Statistical Properties of the EGC Output SNR Over Correlated Nakagami-m Fading Channels

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***Abstract*—Previous studies have addressed with the performance analysis of L-branch equal gain combining (EGC) systems over independent Rayleigh- or Nakagami-m fading channels. In this paper, important statistical parameters, such as the k-moment, the k-central moment, the skewness, and the kurtosis of the output signal-to-noise ratio (SNR), in predetection EGC systems operating over correlated Nakagami-m fading channels are studied. Simple closed-form expressions for the mean and variance of the output SNR are presented and it is shown that our general results reduce to the specific noncorrelative fading case previously published. Moreover, significant performance criteria such as the amount of fading and the spectral efficiency in the low power regime, are investigated. Numerical results and simulations are presented to check the validity of the proposed mathematical analysis and to point out the impact of the fading correlation, the input SNRs unbalancing as well as the fading severity, on the EGC performance.**

***Index Terms*—Amount of fading (AoF), antenna diversity, correlative fading, equal gain combining (EGC), moments, Nakagami-m fading channels.**

I. INTRODUCTION

ANTENNA diversity can be efficiently used to reduce the effects of fading and to improve the performance of wireless communications systems. Various techniques are used to combine the signals from multiple diversity branches. The most popular of them are maximal-ratio combining (MRC), equal-gain combining (EGC), selection combining (SC), and switch combining (SWC) [1], [2]. EGC is of great practical interest because it provides an intermediate solution as far as the performance and the implementation complexity are concerned. In predetection EGC, each signal branch is weighted with the same factor, irrespective of the signal amplitude. Moreover, cophasing of all signals is needed to avoid signal cancellation.

The performance of EGC, assuming independent channel fading, has been studied extensively in the literature, although the published results concerning EGC receivers are less, compared with those for other diversity methods, such as MRC or SC. This lack is mainly due to the difficulty of finding the probability density function (pdf) and cumulative distribution function (cdf) of the EGC output signal-to-noise ratio (SNR) [3]. However, independent fading is not always realized in practice due to insufficient antenna spacing. Therefore, it is important to understand how the correlation between received signals affects the offered diversity gain. From reviewing the literature, there are few approaches for the performance evaluation of predetection EGC over correlated fading channels. In [4], a formula for the error probability of EGC with orthogonal binary frequency shift keying (BFSK) operating in correlative Rician time-selective fading is proposed, while in [5] the BER performance of dual MRC, EGC, and SC over correlated Rayleigh channels is studied. Mallik *et al.* in [6] presented a useful approach to the performance analysis of binary signals with coherent detection employing dual predetection EGC over correlated Rayleigh channels. Iskander and Mathiopoulos in [7] derived an infinite series expression for the pdf of the sum of two correlated Nakagami-m variables to express the error rate of M-ary quadrature amplitude modulation (M-QAM) in terms of infinite sum of Lauricella hypergeometric functions. In other previous related works, average combined SNR or higher moments are used to evaluate diversity systems operating over independent [1] or correlated fading channels [8]–[10].

In this paper, closed-form expressions for important statistical parameters such as the k-moment and the k-central moment of the output SNR, in predetection EGC systems operating over correlated Nakagami-m fading channels are derived. This approach can be efficiently used to overcome the infeasible derivation of the pdf of the output SNR, by evaluating parameters as the mean, the variance, the skewness and the kurtosis that characterize the behavior of the distribution [11]. Furthermore, significant performance criteria such as the amount of fading (AoF) and the spectral efficiency in the low-power regime of the EGC are studied. Finally, simulations are performed to check the accuracy of the proposed mathematical analysis while numerical results are used to point out the impact of the input SNR unbalance as well as the fading correlation and fading severity on the EGC performance.

The organization of the paper is as follows. In Section II, useful closed-form expressions are derived for the k-moment, the k-central moment, the AoF, and the spectral efficiency in the low-power regime of the EGC output SNR. In Section III, simulations and numerical results are presented to illustrate the proposed mathematical analysis. Comments about the impact of the correlation and the input SNR unbalancing to the EGC performance are also included in Section IV. Finally, some concluding remarks are offered in Section IV.

