

# Energy Detection in Full-Duplex Systems with Residual RF Impairments over Fading Channels

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**Abstract**—We study the impact of residual radio frequency (RF) impairments and fading on the spectrum sensing performance of a classical energy detector (ED), which is employed in a full-duplex wireless system. Specifically, we present novel closed-form expressions for the false-alarm and detection probabilities, assuming Nakagami- $m$  fading, and the impact of residual RF impairments. The results reveal the importance of taking into account the wireless environment and the ED’s capabilities, when evaluating the ED performance and setting its threshold.

**Index Terms**—Fading channels, Full-duplex, Spectrum sensing.

## I. INTRODUCTION

**S**ENSING of unoccupied frequency bands is of high interest in several emerging wireless systems and techniques, such as cognitive radio (CR), carrier aggregation (CA), and ultra-wide band [1], [2]. As a result, great amount of effort has been put to derive spectrum sensing solutions and investigate their performance [3]–[7]. These solutions require the use of flexible and re-configurable transceivers, while still the users ask for low-cost and low-power consumption devices. In this context, the direct conversion architecture provides an attractive front-end (FE) solution. However, direct-conversion transceivers are typically sensitive to radio frequency (RF) FE imperfections, such as in-phase and quadrature imbalance (IQI), phase noise (PHN), and non-linearities, which can significantly degrade the performance of the wireless system.

In spite of the paramount importance of the RF FE impairments on the spectrum sensing performance, their effect was overlooked in the vast majority of the analysis. In more detail, in [8], the authors quantified the effect of IQI in the spectrum sensing performance of full-duplex (FD) CR networks (i.e., the CR device transmit and sense the spectrum simultaneously), assuming deterministic wireless channels. Finally, in [9], the authors studied the impact of IQI in half-duplex (HD) CRs, assuming additive white Gaussian channels and zero-mean complex Gaussian (ZMCG) primary user transmitted signal.

To the best of the authors’ knowledge, the joint effect of fading, residual RF FE impairments, and partial self-interference suppression (SIS) have not been addressed in the open literature. Motivated by this, this letter is devoted in investigating the joint impact of fading and residual RF FE impairments in FD wireless systems, which suffer from partial SIS. In particular, we present the signal model that describes the joint effects of residual RF FE impairments and partial SIS, assuming that the classical ED is employed. Based on this

model, novel closed-form expressions for the quantification of the effect of residual RF FE impairments, partial SIS and fading on the spectrum sensing performance of the classical ED are provided. Specifically, assuming Nakagami- $m$  fading channels and ZMCG transmitted signals, we evaluate the detection and false-alarm probabilities in closed-form.

**Notations:** Operators  $E[\cdot]$  and  $\exp(x)$  denote the statistical expectation and the exponential function, respectively, while  $\Re\{x\}$  and  $\Im\{x\}$  represent the real and imaginary part of  $x$ . The lower [10, Eq. (8.350/1)] and upper incomplete Gamma functions [10, Eq. (8.350/2)] are represented by  $\gamma(\cdot, \cdot)$  and  $\Gamma(\cdot, \cdot)$ , respectively, while the Gamma function [10, Eq. (8.310)] is denoted by  $\Gamma(\cdot)$ . Finally,  $\Gamma(a, x, b, \beta) = \int_x^\infty t^{a-1} \exp(-t - bt^{-\beta}) dt$  is the extended incomplete Gamma function defined in [11, Eq. (6.2)].

## II. SYSTEM AND SIGNAL MODEL

We assume that the secondary users (SUs) operate in FD mode, inside the coverage area of the primary user (PU). Each SU device is considered to suffer from transmit and receive RF impairments. Moreover, it is assumed that each SU has partial SIS capability, measured by the degree of SIS ( $a \in [0, 1]$ ).

