

# Performance Analysis of Dual Selection Diversity in Correlated Weibull Fading Channels

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**Abstract**—Ascertaining the importance of the dual selection combining (SC) receivers and the suitability of the Weibull model to describe mobile fading channels, we study the performance of a dual SC receiver over correlated Weibull fading channels with arbitrary parameters. Exact closed-form expressions are derived for the probability density function, the cumulative distribution function, and the moments of the output signal-to-noise ratio (SNR). Important performance criteria, such as average output SNR, amount of fading, outage probability, and average bit-error probability for several modulation schemes are studied. Furthermore, for these performance criteria, novel closed-form analytical expressions are derived. The proposed analysis is complemented by various performance evaluation results, including the effects of the input SNR's unbalancing, fading severity, and fading correlation on the overall system's performance. Computer simulation results have verified the validity and accuracy of the proposed analysis.

**Index Terms**—Amount of fading (AoF), bit-error rate (BER), correlated fading, outage probability, selection combining (SC), Weibull fading channels.

## I. INTRODUCTION

ONE OF THE simplest and yet most efficient techniques to overcome the destructive effects of fading in wireless communication systems is diversity. For all diversity techniques, including equal gain combining (EGC), maximal ratio combining (MRC), selection combining (SC), and a combination of MRC and SC, referred to as generalized selection combining (GSC) [1], the receiver processes the obtained diversity signals in a fashion that maximizes the system's power efficiency. Among these diversity techniques, SC is the least complicated, since the processing is performed only on one of the diversity branches. Traditionally, in SC, the combiner chooses the branch with the highest signal-to-noise ratio (SNR), which corresponds to the strongest signal, if equal noise power is assumed among the different branches [1].

The performance of diversity receivers has been studied extensively in the literature for several well-known fading statistical models, such as Rayleigh, Rice, Nakagami- $m$  and Nakagami- $q$ , for both independent and correlative fading [1]. Past work concerning the performance of dual diversity receivers with correlative fading can be found in many publications, including [2]–[7]. For example, Malik *et al.* [2] presented an efficient approach in analyzing the performance of coherent detection for binary signals with dual diversity in correlative Rayleigh fading. More recently, Karagiannidis *et al.* [3] derived a convergent infinite sum expression for the characteristic function of two correlated Nakagami- $m$  variables and extended the results of [2] to the Nakagami- $m$  fading case. In [4], useful expressions for the outage probability and average bit-error probability (ABEP) were presented for a dual selection diversity system with correlated slow Rayleigh and Nakagami- $m$  fading, while in [5], the average output SNR was evaluated. A study of dual MRC and SC receivers over correlated Rayleigh channels is presented in [6]. Finally, in [7], the average output SNR, the amount of fading (AoF), and the outage probability were investigated for dual receivers operating in correlative lognormal fading.

Another fading channel model, namely the Weibull model, has not received as much attention as the above-mentioned fading models, despite the fact that it exhibits an excellent fit to experimental fading channel measurements, for indoor [8], [9] and outdoor environments [10], [11]. Only very recently, the topic of communications over Weibull fading channels began to receive renewed interest. For example, considering the performance of diversity receivers over Weibull fading channels, an analysis for the evaluation of the GSC performance over independent Weibull fading channels was presented in [12]. In this analysis, the first two moments and the AoF at the output of the GSC combiner were derived. More recently, two other contributions, dealing with switched [13] and selection diversity [14], were presented by Sagias *et al.* In [13], assuming that the receiver employs switched diversity, expressions for the average SNR, the AoF, and the switching rate at the output of the combiner were derived. Finally, in [14], important performance measures such as the outage probability and the average output SNR were studied, for  $L$ -branch SC receivers over independent Weibull fading channels.

In this letter, we analytically evaluate the performance of a dual SC receiver over *correlated* Weibull fading channels with arbitrary parameters. Exact closed-form expressions are derived for the probability density function (pdf), the cumulative distribution function (cdf), and the moments of the dual-SC output

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SNR. Capitalizing on these formulas, novel closed-form analytical expressions are derived for the average output SNR, the AoF, the outage probability, and the ABEP for several modulation schemes. The proposed mathematical analysis is also validated by means of computer simulation.

