This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/LCOMM.2019.2943859, IEEE Communications Letters

3-Color Shift Keying for Indoor Visible Light Communications

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***Abstract*—We introduce the 3-color shift keying (CSK) modulation for indoor visible light communications (VLC) that is based on the utilization of three symbols, each of which corresponds to a different primary color. In addition, we propose a novel technique that uses sequences of symbols to transmit the information in order to optimize the achieved data rate, while retaining the mean chromaticity constraint that is crucial in VLC systems. Furthermore, the design parameters of 3-CSK are studied and optimized, while an analytical expression for the bit-error-rate is derived. Finally, simulations are used to validate the analysis and to extract insights for the modulation design. The results reveal that 3-CSK offers improved error performance and lower power consumption compared to 4-CSK and On-Off Keying, while it retains a relatively high data rate.**

I. INTRODUCTION

VISIBLE light communication (VLC) has been proposed as an emerging technology for several scenarios such as indoor, outdoor, vehicle-to-vehicle (V2V) and underwater, while it is one of the wireless standards in the fifth generation (5G) of wireless networks [1]. VLC uses light emitting diodes (LEDs) in the visible region of the optical spectrum for simultaneous room illumination and data transfer [2]. In VLC the transmission scheme must be designed with respect to the visual perception of the light, especially for indoor applications. As a result, the average illumination should be of white color, while the average LED's intensity must remain constant to avoid flickering and to meet the safety requirements for the human eyes [3]. In this context, white (W) color can be generated by simultaneously using red (R), green (G) and blue (B) LEDs. The use of multi-color LEDs is the main principle behind color shift keying (CSK), which can be implemented in VLC as a multi-light source modulation scheme. Furthermore, CSK shares the basic concept of frequency shift keying (FSK), since the bit patterns are encoded into color (wavelength) bands [4].

In the existing literature, the most common CSK modulation schemes are 4-CSK, 8-CSK and 16-CSK that use 4, 8 and 16 symbols, respectively. The error performance of CSK in the presence of additive white Gaussian noise (AWGN) has been investigated in [5], where an analytical expression for the bit-error-rate (BER) was derived. Moreover, the need to increase the minimum Euclidean distance between the symbols at the transmitter has been highlighted in [6], where the authors introduced a CSK format based on four colours instead of three, which is used in the IEEE 802.15.7 standard [2]. In the same work, the proposed system achieved a significant electrical SNR gain of 4.4 dB compared to the typical CSK system. Thus, the motivation behind the proposed in this paper 3-CSK modulation has been the communications perspective deficiencies of the CSK, as described in [4].

The contribution of this work can be summarised as follows:

- We present a novel CSK scheme, hereinafter termed as 3-CSK, that utilizes only three symbols each of which corresponds to a different primary color .
- The data transmission with the use of three symbols is particularly challenging because it is coupled with the requirement to maintain the visual perception of light. To this end, we propose an efficient method to transmit information as sequences of symbols in order to optimize the achieved data rate.
- We present an analytical expression for the BER of 3-CSK, which is expressed as a function of the number of symbols per sequence.
- We compare 3-CSK, 4-CSK, and On-Off Keying (OOK) in terms of data rate and BER. The results reveal that 3-CSK is an attractive alternative for a wide range of applications, where the implementation complexity, communication reliability, energy consumption and data rate may vary.

II. THE 3-CSK MODULATION

In 4-CSK, each of the three peripheral symbols correspond to the center of the RGB color bands on the xy color coordinates, while the fourth symbol, W, is chosen in order to maximise the distance from the other three symbols.