The two hypotheses, namely absence/presence of PU signal, is denoted by the parameter  $\theta \in \{0, 1\}$ . Suppose the  $n$ -th sample of the PU signal,  $x(n)$ , is conveyed over a flat fading wireless channel, with baseband equivalent channel gain,  $h(n)$ , and additive noise,  $w(n)$ . Moreover, we assume that the SU transmitted signal,  $s(n)$ , is conveyed over a flat fading wireless channel, with base-band equivalent channel gain,  $h_s(n)$ . Hence, the  $n$ -th sample of the baseband equivalent received signal at the SU can be obtained as

$$y(n) = \theta h_i(n) (x(n) + \eta_{\text{PU}}(n)) + \tilde{\theta} a h_s(n) (s(n) + \eta_{\text{SU}}^t(n)) + \eta_{\text{SU}}^r(n) + w(n), \quad (1)$$

where  $\tilde{\theta} \in \{0, 1\}$  denotes the absence/presence of transmitted signal from the SU, whereas  $\eta_{\text{PU}}(n)$ ,  $\eta_{\text{SU}}^t(n)$  and  $\eta_{\text{SU}}^r(n)$  represent the distortion due to the residual RF FE impairments of the PU, the SU transmitter, and receiver, respectively. Note that for given channel realizations, these parameters can be modeled as independent ZMCG processes with variances

$$\sigma_{\eta_{\text{PU}}}^2 = \kappa_{\text{PU}}^2 \frac{\sigma_x^2}{2}, \quad \sigma_{\eta_{\text{SU}}^t}^2 = (\kappa_{\text{SU}}^t)^2 \frac{\sigma_s^2}{2}, \quad (2)$$

$$\sigma_{\eta_{\text{SU}}^r}^2 = (\kappa_{\text{SU}}^r)^2 \left( |h_i(n)|^2 \frac{\sigma_x^2}{2} + |h_s(n)|^2 \frac{\sigma_s^2}{2} \right), \quad (3)$$

respectively, where the design parameters  $\kappa_{\text{BS}}, \kappa_i \geq 0$  characterize the level of impairments in the BS and UE, respectively [12]. These parameters are interpreted as the error

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vector magnitudes (EVMs), which is specified in the range of  $[0.08, 0.175]$  [13, Sec. 14.3.4].

### III. FALSE-ALARM AND DETECTION PROBABILITIES

In a classical ED, the energy of the received signal is used to determine whether a channel is idle or busy. In particular, the ED calculates the test statistics for the channel as

$$T = \frac{1}{N_s} \sum_{m=0}^{N_s-1} |y(n)|^2, \quad (4)$$

where  $N_s$  denotes the number of the complex samples used for sensing. This test statistics are compared against a threshold,  $\gamma$ , to yield the sensing decision, i.e., if  $T \geq \gamma$ , the ED decides that the channel is busy, or idle, otherwise.

Based on (1)-(3), and taking into account that  $E[\Re\{y\}\Im\{y\}] = 0$ , as well as that

$$\sigma^2 = E[\Re\{y\}^2] = E[\Im\{y\}^2] = \theta A_1 |h_i|^2 + \tilde{\theta} A_2 |h_s|^2 + A_3, \quad (5)$$

where

$$A_1 = \left(1 + \kappa_{\text{PU}}^2 + (\kappa_{\text{SU}}^r)^2\right) \frac{\sigma_x^2}{2}, \quad (6)$$

$$A_2 = a^2 \left(1 + (\kappa_{\text{SU}}^t)^2 + (\kappa_{\text{SU}}^r)^2\right) \frac{\sigma_s^2}{2}, \quad (7)$$

$$A_3 = \frac{\sigma_w^2}{2}, \quad (8)$$

we observe that, for a given channel realization,  $H = \{h_i, h_s\}$  and channel occupancy set,  $\Theta = \{\theta, \tilde{\theta}\}$ , the received energy follows chi-square distribution with  $2N_s$  degrees of freedom. Note that, since  $N_s$  is an integer, according to [10, Eq. (8.352/3)], the cumulative distribution function (CDF) of  $T_i$  can be obtained as

$$F_{T_i}(y|H, \Theta) = 1 - \sum_{k=0}^{N_s-1} \frac{1}{k!} \left(\frac{N_s y}{2\sigma^2}\right)^k \exp\left(-\frac{N_s y}{2\sigma^2}\right). \quad (9)$$

