frac{\rho^2}{\rho^2+1}, \frac{\rho^2}{\rho^2+1}\right] \quad (13)$$

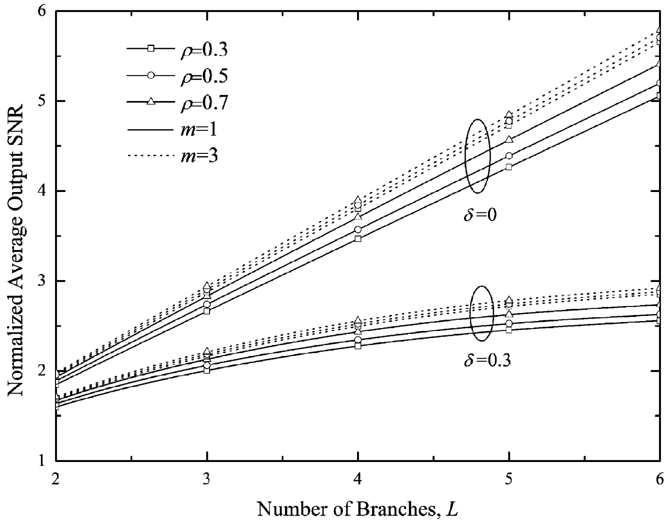

Fig. 1. Normalized average output SNR for exponentially decaying SNR delay profile and exponential correlation model.

2) Correlative Fading: Setting $n = 1$ in (5), it can be easily recognized that only terms of the form $E\left\langle \gamma_i^{1/2}\gamma_j^{1/2}\right\rangle$ appear in (5). Using (10) with $n_i = n_j = 1/2$ and [20, (15.3.3)], the average output SNR of a predetection EGC receiver with L correlated branches can be expressed as

$$\overline{\gamma}_{\text{out}}|_{\rho>0} = \frac{\Gamma^2\left(m+\frac{1}{2}\right)}{\Gamma^2(m)\,mL}\sum_{i,j=1}^{L}\sqrt{\overline{\gamma}_i\,\overline{\gamma}_j}\times {}_2F_1\left(-\frac{1}{2},-\frac{1}{2};m;\rho_{i,j}\right). \quad (21)$$

Equation (21) is a new and simple closed-form expression that can be directly used for an arbitrary number of branches L, arbitrary values of the fading severity parameter m, unequal branch powers, and arbitrary correlation of the diversity paths. For the Rayleigh fading case $m = 1$, it can be written as

$$\overline{\gamma}_{\text{out}}|_{\rho>0} = \frac{\pi}{4L}\sum_{i,j=1}^{L}\sqrt{\overline{\gamma}_i\overline{\gamma}_j}\;{}_2F_1\left(-\frac{1}{2},-\frac{1}{2};1;\rho_{i,j}\right) \quad (22)$$

while for uniform input SNR profile, it is simplified to

$$\overline{\gamma}_{\text{out}}|_{\rho>0} = \frac{\overline{\gamma}\Gamma^2\left(m+\frac{1}{2}\right)}{Lm\Gamma^2(m)}\sum_{i,j=1}^{L}{}_2F_1\left(-\frac{1}{2},-\frac{1}{2};m;\rho_{i,j}\right) \quad (23)$$

and for Rayleigh fading channels to

$$\overline{\gamma}_{\text{out}}|_{\rho>0} = \frac{\overline{\gamma}\pi}{4L}\sum_{i,j=1}^{L}{}_2F_1\left(-\frac{1}{2},-\frac{1}{2};1;\rho_{i,j}\right). \quad (24)$$

Note that when $\rho = 0$, (23) reduces to [10, eq. (9.48)]. To the best of the author's knowledge, (19)–(24) are new.

To illustrate the above mathematical analysis, Fig. 1 plots the normalized average output SNR as a function of the number of branches L, assuming an exponentially decaying SNR profile ($\overline{\gamma}_i = \overline{\gamma}_1 e^{-\delta(i-1)}, i = 1, 2 \ldots, L$) and an exponential correlation model between the diversity branches ($\rho_{i,j} = \rho^{|i-j|}$). Note that for $\delta = 0$, this model corresponds to the important scenario of multichannel reception from equispaced diversity antennas, in which the correlation between the pairs of combined signals decays as the spacing between the antennas increases [10, p. 327].