In the proposed 3-CSK modulation the central symbol is removed and only the three peripheral symbols are utilized, resulting in three decision regions. The constellations of 4-CSK and 3-CSK are presented in Fig. 1, along with their decision regions. The chromaticity values (x, y) are transformed into the intensity values, $[P_r, P_g, P_b]$, according to the transformation rule [2]:

$$x = P_r\ x_R + P_g\ x_G + P_b\ x_B, \tag{1}$$

$$y = P_r\ y_R + P_g\ y_G + P_b\ y_B, \tag{2}$$

$$P_T = P_r + P_g + P_b, \tag{3}$$

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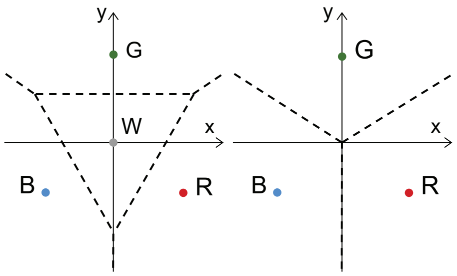
Digital Object Identifier xx.xxxx/xxxxx.xxxx.xxxxxxx



(a) 4-CSK (b) 3-CSK

Fig. 1: Constellations and decision regions.

where (x_R, y_R), (x_G, y_G), (x_B, y_B) are the chromaticity values of the three vertices corresponding to the RGB bands. In addition, it is highly important the average intensity of all symbols to be constant in order to prevent flickering. As a result, the symbols are located on a plane inside the 3D signal space with a constant total intensity, P_T. This enables the use of the 3D-to-2D linear transformation [5]

$$\begin{bmatrix} P_x \\ P_y \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \end{bmatrix} \begin{bmatrix} P_r \\ P_g \\ P_b \end{bmatrix}. \tag{4}$$

A. *Sequential Transmission*

Since the signal space of 3-CSK consists of three symbols, it is impossible to map each possible bit string to a symbol. To deal with this issue, we propose to use sequences of the N successive symbols in order to transmit the information. Specifically, the number of the different sequences that can be constructed is 3^N, with each sequence containing a total of $M = \lfloor \log_2 3^N \rfloor$ bits, where $\lfloor \cdot \rfloor$ is the floor operator. In this context, the necessary number of sequences needed to transmit a bit string of M bits is 2^M. Furthermore, the sequence is transmitted as N successive symbols and each received symbol is demapped to a symbol of the 3-CSK signal space (Fig. 1b) by using the minimum euclidean distance criterion. Finally, the sequence is mapped to an M-bit string.

It should be emphasized that the mapping of the used sequences to the bit strings is implemented using Gray mapping. All the sequences are listed in an array, where each two consecutive entries differ by one symbol. Each entry is mapped to the corresponding entry in the Gray coded bit-strings. The sequences that are not used do not have a corresponding bit string and therefore are mapped to the bit-string of the last used sequence in the sequences' list. However, the mean chromaticity of the transmitted symbols is not of white color, due to the use of only a portion of the 3^N possible combinations of symbols, which alters the visual perception of the light. In order to mitigate this phenomenon, and assuming that all sequences are equiprobable, the $3^N - 2^M$ unused combinations are transmitted once every 2^M sequences.

The energy of a transmitted sequence is

$$E_{\text{seq}} = N E_s, \tag{5}$$

where E_s is the energy of each symbol, which due to the symmetry of the constellation is equal to the average energy of the symbols. Also, for every 3^N sequences only the 2^M contain information and, thus, a total of $M 2^M$ information bits are transmitted. Considering that each symbol of a sequence requires one time unit to be transmitted, it follows that the data rate of 3-CSK can be expressed as

$$R_{3-\text{CSK}} = \frac{M 2^M}{N\ 3^N} \ \text{(bits/channel use)}, \tag{6}$$

whereas the maximum theoretical data rate is obtained for $M = \log_2 3^N$ and is equal to $R_{3-\text{CSK}_{\max}} = 1.585$.

B. *Selection of* N

Based on the above, the number of symbols included in one sequence greatly affects the performance of 3-CSK. It should be emphasized that, as N increases the implementation complexity increases exponentially, which can have detrimental effect on the system's performance. The computational complexity is determined by the number of possible transmitted sequences, $2^{\lfloor \log_2 3^N \rfloor}$, and it quantifies the required computational power needed for a 3-CSK system with N symbols per sequence to decode the received sequence. Taking this into account, an equilibrium is formed that aims to maximize the achievable data rate of 3-CSK, while maintaining low implementation complexity. Also, in order to select the optimal N, the performance degradation, due to the fact that not all transmitted sequences contain information, needs to be quantified.