**Theorem 1.** *The CDF of the test statistics can be obtained as*

$$F_{T_i}(y|\Theta_{00}) = 1 - \sum_{k=0}^{N_s-1} \frac{1}{k!} \left(\frac{N_s x}{2A_3}\right)^k \exp\left(-\frac{N_s y}{2A_3}\right), \quad (10)$$

for  $\Theta = \Theta_{00} = [0, 0]$ ,

$$F_{T_i}(y|\Theta_{01}) = 1 - \frac{1}{(m_s - 1)!} \exp\left(\frac{m_s A_3}{A_2 \sigma_{h_s}^2}\right) \times \sum_{k=0}^{N_s-1} \sum_{l=0}^{m_s-1} \frac{1}{k!} \binom{m_s-1}{l} (-A_3)^{m_s-1-l} \left(\frac{m_s}{A_2 \sigma_{h_s}^2}\right)^{m_s+k-l-1} \times \left(\frac{N_s}{2}\right)^k x^k \Gamma\left(l-k+1, \frac{m_s A_3}{\sigma_{h_s}^2}, \frac{N_s m_s x}{2A_2 \sigma_{h_s}^2}, 1\right), \quad (11)$$

for  $\Theta = \Theta_{01} = [0, 1]$ ,

$$F_{T_i}(y|\Theta_{10}) = 1 - \frac{1}{(m_i - 1)!} \exp\left(\frac{m_i A_3}{A_1 \sigma_{h_i}^2}\right) \times \sum_{k=0}^{N_s-1} \sum_{l=0}^{m_i-1} \frac{1}{k!} \binom{m_i-1}{l} (-A_3)^{m_i-1-l} \left(\frac{m_i}{A_1 \sigma_{h_i}^2}\right)^{m_i+k-l-1} \times \left(\frac{N_s}{2}\right)^k x^k \Gamma\left(l-k+1, \frac{m_i A_3}{\sigma_{h_i}^2}, \frac{N_s m_i x}{2A_1 \sigma_{h_i}^2}, 1\right), \quad (12)$$

for  $\Theta = \Theta_{10} = [1, 0]$ , and

$$F_{T_i}(y|\Theta_{11}) = 1 - \sum_{p=0}^{N_s-1} \sum_{l=1}^2 \sum_{k=1}^{a_l} \sum_{n=0}^{k-1} \binom{k-1}{n} \frac{1}{p!} \times (-A_3)^{k-1-n} \frac{N_s^p \Xi_{l,k}}{2^p b_l^{k+p-n-1} (k-1)!} x^p \exp\left(\frac{A_3}{b_l}\right) \times \Gamma\left(n-p+1, \frac{A_3}{b_l}, \frac{N_s}{2b_l} x, 1\right), \quad (13)$$

for  $\Theta = \Theta_{11} = [1, 1]$ . Note that in (13),

$$a = \{a_1, a_2\} = \{m_i, m_s\} \quad (14)$$

and

$$b = \{b_1, b_2\} = \left\{ \frac{A_1 \sigma_{h_i}^2}{2m_1}, \frac{A_2 \sigma_{h_s}^2}{2m_s} \right\}, \quad (15)$$

whereas  $\Xi_{l,k}$  can be obtained as in [14, Eqs. (8) and (9)].