## II. STATISTICAL PROPERTIES OF THE OUTPUT SNR

### A. System and Channel Model

We consider a dual diversity receiver operating in a Weibull fading environment. The baseband received signal in the  $i$ th ( $i = 1, 2$ ) antenna is  $z_i = sr_i + n_i$ , where  $s$  is the transmitted symbol,  $r_i$  is the fading envelope, modeled as a Weibull random variable (rv), and  $n_i$  is the additive white Gaussian noise (AWGN). The usual assumption is made for the AWGN that it is uncorrelated between the two diversity branches and has a single-sided power spectral density  $N_0/2$ . The Weibull distribution can be regarded as an approximation to the generalized Nakagami distribution of the same order as the Nakagami- $m$  distribution [15], and its pdf is given by [16]

$$f_{r_i}(r_i) = \frac{\beta}{\omega_i} \left(\frac{r_i}{\omega_i}\right)^{\beta-1} \exp\left[-\left(\frac{r_i}{\omega_i}\right)^\beta\right]. \quad (1)$$

In the above equation,  $\omega_i = \sqrt{r_i^2/\Gamma(1+2/\beta)}$ ,  $\Gamma(\cdot)$  is the Gamma function [17, (8.310/1)],  $r_i^2$  is the average signal power, and  $\beta$  is the Weibull fading parameter ( $\beta \geq 0$ ). As the value of  $\beta$  increases, the severity of the fading decreases. For the special case of  $\beta = 2$ , (1) reduces to the well-known Rayleigh pdf. It is convenient to define the function  $d_\tau = 1 + \tau/\beta$ , where, in general,  $\tau$  is a nonnegative real variable. The corresponding cdf and the moments are given by [16]

$$F_{r_i}(r_i) = 1 - \exp\left[-\left(\frac{r_i}{\omega_i}\right)^\beta\right] \quad (2)$$

and

$$E\langle r_i^n \rangle = \omega_i^n \Gamma(d_n) \quad (3)$$

respectively, where  $E\langle \cdot \rangle$  denotes expectation and  $n$  is a positive integer. To model the correlation between the two diversity paths, the bivariate distribution of  $r_1$  and  $r_2$  is needed. Among the various distributions belonging to the family of the Weibull bivariate distributions [18], [19], the most suitable distribution to model the *correlative* fading paths must satisfy the following criteria: its marginal pdfs should be two-parameters Weibull distributions, and the range of values for its correlation coefficient should be  $[0,1]$ . Such a bivariate distribution is derived in [18] as a mixture of its marginals, which are also two-parameters Weibull distributions. The complementary cdf (or survival function) of this bivariate distribution can be mathematically expressed in the following form [19]:

$$\tilde{F}_{r_1, r_2}(r_1, r_2) = \exp\left\{-\left[\left(\frac{r_1}{\omega_1}\right)^{\beta/\delta} + \left(\frac{r_2}{\omega_2}\right)^{\beta/\delta}\right]^\delta\right\} \quad (4)$$

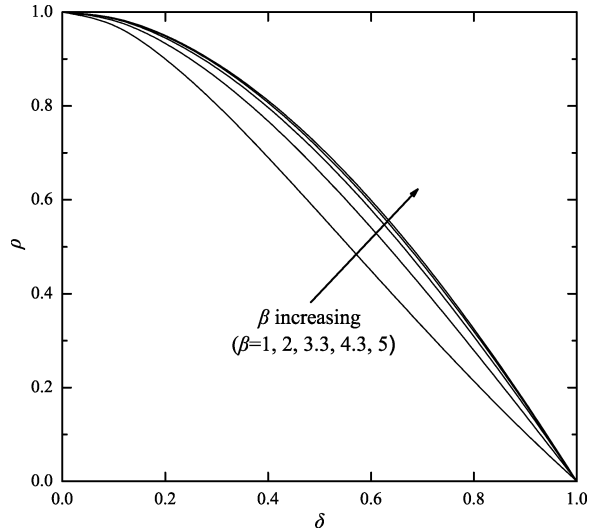


Fig. 1. Correlation coefficient  $\rho$  versus  $\delta$ , for various values of  $\beta$ .

where the dependence factor  $\delta$  ( $0 < \delta \leq 1$ ) is related to the correlation coefficient  $\rho = \text{cov}(r_1, r_2)/\sqrt{\text{var}(r_1)\text{var}(r_2)}$  as follows [18]:

$$\rho = \frac{\Gamma^2(d_\delta)\Gamma(d_2) - \Gamma^2(d_1)\Gamma(d_{2\delta})}{\Gamma(d_{2\delta})[\Gamma(d_2) - \Gamma^2(d_1)]}. \quad (5)$$