TABLE I
DEGREE OF PADÉ APPROXIMANTS FOR MATCHING THE ASEP OF BPSK AT THE SIXTH SIGNIFICANT DIGIT WITH SEVERAL PUBLISHED METHODS ($\gamma_1 = \gamma_2 = \gamma_3$)

	$L=2, m=1$		$L=3, \rho=0$	
SNR/symbol (dB)	$\rho = 0.3$ [8, eq. (15)]	$\rho = 0.6$ [8, eq. (15)]	$m = 1$ [2, eq. (21)]	$m = 2$ [2, eq. (21)]
0	[3/4]	[6/7]	[3/4]	[5/6]
5	[4/5]	[6/7]	[4/5]	[5/6]
10	[6/7]	[7/8]	[4/5]	[6/7]
15	[6/7]	[7/8]	[4/5]	[7/8]*

*Matching at the 9th significant digit

B. ASEP

Using the Padé approximations method presented in Section II-C, and the well-known MGF-based unified approach for the average error analysis of digital communications systems over generalized fading channels [10, Ch. 1], the ASEP can be evaluated directly for noncoherent and differential binary signaling (noncoherent binary frequency-shift keying, differential binary phase-shift keying), since for all the other cases (binary phase-shift keying (BPSK), M-ary PSK, M-ary quadrature amplitude modulation, M-ary differential PSK), single integrals with finite limits and integrands composed of elementary (exponential and trigonometric) functions have to be readily evaluated via numerical integration. Note that the Padé approximants approach can be efficiently used for arbitrary values of m and $\rho_{i,j}$, while the corresponding moments exist and are perfectly known in closed form.

In order to validate and to show the simplicity of the proposed approach, a comparison was made with already published results. Table I presents the degree of Padé approximants that need to be evaluated to match the ASEP at the sixth significant digit with previously published exact formulas, both for independent and correlated (dual diversity) fading and under the same values for the system's parameters. For the dual combining case, the results are compared with [8, Eq. (15)]. A mean number of 15 terms are used for the convergence of the infinite sum in [8, Eq. (15)] for $\rho = 0.3$ and 30 for $\rho = 0.7$. For $L = 3$, the results are compared with [2, Eq. (21)] for $m = 1$, and with [6, App. D] for $m = 2$. It is clear that Padé approximants of low order are needed to accurately approximate exact results. Moreover, this method works efficiently for arbitrary values for the system's parameters. Because of the absence in the literature of studies of ASEP for EGC receivers over correlated fading channels with $L > 2$, Fig. 2 plots the average BER performance of BPSK employing three and four branches EGC versus the average input SNR, for $m = 2$ and i.d. input paths with exponential correlation. While, to the best of the author' knowledge, such curves are presented for first time in the literature, Monte Carlo simulations were performed, and the results are also depicted in Fig. 2 for comparison purposes.

C. Outage Probability

If γ_{th} is a certain specified threshold ratio, then for noise-limited systems, outage probability is defined as the probability

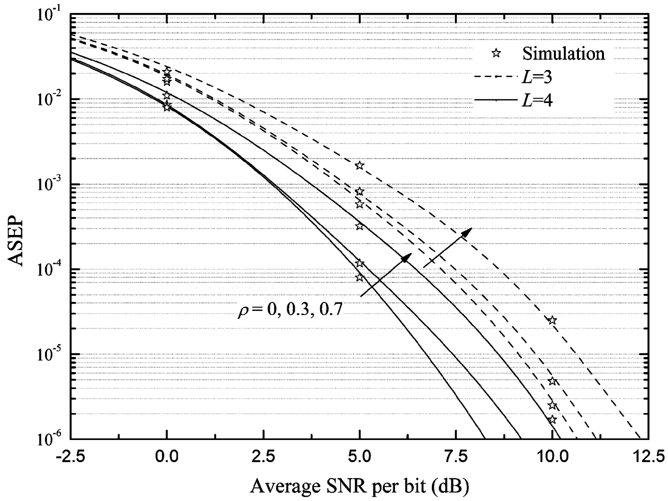

Fig. 2. Error probability of BPSK for EGC with three and four exponentially correlated i.d. branches with $m = 2$.