*Proof:* Please refer to Appendix. ■

Based on Theorem 1, the detection and false-alarm probabilities can be respectively evaluated as

$$P_d(\gamma) = P_r(T_i \geq \gamma | \theta = 1) = P_r(\Theta_{11})(1 - F_{T_i}(y|\Theta_{11})) + P_r(\Theta_{10})(1 - F_{T_i}(y|\Theta_{10})), \quad (16)$$

$$P_{fa}(\gamma) = P_r(T_i \geq \gamma | \theta = 0) = P_r(\Theta_{01})(1 - F_{T_i}(y|\Theta_{01})) + P_r(\Theta_{00})(1 - F_{T_i}(y|\Theta_{00})), \quad (17)$$

where  $P_r(\Theta)$  represents the probability of the occupancy set,  $\Theta$ .

**Special case 1 (Rayleigh fading):** In the special case, where  $|h_i|$  and  $|h_s|$  are Rayleigh distributed, the CDF of the energy test statistics for a given channel occupancy set  $\Theta \neq \Theta_{00}$  can be obtained by setting  $m_i = m_s = 1$  into (11)-(13) as

$$F_{T_i}(y|\Theta_{01}) = 1 - \exp\left(\frac{m_s A_3}{A_2 \sigma_{h_s}^2}\right) \times \sum_{k=0}^{N_s-1} \frac{1}{k!} \left(\frac{N_s}{2A_2 \sigma_{h_s}^2}\right)^k x^k \Gamma\left(1-k, \frac{A_3}{\sigma_{h_s}^2}, \frac{N_s x}{2A_2 \sigma_{h_s}^2}, 1\right), \quad (18)$$

$$F_{T_i}(y|\Theta_{10}) = 1 - \exp\left(\frac{m_s A_3}{A_1 \sigma_{h_s}^2}\right) \times \sum_{k=0}^{N_s-1} \frac{1}{k!} \left(\frac{N_s}{2A_1 \sigma_{h_s}^2}\right)^k x^k \Gamma\left(1-k, \frac{A_3}{\sigma_{h_s}^2}, \frac{N_s x}{2A_1 \sigma_{h_s}^2}, 1\right), \quad (19)$$

and

$$F_{T_i}(y|\Theta_{11}) = 1 - \sum_{p=0}^{N_s-1} \sum_{l=1}^2 \frac{1}{p!} \frac{N_s^p \Xi_{l,1}}{2^p b_l^p} x^p \exp\left(\frac{A_3}{b_l}\right) \times \Gamma\left(1-p, \frac{A_3}{b_l}, \frac{N_s}{2b_l} x, 1\right), \quad (20)$$

whereas, for  $\Theta = \Theta_{00}$ , the CDF of the energy test statistics can be obtained as in (10). By substituting (10) and (18) into (17), we derive the false-alarm probability, when the coefficients of the PU-SU and the self-interference channels are assumed to be Rayleigh distributed, whereas by substituting (19) and (20) into (16), we obtained the corresponding detection probability.

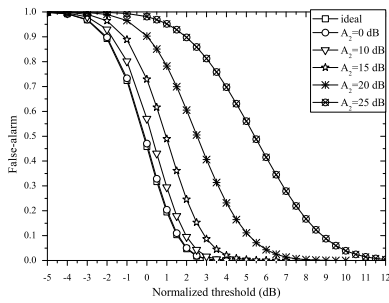


Fig. 1:  $P_{fa}$  against  $\gamma/\sigma_w^2$  for different values of  $A_2$ .

**Special case 2 (HD system):** In the case in which the spectrum sensing device operate in HD mode,  $\tilde{\theta} = 0$ ; hence, the detection and false-alarm probabilities can be respectively expressed as

$$P_d(\gamma) = 1 - F_{T_i}(y|\Theta_{10}) \text{ and } P_{fa}(\gamma) = 1 - F_{T_i}(y|\Theta_{00}). \quad (21)$$

Note that these expressions can also be used to quantify the spectrum sensing capability of a spectrum sensing device, which operates in FD mode with complete SIS.