Since  $\rho$  does not directly appear in (4), Fig. 1 plots  $\rho$  as a function of  $\delta$  for several values of  $\beta$ . For  $\rho = 0$  (i.e.,  $\delta = 1$ ), (4) can be expressed as the product of two single Weibull complementary cdfs. Substituting (2) and (4) in [20, eq. (6.22)], the cdf of  $r_1$  and  $r_2$  is derived as

$$F_{r_1, r_2}(r_1, r_2) = 1 + \exp\left\{-\left[\left(\frac{r_1}{\omega_1}\right)^{\beta/\delta} + \left(\frac{r_2}{\omega_2}\right)^{\beta/\delta}\right]^\delta\right\} - \exp\left[-\left(\frac{r_1}{\omega_1}\right)^\beta\right] - \exp\left[-\left(\frac{r_2}{\omega_2}\right)^\beta\right]. \quad (6)$$

Differentiating (6), the joint pdf<sup>1</sup> of  $r_1$  and  $r_2$  is obtained as

$$\begin{aligned} f_{r_1, r_2}(r_1, r_2) &= \frac{\beta^2}{\omega_1 \omega_2 \delta} \left(\frac{r_1 r_2}{\omega_1 \omega_2}\right)^{\beta/\delta - 1} \left[\left(\frac{r_1}{\omega_1}\right)^{\beta/\delta} + \left(\frac{r_2}{\omega_2}\right)^{\beta/\delta}\right]^{\delta - 2} \\ &\times \left\{1 - \delta + \delta \left[\left(\frac{r_1}{\omega_1}\right)^{\beta/\delta} + \left(\frac{r_2}{\omega_2}\right)^{\beta/\delta}\right]^\delta\right\} \\ &\times \exp\left\{-\left[\left(\frac{r_1}{\omega_1}\right)^{\beta/\delta} + \left(\frac{r_2}{\omega_2}\right)^{\beta/\delta}\right]^\delta\right\}. \end{aligned} \quad (7)$$

Note that when the diversity input channels are uncorrelated, (7) is also expressed as the product of two single Weibull pdfs.

<sup>1</sup>It should be noted that the bivariate Weibull and Rayleigh distributions belong to different families of multivariate distributions, namely the chi-squared and exponential, respectively [19]. Thus, although for  $\beta = 2$  (1) reduces to the Rayleigh pdf, for  $\beta = 2$  (7) does not reduce to the bivariate Rayleigh pdf [1, eq. (6.2)].

### B. CDF and PDF of the Output SNR

The instantaneous SNR per symbol for each diversity channel can be expressed as  $\zeta_i = r_i^2 E_s / N_0$ , where  $E_s = E \langle s^2 \rangle$  is the transmitted symbols' energy. The corresponding average SNR per symbol for each diversity branch is  $\bar{\zeta}_i = r_i^2 E_s / N_0 = \Gamma(d_2) \omega_i^2 E_s / N_0$ . Setting  $a = 1/\Gamma(d_2)$ , the joint cdf of  $\zeta_1$  and  $\zeta_2$  can be obtained directly from (6) as

$$F_{\zeta_1, \zeta_2}(\zeta_1, \zeta_2) = F_{r_1, r_2} \left( \omega_1 \sqrt{\frac{\zeta_1}{a\bar{\zeta}_1}}, \omega_2 \sqrt{\frac{\zeta_2}{a\bar{\zeta}_2}} \right). \quad (8)$$

Defining the instantaneous SNR at the SC output as  $\zeta_{sc} \triangleq \max(\zeta_1, \zeta_2)$  and setting  $\zeta_1 = \zeta_2 = \zeta_{sc}$  in the above equation, the cdf of  $\zeta_{sc}$  can be directly obtained as

$$F_{\zeta_{sc}}(\zeta_{sc}) = \exp(-D\zeta_{sc}^{\beta/2}) - \exp\left[-\left(\frac{\zeta_{sc}}{a\bar{\zeta}_1}\right)^{\beta/2}\right] + 1 - \exp\left[-\left(\frac{\zeta_{sc}}{a\bar{\zeta}_2}\right)^{\beta/2}\right] \quad (9)$$

where  $D = a^{-\beta/2} \left[ \bar{\zeta}_1^{-\beta/(2\delta)} + \bar{\zeta}_2^{-\beta/(2\delta)} \right]^\delta$ . Differentiating (9), the pdf of  $\zeta_{sc}$  can be derived as