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II. Statistical Properties of the EGC Output SNR

We assume that the transmitted signal is received over correlated slowly varying Nakagami-m flat fading channels [12]. Additive white Gaussian noise (AWGN) is added to each signal. The AWGN is assumed to be statistically independent from branch to branch with a power spectral density N_0 equal for all paths and also independent of the fading amplitudes $a_1, \ldots, a_L$. The output SNR of the L branches predetection EGC receiver is [1, eq. (9.46)]

$$\gamma_{\text{out},L} = \frac{E_s}{LN_0}(a_1 + \cdots + a_L)^2 \tag{1}$$

with E_s the energy per symbol and $a_1, \ldots, a_L$ be the fading amplitudes, modeled as correlated Nakagami-m random variables (RVs).

A. Moments

By definition, the k-moment of the output SNR can be written as [13, eq. (2.80)]

$$\begin{aligned} E\left\langle \gamma_{\text{out},L}^k \right\rangle &= E\left\langle \left[\frac{E_s}{LN_0}(a_1 + \cdots + a_L)^2\right]^k \right\rangle \\ &= \left(\frac{E_s}{LN_0}\right)^k E\left\langle (a_1 + \cdots + a_L)^{2k} \right\rangle \end{aligned} \tag{2}$$

where $E\langle\cdot\rangle$ means expectation. Expanding $(a_1 + \cdots + a_L)^{2k}$ with the use of the multinomial theorem [14, eq. (24.1.2)] and taking into account that [1, eq. (2.23)]

$$E\left\langle \gamma_i^k \right\rangle = \frac{\Gamma(m+k)}{\Gamma(m)m^k}\overline{\gamma}_i^k \tag{3}$$

with $\gamma_i = E_s a_i^2/N_0, i = 1, \ldots, L$ being the instantaneous SNR at the ith input branch of the combiner and $\overline{\gamma}_i$ the mean γ_i, (2) can be written after algebraic manipulations as (4), located at the bottom of the page, where $\delta(\cdot)$ is the well-known delta function defined as

$$\delta(i,j) = \begin{cases} 1, & i = j \\ 0, & i \neq j. \end{cases} \tag{5}$$

In order to extract useful forms for the moments of the EGC output SNR, it is necessary to evaluate $E\left\langle a_1^{n_1} \ldots a_L^{n_L} \right\rangle$ which is defined as

$$\begin{aligned} E\left\langle a_1^{n_1} \ldots a_L^{n_L} \right\rangle = \int_0^\infty \cdots \int_0^\infty a_1^{n_1} \ldots a_L^{n_L} \\ \times f_{a_1,\ldots,a_L}(a_1, \ldots, a_L) da_1, \ldots, da_L \end{aligned} \tag{6}$$

with $f_{a_1,\ldots,a_L}(a_1, \ldots, a_L)$ being the multivariate (joint) Nakagami-m distribution of $a_1, \ldots, a_L$. When $L = 2$, $f_{a_1,a_2}(a_1, a_2)$ is the Nakagami-m bivariate distribution [12, eq. (126)] but a known formulation for $f_{a_1}, \ldots, a_L(a_1, \ldots, a_L)$ when $L > 2$, does not exist in the literature. However, recently the authors in [15] and [16] presented an efficient approximation to the multivariate Nakagami-m distribution with arbitrary correlation. According to this approach $f_{a_1,\ldots,a_L}(a_1, \ldots, a_L)$ can be expressed as

$$\begin{aligned} f_{a_1,\ldots,a_L}(a_1, \ldots, a_L) &= \frac{|\mathbf{W}|^m a_1^{m-1} a_L^m e^{-\sum_{n=1}^{L} w_{n,n} a_n^2/2}}{2^{m-1}\Gamma(m)} \\ &\times \prod_{n=1}^{L-1} \left[\frac{a_n I_{m-1}(|w_{n,n+1}| a_n a_{n+1})}{|w_{n,n+1}|^{(m-1)}}\right] \end{aligned} \tag{7}$$