**Special case 3 (Ideal):** In the case of ideal PU's and SU's RF FE, as well as complete SIS, the detection and false-alarm probabilities can be expressed as (21), by setting  $\kappa_{PU} = \kappa_{SU}^r = \kappa_{SU}^t = 0$ , or equivalently  $A_1 = \sigma_x^2/2$ , and  $A_2 = 0$ .

#### IV. NUMERICAL RESULTS AND DISCUSSION

This section is devoted in illustrating the impact of fading, residual RF FE impairments, and partial SIS on the spectrum sensing performance of the ED. For all figures,  $N_s = 10$ , whereas, without loss of generality, it is assumed that  $\sigma_{h_i}^2 = \sigma_{h_s}^2 = \sigma_w^2 = 1$ . Additionally, note that in the following illustrations, the solid curves represent analytical values obtained through the derived formulas, while the markers represent simulation results.

In Fig. 1, the false-alarm probability is demonstrated as a function of the normalized threshold,  $\gamma/\sigma_w^2$ , for different values of  $A_2$ , when  $m_s = 2$ ,  $m_i = 1$ ,  $\kappa_{PU} = 0.12$ ,  $\sigma_x^2/\sigma_w^2 = 0$  dB, and  $P_r(\Theta_{00}) = 1 - P_r(\Theta_{01}) = 0$ . Note that based on (7),  $A_2$  stands for the joint effect of the SU's imperfections and its partial SIS capability. As expected for a given  $A_2$ , as the normalized threshold increases, the false-alarm probability decreased, while, for a fixed normalized threshold, as  $A_2$  increases, the false-alarm probability also increases. For instance, for normalized threshold equals 2.5 dB, the false-alarm probability is 0.25 and 0.5, for  $A_2 = 15$  dB and  $A_2 = 20$  dB, respectively. Additionally, from this figure we observe that for a given false-alarm probability requirement, as  $A_2$  increases, the normalized threshold should be increased. For example, if the false-alarm probability is set to 0.1, for  $A_2 = -10$  dB, the normalized threshold should be set to 1.95 dB, whereas, for the same false-alarm probability requirement and  $A_2 = -10$  dB, the normalized threshold should be set to 13.5 dB. This indicates the importance of taking into consideration the joint effect of the SU's transceivers imperfections and its partial SIS capability.

Fig. 2 depicts the ROCs for different values of  $a$  and  $\sigma_s^2/\sigma_w^2$ , when  $m_i = m_s = 1$ ,  $\kappa_{PU} = 0.17$ ,  $\kappa_{SU}^t = 0.11$ ,  $\kappa_{SU}^r =$

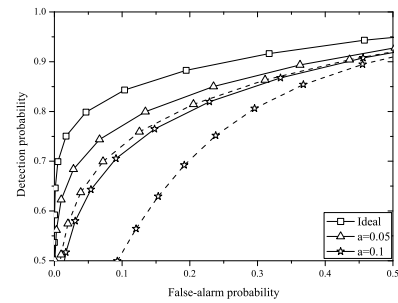


Fig. 2: ROCs for different values of  $a$ , when  $\sigma_s^2/\sigma_w^2 = 15$  dB (continuous lines) and  $\sigma_s^2/\sigma_w^2 = 20$  dB (dashed lines).

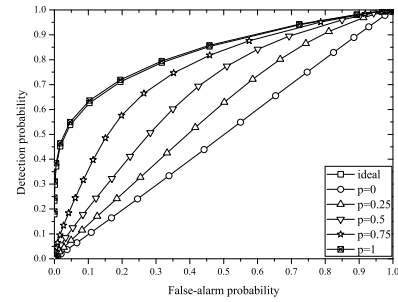


Fig. 3: ROCs for different values of  $p$ .