$$f_{\zeta_{sc}}(\zeta_{sc}) = \frac{\beta}{2} \left\{ \frac{1}{a\bar{\zeta}_1} \left(\frac{\zeta_{sc}}{a\bar{\zeta}_1}\right)^{(\beta/2)-1} \exp\left[-\left(\frac{\zeta_{sc}}{a\bar{\zeta}_1}\right)^{\beta/2}\right] + \frac{1}{a\bar{\zeta}_2} \left(\frac{\zeta_{sc}}{a\bar{\zeta}_2}\right)^{(\beta/2)-1} \exp\left[-\left(\frac{\zeta_{sc}}{a\bar{\zeta}_2}\right)^{\beta/2}\right] - D\zeta_{sc}^{(\beta/2)-1} \exp\left[-D\zeta_{sc}^{\beta/2}\right] \right\}. \quad (10)$$

### C. Moments of the Output SNR

By definition, the moments of the output SNR are [20]

$$E \langle \zeta_{sc}^n \rangle \triangleq \int_0^\infty \zeta_{sc}^n f_{\zeta_{sc}}(\zeta_{sc}) d\zeta_{sc}. \quad (11)$$

By substituting (10) into (11) and using [17, eq. (3.326/2)], after some straightforward simplifications, the  $n$ th moment of  $\zeta_{sc}$  can be derived in a closed-form expression as

$$E \langle \zeta_{sc}^n \rangle = \left[ \bar{\zeta}_1^n + \bar{\zeta}_2^n - \left( \bar{\zeta}_1^{-\frac{\beta}{2\delta}} + \bar{\zeta}_2^{-\frac{\beta}{2\delta}} \right)^{-\frac{2\delta}{\beta} n} \right] \frac{\Gamma(d_2 n)}{\Gamma^n(d_2)}. \quad (12)$$

## III. PERFORMANCE ANALYSIS

### A. Average Output SNR

The average output SNR  $\bar{\zeta}_{sc}$  is obtained by setting  $n = 1$  in (12) as

$$\bar{\zeta}_{sc} = \bar{\zeta}_1 + \bar{\zeta}_2 - \left( \bar{\zeta}_1^{-\frac{\beta}{2\delta}} + \bar{\zeta}_2^{-\frac{\beta}{2\delta}} \right)^{-\frac{2\delta}{\beta}}. \quad (13)$$

It should be also noted that for independent and identically distributed (i.i.d.) input paths (e.g.,  $\delta = 1$  and  $\bar{\zeta}_1 = \bar{\zeta}_2$ ), (13) is identical to [12, eq. (8)] for the GSC(2,1) (selecting one out of two input paths).

### B. Amount of Fading

Using (12), the AoF of the SC's output, defined as the ratio of the variance to the square mean SC output SNR, can be easily expressed in a simple closed-form expression as

$$\text{AoF} \triangleq \frac{\text{var}(\zeta_{sc})}{\bar{\zeta}_{sc}^2} = \frac{E \langle \zeta_{sc}^2 \rangle}{\bar{\zeta}_{sc}^2} - 1. \quad (14)$$

It is noted that, for i.i.d. input paths, the expression obtained using (14) for the AoF is the same as [12, eq. (10)] for GSC(2,1).

### C. Outage Probability

The outage probability  $P_{\text{out}}$  is defined as the probability that the SC output SNR falls below a given threshold  $\zeta_{\text{th}}$ . Since this probability is simply the probability that neither  $\zeta_1$  nor  $\zeta_2$  exceeds  $\zeta_{sc}$ , the  $P_{\text{out}}$  is obtained by replacing  $\zeta_{sc}$  with  $\zeta_{\text{th}}$  in (9) as

$$P_{\text{out}} = F_{\zeta_{sc}}(\zeta_{\text{th}}). \quad (15)$$

### D. Average Bit-Error Probability

The most straightforward approach to obtain the ABEP  $\bar{P}_{\text{be}}$  is to average the conditional BEP  $P_{\text{be}}$  over the pdf of the output SNR [1] as

$$\bar{P}_{\text{be}} = \int_0^\infty P_{\text{be}}(\zeta) f_{\zeta_{sc}}(\zeta) d\zeta. \quad (16)$$