Definition 1. The sequence *Fill Factor (FF)* is an indicator of how close to the upper bound is the achieved data rate for a specific sequence length, N. This is defined as the ratio of the achievable data rate, $R_{3-\text{CSK}}$, to the upper bound of the data rate, $R_{3-\text{CSK}_{\max}}$.

The number of bits per sequence, the achievable data rate, as calculated through (6), and the FF for $N \in [2, 100]$ are presented in Table I. The table is sorted for increasing N.

TABLE I: 3-CSK performance with regard to N

N	**M**	**$R_{3\text{-CSK}}$**	**FF (%)**	**N**	**M**	**$R_{3\text{-CSK}}$**	**FF (%)**
2	**3**	**1.3333**	**84.12**	8	12	0.9364	59.08
3	4	0.7901	49.85	9	14	1.2948	81.69
4	6	1.1852	74.78	10	15	0.8324	52.52
5	7	0.7374	46.52	11	17	1.1435	72.14
6	9	1.0535	66.47	**12**	**19**	**1.5620**	**98.55**
7	**11**	**1.4716**	**92.85**	**53**	**84**	**1.5816**	**99.79**

From Table I, it is remarkable that the complexity increases with N, but the FF is not an increasing function of N. More specifically, the set of values $\mathcal{N}_{\text{d}} = \{2, 7, 12, 53\}$ dominates all other values of $N \in [2, 100]$ in terms of complexity and performance, which also justifies the exclusion of all other values. Also, it deserves to be noticed that the option $N = 53$ offers a minor gain to performance compared to $N = 12$, despite the dramatical increase of complexity, while the later offers a good balance between the two objectives. It is important to highlight that the value of N, as selected in this paper, is intentionally sub-optimal. In more detail, although some values of N provide better error performance for the proposed modulation, they require higher computational power for the interpretation of the received signal, and therefore are excluded.

Finally, the main difference between the implementation of the 4-CSK and the 3-CSK modulations derives from the elimination of the white symbol of the former. This modification simplifies the decision regions of the transmitted symbols and, thus, the modulation-demodulation procedure. However, it introduces the need for sequential transmission, as stated above. As a result, the addition of a encoder at the transmitter and a decoder at the receiver is necessary.

III. Performance Analysis

In this section, we derive an exact analytical expression for the BER of 3-CSK over AWGN channels. After applying the aforementioned linear transformation from the 3D to 2D signal space, we evaluate the symbol transition probability [5].

A. Transition Probability

In [5], the 4-CSK symbols are located in an equilateral triangle area on the plane, given by the transformation in (4). Considering that 3-CSK utilizes the peripheral symbols of 4-CSK, the same transformation is applicable. The xy coordinate system is constructed in the 2D plane and its symmetry enables the use of the transition probability analysis. In this 2D signal space, the transition probability is defined as the probability that a specific transmitted symbol, S_t, is decoded as another specific symbol, S_r. For instance, the symbol PEP may be equal to the probability to receive B while R is transmitted. The transition probability is equal to the symbol pairwise error probability (PEP), $P_e(S_r|S_t)$, and can be computed by using a technique called *decision region partitioning* [7]. According to this technique, the transition probability for any decision region can be calculated based on the probability that the noise superimposed on the transmitted symbol falls into basic shapes of two type, I and II. These two types of decision regions are characterized by the parameters α, β, γ and δ, where α, β and γ are geometric parameters as labeled in Fig. 2 and $\delta = 1.2\,\left(\overline{WB}/P_T\right)^2$. Based on these values, the transition probability of the type II decision regions is given by [7]

$$\begin{aligned} P_e(S_r|S_t) &= \int_\alpha^\gamma \int_{R(\theta)}^\infty p_n(r,\theta)\,dr\,d\theta \\ &= \left| \frac{1}{2\pi}\int_\alpha^\gamma \exp\left[-\frac{\delta\gamma_s \sin^2\beta}{\sin(|\theta-\alpha|+\beta)}\right] d\theta \right| \triangleq |G(\alpha,\gamma,\beta,\delta)|, \end{aligned} \tag{7}$$

where $R(\theta) = \frac{\sqrt{\alpha\gamma}\sin(\beta)}{\sin(|\theta-\alpha|+\beta)}$, γ represent the signal-to-noise ratio (SNR) at the receiver and p_n denotes the probability density function (PDF) of the AWGN. It should be emphasized that all transition probabilities in 3-CSK are equal, due to the symmetry of the constellation. Also, note that the definite integral in (7) can be evaluated numerically in closed form with the desired accuracy via the Gauss Formula [8, eq. 25.4.30].