0.12,  $\sigma_x^2/\sigma_w^2 = 5$  dB, and  $P_r(\Theta_{00}) = P_r(\Theta_{01}) = P_r(\Theta_{10}) = P_r(\Theta_{11}) = 0.5$ . For a fixed  $\sigma_s^2/\sigma_w^2$ , as  $a$  increases, the leakage from the transmitter to the ED increases; hence, the spectrum sensing capability of the ED is constrained. For instance, in the ideal case, for false-alarm probability equals 0.1, the detection probability is 0.84, whereas, in the non-ideal case in which  $\sigma_s^2/\sigma_w^2 = 15$  dB, the detection probabilities are respectively 0.77, for  $a = 0.05$ , and 0.71, for  $a = 0.1$ . Similarly, for a fixed  $a$ , as  $\sigma_s^2/\sigma_w^2$  increases, the level of self-interference also increase; therefore, the spectrum sensing capability of the ED decreases. This indicates that the selection of the transmission parameters as well as the SIS capabilities of the spectrum sensing device affect the capabilities of the ED.

Fig. 3 demonstrates the ROCs for different values  $p = P_r(\Theta_{00}) = P_r(\Theta_{10}) = 1 - P_r(\Theta_{01}) = 1 - P_r(\Theta_{11})$ , when  $m_i = 1$ ,  $m_s = 3$ ,  $a = 0.1$ ,  $\kappa_{PU} = 0.12$ ,  $\kappa_{SU}^t = 0.14$ ,  $\kappa_{SU}^r = 0.17$ ,  $\sigma_x^2/\sigma_w^2 = 0$  dB, and  $\sigma_s^2/\sigma_w^2 = 20$  dB. Note that  $p$  denotes the probability that the SU's transmitter is idle. From this figure, it is evident that as  $p$  decreases, the probability of SU's transmission interference increases; hence, the spectrum sensing capability of the ED is constrained. Interestingly, for the special case in which  $p = 1$ , i.e., the spectrum sensing device operates in HD mode, the spectrum sensing performance of the ED outperforms even the performance of the corresponding ideal ED. This is due to the fact that, in the absence of SU transmission, the residual RF FE impairments of the PU transmitter and the SU's receiver cause an increase of the power at the ED, which is described by (6). In other words, residual RF FE impairments can positively affect the spectrum sensing performance of the HD system.

#### V. CONCLUSIONS

This paper investigated the joint effect of residual RF impairments, fading, and partial SIS, in the spectrum sens-

ing capability of a FD system. The results highlighted the importance of taking into consideration the joint effect of the SU's transceivers imperfections and its partial SIS capability in order to appropriately select the energy detection threshold. Finally, the degradation, due to the imperfections and partial SIS, as well as due to the uncertainty of the transmission state of the SU was quantified and demonstrated.

#### APPENDIX A

##### PROOF OF THEOREM 1

For  $\Theta = \Theta_{00} = [0, 0]$ , according to (5),  $\sigma^2 = A_3$ . By substituting this expression into (9), we get (10).

For  $\Theta = \Theta_{01} = [0, 1]$ ,

$$\sigma_{\Theta_{01}}^2 = A_2|h_s|^2 + A_3. \quad (22)$$

Since  $|h_s|$  follows Nakagami- $m$  distribution,  $|h_s|^2$  follows Gamma distribution; thus,  $\sigma_{\Theta_{01}}^2$  also follows Gamma distribution with probability density function (PDF) obtained as

$$f_{\sigma_{\Theta_{01}}^2}(y) = \frac{1}{\Gamma(m_s)} \left( \frac{m_s}{A_2\sigma_{h_s}^2} \right)^{m_s} (y - A_2)^{m_s-1} \times \exp\left(\frac{m_s A_3}{A_2\sigma_{h_s}^2}\right) \exp\left(-\frac{m_s}{A_2\sigma_{h_s}^2}y\right), \text{ for } y \geq A_3. \quad (23)$$