Using well-known expressions for the  $P_{\text{be}}$  found in [4], it is easy to realize that for differential binary phase-shift keying (DBPSK) and noncoherent binary frequency-shift keying (NBFSK), (16) requires evaluation of integrals of the form

$$\Upsilon(\xi) = \int_0^\infty x^{(\beta/2)-1} \exp(-x) \exp(-\xi x^{\beta/2}) dx. \quad (17)$$

Similarly, for  $M$ -amplitude modulation (AM),  $M$ -PSK,  $M$ -quadrature amplitude modulation (QAM), BFSK, and  $M$ -DPSK, it is required to evaluate finite integrals of the form

$$\int_{\lambda_1}^{\lambda_2} \Upsilon[\xi(\varphi)] d\varphi \quad (18)$$

where the particular values of  $\lambda_1$  and  $\lambda_2$  depend upon the modulation scheme. The integral in (17) can be evaluated in closed-form as follows. By expressing the exponential function as a Meijer's  $G$ -function [17, eq. (9.301)], i.e.,  $\exp[-g(x)] = G_{0,1}^{1,0} \left[ g(x) \middle| \begin{matrix} - \\ 0 \end{matrix} \right]$  [21, eq. (11)], where  $g(\cdot)$  is an arbitrary function, the integral in (17) can be written as

$$\Upsilon(\xi) = \int_0^\infty x^{(\beta/2)-1} G_{0,1}^{1,0} \left[ x \middle| \begin{matrix} - \\ 0 \end{matrix} \right] G_{0,1}^{1,0} \left[ \xi x^{\beta/2} \middle| \begin{matrix} - \\ 0 \end{matrix} \right] dx. \quad (19)$$

Using [21, eq. (21)], the above equation can be expressed in closed form as

$$\Upsilon(\xi) = \frac{\left(\frac{k}{l}\right)^{1/2} l^{\beta/2}}{(2\pi)^{(k+l/2)-1}} G_{l,k}^{k,l} \left[ \xi^k \frac{l}{k^k} \middle| \begin{matrix} \{1-(\beta/2)+n/l\}_{n=0,1,\dots,l-1} \\ \{m/k\}_{m=0,1,\dots,k-1} \end{matrix} \right] \quad (20)$$

with

$$\frac{l}{k} = \frac{\beta}{2} \quad (21)$$

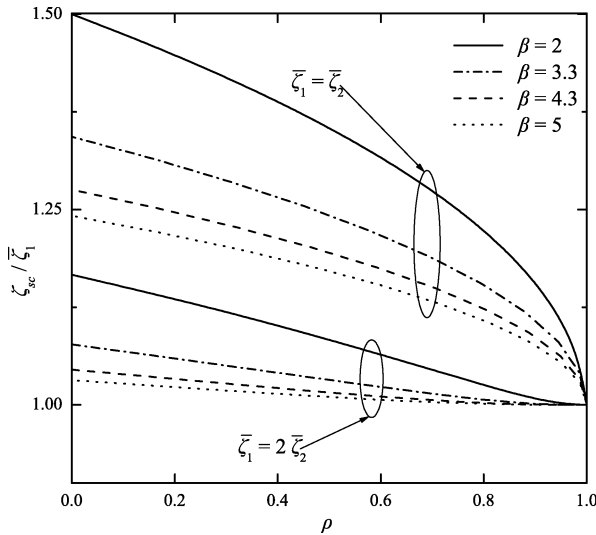


Fig. 2. First-branch normalized average output SNR,  $\zeta_{sc}/\bar{\zeta}_1$ , versus the correlation coefficient  $\rho$ , for equal ( $\bar{\zeta}_1 = \bar{\zeta}_2$ ) and unequal ( $\bar{\zeta}_1 = 2\bar{\zeta}_2$ ) input SNRs and for several values of  $\beta$ .

and  $k$  and  $l$  positive integers. Depending upon the value of  $\beta$ , a set with minimum values of  $k$  and  $l$  can be properly chosen in order for (21) to be valid (e.g., for  $\beta = 4.3$ , we have to choose  $k = 20$  and  $l = 43$ ). For the special case where  $\beta$  is an integer, setting  $k = 2$  and  $l = \beta$ , (20) reduces to

$$\Upsilon(\xi) = \sqrt{\frac{2}{\beta}} \left( \frac{\beta}{2\pi} \right)^{\beta/2} G_{\beta,2}^{2,\beta} \left[ \frac{\xi^2}{4} \beta^\beta \left\{ \begin{matrix} (1-(\beta/2)+n)/\beta \\ 0, (1/2) \end{matrix} \right\}_{n=0,1,\dots,\beta-1} \right]. \quad (22)$$