with $\mathbf{W}$ being the inverse of the correlation matrix $\mathbf{\Sigma}$, i.e $\mathbf{W} = \mathbf{\Sigma}^{-1}$ with elements $w_{i,j}$, $1 \leq i, j \geq L$. For the special case—important in practical applications such as in linear antenna arrays [1], [17]—of exponentially correlated fading, i.e., $\mathbf{\Sigma}_{i,j} \equiv \rho^{|i-j|}$, (7) can be directly used to evaluate $f_{a_1,\ldots,a_L}(a_1, \ldots, a_L)$. If $\mathbf{\Sigma}$ is an arbitrary correlation matrix, then its entries must be approximated with the elements of a Green's matrix [18], $\mathbf{C}$, in order $\mathbf{W} = \mathbf{C}^{-1}$ being tridiagonal. Without loss of generality and for simplification purposes, it is assumed in (7) that $\Omega_i = 2\sigma_i^2$, with $\sigma_i^2 = 1$ being the variance of the input signal at the ith branch. Following the same procedure as in [12] for the calculation of $E\left\langle a_1^{n_1} a_2^{n_2} \right\rangle$, $E\left\langle a_1^{n_1} \ldots a_2^{n_2} \right\rangle$ can be expressed as (8), located at the bottom of the following page, where $|\mathbf{W}|$ is the norm of $\mathbf{W}$. Hence, using (4) and (8), the k-moment of the output SNR can be evaluated in its general form. The infinite series in (8) have a fast convergence and a mean number of 10 terms for each sum are sufficient for accuracy at the fifth significant figure assuming exponential correlation between the branches and signals envelopes ranging from 1 to 10 [15].

Now, using (4), $E\left\langle \gamma_{\text{out},L}^k \right\rangle$ will be evaluated for $k = 1$, which corresponds to the important statistical parameter of the average output SNR. In this case, it is easily recognized that only terms in the form of $E\langle a_i a_j \rangle$ appear in (4). These terms can be written as [12, eq. (137)]

$$\begin{aligned} E\langle a_i a_j \rangle = \frac{\sqrt{\Omega_i \Omega_j}}{m[\Gamma(m)]^2} \left[\Gamma\left(m + \frac{1}{2}\right)\right]^2 \\ \times {}_2F_1\left(-\frac{1}{2}, -\frac{1}{2}; m; \rho_{i,j}\right) \end{aligned} \tag{9}$$

$$E\left\langle \gamma_{\text{out},L}^k \right\rangle = \begin{cases} 1, & \text{for } k = 0 \\ \frac{\Gamma(m+k)}{L^k\Gamma(m)m^k} \sum_{i=1}^{L} \overline{\gamma}_i^k + \left(\frac{E_s}{LN_0}\right)^k \sum_{\substack{n_1,\ldots,n_L=0 \\ n_1+\cdots+n_L=2k}}^{2k} \left[\frac{(2k)!}{n_1! \ldots n_L!} E\left\langle a_1^{n_1} \ldots a_L^{n_L} \right\rangle \prod_{j=1}^{L} [1 - \delta(n_j, 2k)]\right], & \text{for } k > 0 \end{cases} \tag{4}$$

where $\Omega_i = \overline{a}_i^2$, with $\overline{a}_i^2$ being the average signal power of the ith branch, $\rho_{i,j}$ is the power correlation coefficient between the signal envelopes a_i, a_j defined as

$$\rho_{i,j} = \frac{\mathrm{cov}\left(a_i^2, a_j^2\right)}{\sqrt{\mathrm{var}\left(a_i^2\right)\mathrm{var}\left(a_j^2\right)}}, \quad 0 \le \rho_{i,j} < 1 \tag{10}$$

and ${}_2F_1(x_1, x_2, y_1, z)$ being the Gauss hyper-geometric function [14, eq. (15.1.1)]. Using (4) and (9), the average output SNR (first moment) of a predetection EGC receiver with L correlated branches can be expressed as

$$\overline{\gamma}_{\mathrm{out},L} = \frac{1}{L}\sum_{l=1}^{L}\overline{\gamma}_l + \frac{\left[\Gamma\left(m+\frac{1}{2}\right)\right]^2}{Lm[\Gamma(m)]^2}\sum_{i,j=1}^{L}\sqrt{\overline{\gamma}_i\overline{\gamma}_j} \times {}_2F_1\left(-\frac{1}{2}, -\frac{1}{2}; m; \rho_{i,j}\right)[1-\delta(i,j)]. \tag{11}$$

Equation (11) is an important new and simple closed-form expression that can be directly used for arbitrary number of branches L, arbitrary values of the fading severity parameter m, unequal branch powers and arbitrary correlation between the diversity paths. Moreover, if these paths are uncorrelated ($\rho_{i,j} = 0$ for every i, j) and identically distributed ($\overline{\gamma}_i = \overline{\gamma}, i = 1, \ldots, L$), (11) reduces to the well-known published formula for the average output SNR of an EGC receiver with L independent branches [1, eq. (9.48)]. For correlated Rayleigh-fading channels ($m = 1$), (11) is simplified to

$$\overline{\gamma}_{\mathrm{out},L} = \frac{1}{L}\sum_{l=1}^{L}\overline{\gamma}_l + \frac{\pi}{4L}\sum_{i,j=1}^{L}\sqrt{\overline{\gamma}_i\overline{\gamma}_j} \times {}_2F_1\left(-\frac{1}{2}, -\frac{1}{2}; 1; \rho_{i,j}\right)[1-\delta(i,j)]. \tag{13}$$

Note, that for the important practical case of dual diversity the moments of the output SNR can be extracted setting $L = 2$ in (4) and using [12, eq. (137)] as (14), located at the bottom of the page.