Moreover, assuming that  $m_s$  is an integer and after employing the binomial expansion, (23) can be rewritten as

$$f_{\sigma_{\Theta_{01}}^2}(y) = \frac{1}{(m_s - 1)!} \left( \frac{m_s}{A_2\sigma_{h_s}^2} \right)^{m_s} \exp\left(\frac{m_s A_3}{A_2\sigma_{h_s}^2}\right) \times \sum_{k=0}^{m_s-1} \binom{m_s-1}{k} (-A_3)^{m_s-1-k} y^k \exp\left(-\frac{m_s}{\sigma_{h_s}^2 A_2}y\right). \quad (24)$$

Thus, for  $\Theta = \Theta_{01}$ , the unconditional CDF can be obtained as

$$F_{T_i}(y|\Theta_{01}) = \int_{A_3}^{\infty} \left( 1 - \sum_{k=0}^{N_s-1} \frac{1}{k!} \left( \frac{N_s x}{2y} \right)^k \exp\left(-\frac{N_s x}{2y}\right) \right) \times f_{\sigma_{\Theta_{01}}^2}(y) dy, \quad (25)$$

which, after some mathematical manipulations, can equivalently be written as

$$F_{T_i}(y|\Theta_{01}) = 1 - \frac{1}{(m_s - 1)!} \left( \frac{m_s}{A_2\sigma_{h_s}^2} \right)^{m_s} \exp\left(\frac{m_s A_3}{A_2\sigma_{h_s}^2}\right) \times \sum_{k=0}^{N_s-1} \sum_{l=0}^{m_s-1} \frac{1}{k!} \binom{m_s-1}{l} (-A_3)^{m_s-1-l} \left( \frac{N_s x}{2} \right)^k \mathcal{I}(x), \quad (26)$$

where  $\mathcal{I}(x) = \int_{A_3}^{\infty} y^{l-k} \exp\left(-\frac{N_s}{2}xy^{-1} - \frac{m_s}{\sigma_{h_s}^2 A_2}y\right) dy$ , which by setting  $z = \frac{m}{\sigma_{h_s}^2 A_2}y$ , can be rewritten as

$$\mathcal{I}(x) = \left( \frac{A_2\sigma_{h_s}^2}{m_s} \right)^{l-k+1} \Gamma\left(l - k + 1, \frac{m_s A_3}{\sigma_{h_s}^2 A_2}, \frac{N_s m_s x}{2A_2\sigma_{h_s}^2}, 1\right). \quad (27)$$

By substituting (27) into (26), we obtain (11).

For  $\Theta = \Theta_{10} = [1, 0]$ ,  $\sigma_{\Theta_{10}}^2 = A_1|h_i|^2 + A_3$ , i.e., it has the same form as  $\sigma_{\Theta_{01}}^2$ , given by (22). Hence, following the same steps as above, we get (12).

For  $\Theta = \Theta_{11} = [1, 1]$ ,  $\sigma_{\Theta_{11}}^2 = A_1|h_i|^2 + A_2|h_s|^2 + A_3$ , follows squared Nakagami- $m$  distribution with PDF obtained as

$$f_{\sigma_{\Theta_{11}}^2}(y) = \sum_{l=1}^2 \sum_{k=1}^{a_l} \Xi_{l,k} \frac{(y - A_3)^{k-1}}{b_l^k (k-1)!} \exp\left(-\frac{y - A_3}{b_l}\right). \quad (28)$$

By employing the binomial expansion (28), we get

$$f_{\sigma_{\Theta_{11}}^2}(y) = \sum_{l=1}^2 \sum_{k=1}^{a_l} \sum_{n=0}^{k-1} \binom{k-1}{n} (-A_3)^{k-1-n} \frac{\Xi_{l,k}}{b_l^k (k-1)!} \times \exp\left(\frac{A_3}{b_l}\right) y^n \exp\left(-\frac{y}{b_l}\right). \quad (29)$$