As an indicative example, the ABEP for DBPSK with integer values of  $\beta$  and equal SNRs ( $\bar{\zeta}_i = \bar{\zeta}_0, \forall i$ ) is expressed as

$$\bar{P}_{be} = \frac{\beta}{2(a\bar{\zeta}_0)^{\beta/2}} \left\{ \Upsilon \left[ \frac{1}{(a\bar{\zeta}_0)^{\beta/2}} \right] - 2^{\delta-1} \Upsilon \left[ \frac{2^\delta}{(a\bar{\zeta}_0)^{\beta/2}} \right] \right\}. \quad (23)$$

#### IV. NUMERICAL RESULTS

In this section, using the previous mathematical analysis, theoretical results are presented for the performance of dual SC receivers over correlated Weibull fading channels. In Fig. 2, using (13), the first-branch normalized average output SNR  $\zeta_{sc}/\bar{\zeta}_1$  is plotted as a function of  $\rho$ , for equal ( $\bar{\zeta}_1 = \bar{\zeta}_2$ ) and unequal ( $\bar{\zeta}_1 = 2\bar{\zeta}_2$ ) input branches SNRs, and for several values of  $\beta$ . As expected, the diversity gain decreases as  $\rho$  increases. It is interesting to note that the normalized average output SNR degrades more rapidly as  $\rho$  increases, especially for the equal-input SNR case and for lower values of  $\beta$ . For the limiting case of  $\rho = 0$ , the SNR gain of the combiner takes its maximum value, while as  $\rho \rightarrow 1$ , the corresponding gain approaches unity. Additionally, for a fixed  $\rho$ , the normalized output SNR increases as the severity of fading increases (i.e., as  $\beta$  decreases). Similar behavior was observed in [5], where the average SNR of dual SC over correlated Nakagami- $m$  fading channels was studied.

Having numerically evaluated (15), in Fig. 3, the outage probability  $P_{out}$  performance of the dual SC receiver is presented as a function of the normalized threshold with unequal ( $\bar{\zeta}_1 = 5\bar{\zeta}_2$ )

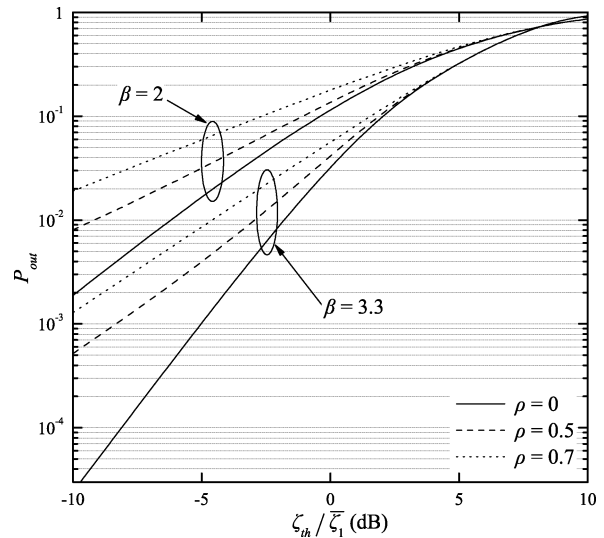


Fig. 3. Outage probability  $P_{out}$  versus the normalized outage threshold  $\zeta_{th}/\bar{\zeta}_1$ , for unequal ( $\bar{\zeta}_1 = 5\bar{\zeta}_2$ ) average SNRs for  $\rho = 0, 0.5, 0.7$  and for  $\beta = 2, 3.3$ .

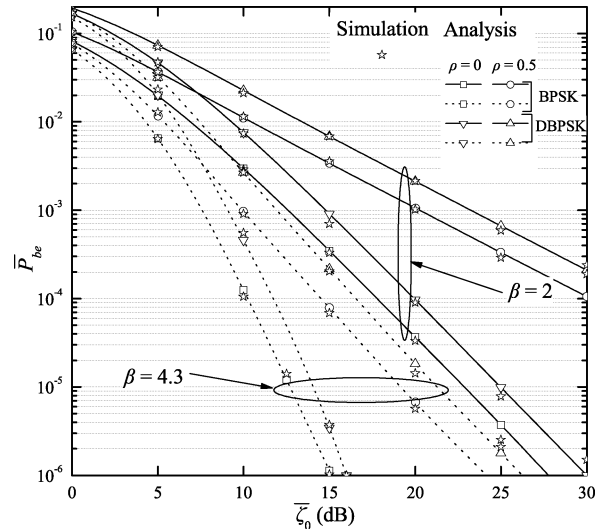


Fig. 4. ABEP of BPSK and DBPSK versus SNR per bit of the first branch, for  $\rho = 0, 0.5$  and for  $\beta = 2, 4.3$ .