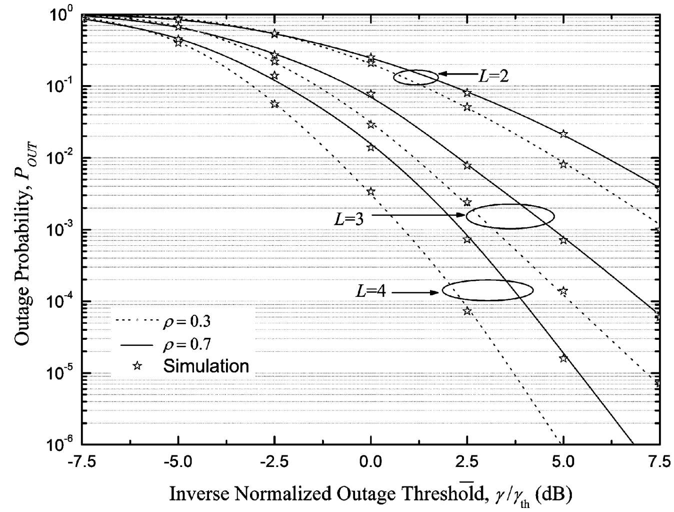

Fig. 3. Outage probability versus the inverse normalized outage threshold, for EGC with two, three, and four exponentially correlated i.d. branches with $m = 2$.

that the instantaneous EGC output SNR $\gamma_{\rm out}$ falls below $\gamma_{\rm th}$ and is expressed as [10, Ch. 1]

$$P_{\rm out} = F_{\gamma_{\rm out}}(\gamma_{\rm th}) = \mathcal{L}^{-1}\left(\frac{\mathcal{M}_{\gamma_{\rm out}}(-s)}{s}\right)\Bigg|_{\gamma_{\rm th}} \tag{25}$$

where $F_{\gamma_{\rm out}}(\gamma)$ and $\mathcal{L}^{-1}(.)$ denote the cdf of the EGC output SNR and the inverse Laplace transform, respectively. Due to the rational form of $\mathcal{M}_{\gamma_{\rm out}}(s)$

$$\mathcal{M}_{\gamma_{\rm out}}(s) \cong \frac{\sum_{i=0}^{A} a_i s^i}{1 + \sum_{i=1}^{B} b_i s^i} = \sum_{i=1}^{B} \frac{\lambda_i}{s + p_i} \tag{26}$$

and using the residue inversion formula, the outage probability can be easily evaluated from (25) as

$$P_{\rm out} = 1 - \sum_{i=1}^{B} \frac{\lambda_i}{p_i} e^{p_i \gamma_{\rm th}} \tag{27}$$

where p_i are the poles of the Padé approximant to $\mathcal{M}_{\gamma_{\rm out}}(s)$, which must have a negative real part, and λ_i are the residues [21]. More about the approximation of pdfs and cdfs using Padé approximants can be found in [17] and [19].

In Fig. 3, the outage probability is plotted versus the inverse normalized outage threshold, $\overline{\gamma}/\gamma_{\rm th}$, for two, three, and four branches EGC over i.d. exponentially correlated Nakagami-m fading channels with $m = 2$. The negative impact of the fading correlation to the outage performance is evident.

IV. Conclusions

This letter presents an alternative moments-based approach to the performance analysis of EGC receivers over independent and correlated Nakagami-m fading channels. The moments of the EGC output SNR were extracted in simple closed form, and the corresponding MGF was approximated with the use of the Padé approximants theory. These results were used to study important performance criteria, such as the average output SNR, the outage probability, and the ASEP for several coherent, noncoherent, and multilevel modulation schemes. Especially for the case of EGC receivers over correlated Nakagami-m fading channels, where the literature is very poor, this letter gives an efficient and simple tool for the their performance analysis.

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