Hence, the unconditional CDF can be evaluated as

$$F_{T_i}(y|\Theta_{11}) = 1 - \sum_{p=0}^{N_s-1} \sum_{l=1}^2 \sum_{k=1}^{a_l} \sum_{n=0}^{k-1} \binom{k-1}{n} \frac{1}{p!} \times (-A_3)^{k-1-n} \frac{N_s^p \Xi_{l,k}}{2^p b_l^k (k-1)!} x^p \exp\left(\frac{A_3}{b_l}\right) \mathcal{J}(x). \quad (30)$$

where  $\mathcal{J}(x) = \int_{A_3}^{\infty} y^{n-p} \exp\left(-\frac{N_s}{2}xy^{-1} - \frac{1}{b_l}y\right) dy$ , or

$$\mathcal{J}(x) = b_l^{n-p+1} \Gamma\left(n - p + 1, \frac{A_3}{b_l}, \frac{N_s}{2b_l}x, 1\right). \quad (31)$$

By substituting (31) into (30), we obtain (13).

#### REFERENCES

- [1] FCC, "Spectrum policy task force report," Nov. 2002.
- [2] D. Porcino and W. Hirt, "Ultra-wideband radio technology: Potential and challenges ahead," *IEEE Commun. Mag.*, vol. 41, no. 7, pp. 66–74, Jul. 2003.
- [3] L. Fan, X. Lei, T. Q. Duong, R. Q. Hu, and M. Elkashlan, "Multiuser cognitive relay networks: Joint impact of direct and relay communications," *IEEE Trans. Wireless Commun.*, vol. 13, no. 9, pp. 5043–5055, Sep. 2014.
- [4] T. Duong, P. L. Yeoh, V. N. Q. Bao, M. Elkashlan, and N. Yang, "Cognitive relay networks with multiple primary transceivers under spectrum-sharing," *IEEE Signal Process. Lett.*, vol. 19, no. 11, pp. 741–744, Nov. 2012.
- [5] Y. Liu, L. Wang, T. T. Duy, M. Elkashlan, and T. Duong, "Relay selection for security enhancement in cognitive relay networks," *IEEE Wireless Commun. Lett.*, vol. 4, no. 1, pp. 46–49, Feb. 2015.
- [6] C. Zhong and T. Ratnarajah, "Performance of user selection in cognitive broadcast channels," *IEEE Trans. Commun.*, vol. 60, no. 12, pp. 3529–3534, Dec. 2012.
- [7] S. Dikmese, S. Srinivasan, M. Shaat, F. Bader, and M. Renfors, "Spectrum sensing and resource allocation for multicarrier cognitive radio systems under interference and power constraints," *EURASIP Journal on Advances in Signal Processing*, vol. 2014, no. 1, p. 68, May 2014.
- [8] A.-A. A. Boulogeorgos, H. B. Salameh, and G. Karagiannidis, "Spectrum sensing in full-duplex cognitive radio networks under hardware imperfections," *IEEE Trans. Veh. Technol.*, vol. 66, no. 3, pp. 2072–2084, Mar. 2016.
- [9] A. Gokceoglu, S. Dikmese, M. Valkama, and M. Renfors, "Energy detection under IQ imbalance with single- and multi-channel direct-conversion receiver: Analysis and mitigation," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 3, pp. 411–424, Mar. 2014.
- [10] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. New York: Academic, 2000.
- [11] A. Chaudhry and S. Zubair, *On a Class of Incomplete Gamma Functions with Applications*. CRC Press, 2001.
- [12] E. Bjornson, M. Matthaiou, and M. Debbah, "A new look at dual-hop relaying: Performance limits with hardware impairments," *IEEE Trans. Commun.*, vol. 61, no. 11, pp. 4512–4525, Nov. 2013.
- [13] H. Holma and A. Toskala, *LTE for UMTS: Evolution to LTE-advanced*, 2nd ed. John Wiley & Sons, 2011.
- [14] G. K. Karagiannidis, N. C. Sagiias, and T. A. Tsiftsis, "Closed-form statistics for the sum of squared Nakagami- $m$  variates and its applications," *IEEE Trans. Commun.*, vol. 54, pp. 1353–1359, Aug. 2006.