B. BER Analysis

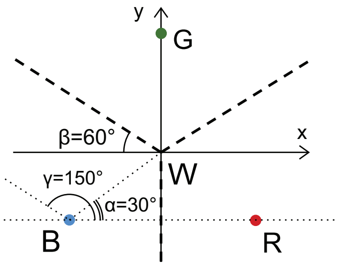


Fig. 2: Transition probability.

Next, we derive the BER of the proposed 3-CSK scheme based on the previous analysis on transition probability. As previously stated, the PEP is equal to the transition probability. Also, each symbol can transition to one of the other two symbols, and thus, it is part of exactly two equal symbol PEPs. As a result, the probability of correct detection when a specific symbol, S_t, is transmitted, can be written as

$$P_c(S_r|S_t) = 1 - 2P_e(S_r|S_t). \tag{8}$$

After evaluating the probabilities of error and correct detection for the transmission of all symbols in 3-CSK, we can combine them to express the probability of receiving the specific symbol S_r, when S_t is transmitted as

$$P(S_r|S_t) = \begin{cases} 1-2\left|G(\frac{\pi}{6},\frac{5\pi}{6},\frac{\pi}{3},0.8)\right|, & \text{if } S_r = S_t \\ \left|G(\frac{\pi}{6},\frac{5\pi}{6},\frac{\pi}{3},0.8)\right|, & \text{if } S_r \neq S_t \end{cases}. \tag{9}$$

Due to the use of sequential transmission, the error performance of 3-CSK depends on the accurate interpretation of the received sequences. As a result, in order to obtain an exact expression for the BER, we need to determine the PEP for the transmitted sequences. It should be highlighted that each sequence contains N independent symbols. Hence, the received sequence will be interpreted incorrectly if at least one of its symbols is wrong. Thus, the sequence PEP is

$$P(Seq_r|Seq_t) = \prod_{i=1}^{N} P(S_{r,i}|S_{t,i}), \tag{10}$$

where $S_{t,i}$ and $S_{r,i}$ denote the i-th symbol of the transmitted, Seq_t, and received, Seq_r, sequences, respectively.

By using (10), an exact analytical expression for the BER for 3-CSK can be derived as the sum of the PEPs of the possible misinterpretations of all the used sequences. However, for each of the 2^M used sequences there are $3^N - 1$ possible sequences that can be misinterpreted at the receiver, with each one leading to different number of incorrect bits. The number of wrong bits can be expressed by the hamming distance between the transmitted and the received code words, $d(Seq_t, Seq_r)$. Finally, the exact analytical expression of the BER, P_b, is

$$P_b = \frac{1}{M}\sum_{t=1}^{2^M}\sum_{\substack{r=1 \\ r\neq t}}^{3^N} d(Seq_t, Seq_r) P(Seq_t) P(Seq_r|Seq_t), \tag{11}$$

where $P(Seq_t)$ is the probability of transmitting a sequence.

Remark 1. It should be noted that, throughout the analysis, 3-CSK is limited by the requirement to maintain the visual perception of light. However, this constraint can be neglected in some applications, such as V2V or underwater

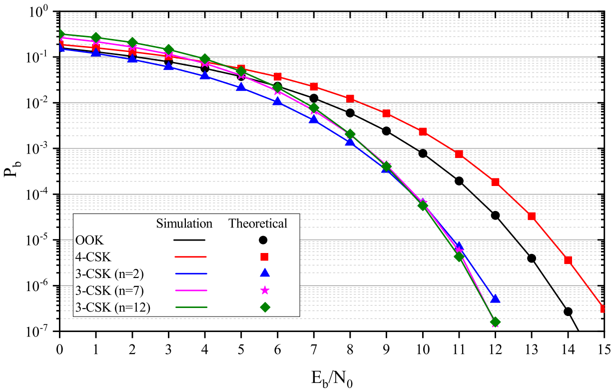


Fig. 3: BER of 4-CSK, OOK and 3-CSK for different N.