input SNRs and for different values of  $\beta$  and  $\rho$ . For comparison purposes, the curve for  $\rho = 0$  is also included as a special case for best performance. The obtained results clearly show that the outage performance degrades with an increase of the fading correlation and/or fading severity.

Using (16)–(22), the ABEP of various coherent and non-coherent binary and multilevel modulation schemes can be obtained. As typical examples, the performances of BPSK/DBPSK and a Gray-encoded 64-QAM, as a function of the average SNR per bit of the first branch, for  $\bar{\zeta}_i = \bar{\zeta}_0$  and for several values of  $\beta$  and  $\rho$  are illustrated in Figs. 4 and 5, respectively. The obtained performance evaluation results show that the error performance improves with an increase of  $\beta$ , while as expected, the diversity gain decreases with increasing values of  $\rho$ . In order to verify these analytical results, computer simulations were also performed. For comparison purposes, these results are included in Figs. 4 and 5, verifying the validity of our theoretical approach. As, to the best of the authors'

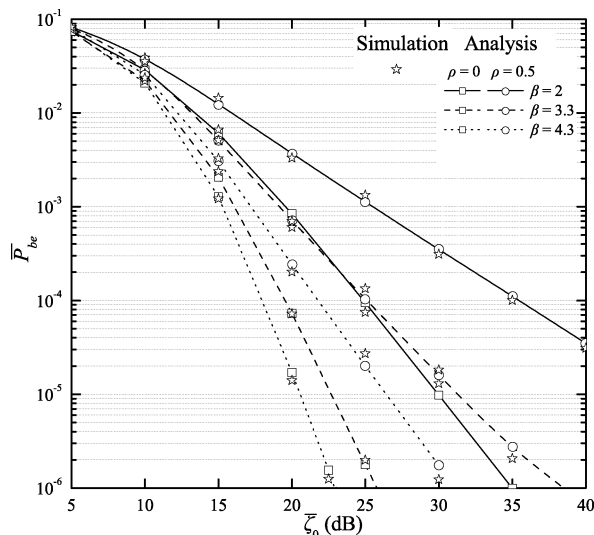


Fig. 5. ABEP of Gray-encoded 64-QAM versus SNR per bit of the first branch, for  $\rho = 0, 0.5$  and for  $\beta = 2, 3.3, 4.3$ .

knowledge, the generation algorithm of the correlated Weibull fading envelopes has not been previously published in the open technical literature, it is presented in the Appendix.

### V. CONCLUSIONS

Closed-form expressions for the pdf, the cdf, and the moments of the combined SNR at the output of a dual receiver employing SC over both correlated and uncorrelated Weibull fading channels were derived. Capitalizing on these expressions, important performance criteria, such as average output SNR, AoF, outage probability, and ABEP, were extracted in closed form. As an illustration of the mathematical formalism, numerical results of these performance criteria were presented, describing their dependence on  $\beta$  and  $\rho$ . Extensive computer simulations validated the mathematical analysis.

### APPENDIX

#### GENERATION OF CORRELATED WEIBULL FADING ENVELOPES

Following the analysis presented in [18], an efficient algorithm for the generation of two correlated groups of Weibull distributed fading envelopes  $r_1$  and  $r_2$ , with a joint pdf defined in (7), is given in three steps below.

- 1) Generate five uniform distributed rvs  $\{U_n \in [0, 1)\}$ ,  $n = 1, 2, 3, 4, 5$ .
- 2) Set  $U = U_1$  and  $V = \begin{cases} -\ln(U_2U_3), & \text{if } U_5 \leq \delta \\ -\ln(U_4), & \text{if } U_5 > \delta \end{cases}$ .
- 3) Set  $(r_1, r_2) = (\omega_1 U^{\delta/\beta} V^{1/\beta}, \omega_2 (1 - U)^{\delta/\beta} V^{1/\beta})$ .

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