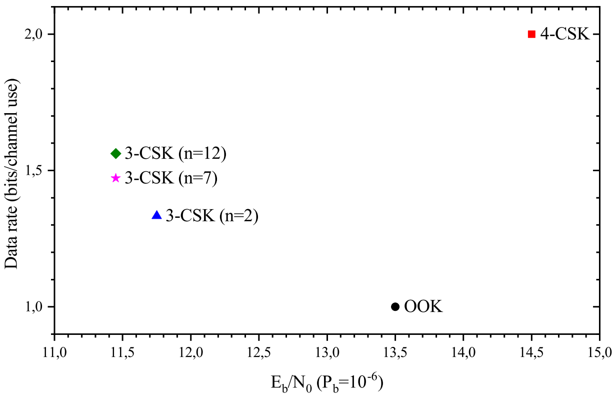


Fig. 4: Data rate as a function of E_b/N_0 for $P_b = 10^{-6}$.

communications. Thus, the need to transmit sequences that do not contain information is eliminated. In other words, the FF of the modulation becomes equal to 100%. As a result, 3-CSK can also achieve improved performance, in terms of data rate.

IV. SIMULATIONS AND DISCUSSION

In this section, the error performance of 3-CSK for data transmission over AWGN channels is discussed. The proposed scheme is compared to 4-CSK and OOK. Simulations have been performed and compared with the mathematical analysis. In order to ensure fairness in the comparison between the different modulation schemes, the energy of each symbol, E_s, is normalized to unity.

Fig. 3 depicts the error performance of 4-CSK, OOK and 3-CSK modulations as a function of the SNR. As stated in Section II-B, the highest error performance with reasonable implementation complexity is achieved for $N = 12$, 7 and 2. Consequently, these values are selected for the simulations. Also, markers represent the simulation outcomes, whereas the continuous curves corresponds to the analytical results. It is observed that analytical and simulation results coincide. In addition, from this figure, it is obvious that 4-CSK is characterised by the worst BER performance, closely followed by the OOK. In more detail, OOK outperforms 4-CSK in terms of BER, while 4-CSK can achieve much higher data rate than OOK. As far as 3-CSK is concerned, the error performance of the optimal N values is similar but higher than 4-CSK and OOK. In addition, 3-CSK achieves BER of 10^{-4} with SNR approximately 3 dB less than 4-CSK and 2 dB less than OOK. Moreover, a crossing point is observed for SNR equal to 10 dB and different values of N. It is evident that $N = 2$ exhibits better BER in the low SNR regime, while, for SNR values higher than 10 dB, the system with $N = 7$ and 12 outperforms the former. This happens because as N increases the probability of incorrect sequence detection increases as well. Hence, in the low SNR regime, 3-CSK with high N is more susceptible to noise than that with lower N values, while for high SNR, the opposite is valid. It should be highlighted that most analyses of VLC systems assume a high SNR regime [9]. Systems with $N = 7$ and 12 achieve similar performance, but the latter results in higher implementation complexity.

However, to ensure a fair comparison it is imperative to quantify the impact of the achievable data rate of the different modulation schemes, under the assumption of equal symbol duration for all modulations. As a result, the achievable data rate of OOK, 4-CSK and the selected N values of 3-CSK are presented in Fig. 4 as a function of E_b/N_0 for BER equal to 10^{-6}. The data rate of the 3-CSK variations can be calculated by (6). As expected the highest data rate is achieved by 4-CSK and the lowest by OOK. Also, for the selected 3-CSK variations, as N increases, the achievable data rate increases as well. Moreover, from this figure it is obvious that 3-CSK outperforms OOK in terms of data rate and power requirements, while, for the same BER, 4-CSK provides higher data rates but with higher power consumption.

In conclusion, 3-CSK combines superior error performance and less power consumption for the same BER, while, at the same time achieving relatively high data rates. This fact alongside the versatility offered by the selection of N, makes 3-CSK an attractive alternative for a wide range of applications, with different requirements in terms of data rate, implementation complexity, and communication reliability.

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