B. Central Moments

The k-central moment of the output SNR, $E\langle(\gamma_{\mathrm{out},L} - \overline{\gamma}_{\mathrm{out},L})^k\rangle$ can be written after using the binomial theorem [14, eq. (3.1.1)], as

$$\begin{aligned} E\left\langle\left(\gamma_{\mathrm{out},L} - \overline{\gamma}_{\mathrm{out},L}\right)^k\right\rangle &= \sum_{n=0}^{k}\frac{k!}{n!\,(k-n)!}E\left\langle\gamma_{\mathrm{out},L}^n\left(-\overline{\gamma}_{\mathrm{out},L}\right)^{k-n}\right\rangle \\ &= \sum_{n=0}^{k}\frac{k!\,(-1)^{k-n}\left(\overline{\gamma}_{\mathrm{out},L}\right)^{k-n}}{n!\,(k-n)!}E\left\langle\gamma_{\mathrm{out},L}^n\right\rangle \end{aligned} \tag{15}$$

where $E\left\langle\gamma_{\mathrm{out},L}^n\right\rangle$ is the nth moment of $\gamma_{\mathrm{out},L}$ given by (4). Setting $k = 2$ in (15), another important statistical parameter, the variance of the output SNR, can be derived as

$$\mathrm{var}\left\langle\gamma_{\mathrm{out},L}\right\rangle = E\left\langle\gamma_{\mathrm{out},L}^2\right\rangle - \left(\overline{\gamma}_{\mathrm{out},L}\right)^2 \tag{16}$$

where $E\left\langle\gamma_{\mathrm{out},L}^2\right\rangle$ is the second moment of $\gamma_{\mathrm{out},L}$, evaluated using (4) with $k = 2$ and $\overline{\gamma}_{\mathrm{out},L}$ is the average output SNR (11).

C. Kurtosis and Skewness

Kurtosis is a measure of the "peakedness" of a distribution, namely, the higher the kurtosis the lower the concentration of the density function around its mode [19]. It takes its minimum value for deterministic variables. For high values of kurtosis, the pdf is called "leptokurtic" and for low ones (near to one), platykurtic. Leptokurtic random variables have typically a "spiky" pdf with heavy tails, i.e., the pdf is relatively large at zero and at large values of the variable, while being small for intermediate values. Since, bit errors or outage in wireless communications systems mainly occurs during deep fades, the tail of the pdf mainly determines these performance measures and knowledge of the kurtosis behavior is important to understand BER or the outage performance. There are several definitions of kurtosis commonly encountered, including Fisher kurtosis (known as the kurtosis excess) and Pearson kurtosis

$$E\left\langle a_1^{n_1}\ldots a_L^{n_L}\right\rangle = \frac{2^{\sum_{j=1}^{L}\frac{n_j}{2}}|\mathbf{W}|^m}{\Gamma(m)}\sum_{i_1,\ldots,i_{L-1}=0}^{\infty}\left\{\left[\prod_{j=1}^{L-1}\frac{|w_{j,j+1}|^{2i_j}}{i_j!\Gamma(m+i_j)}\right] \times\left(w_{1,1}^{m+i_1+n_1/2}w_{2,2}^{m+i_1+i_2+n_2/2}\ldots w_{L-1,L-1}^{m+i_{L-2}+i_{L-1}+n_{L-1}/2}w_{L,L}^{m+i_{L-1}+n_L/2}\right) \times\Gamma\left(m+i_1+\frac{n_1}{2}\right)\Gamma\left(m+i_1+i_2+\frac{n_2}{2}\right)\ldots \Gamma\left(m+i_{L-2}+i_{L-1}+\frac{n_{L-1}}{2}\right)\Gamma\left(m+i_{L-1}+\frac{n_L}{2}\right)\right\} \tag{8}$$

$$E\left\langle\gamma_{\mathrm{out},2}^k\right\rangle = \begin{cases} 1, & \text{for } k=0 \\ \frac{\Gamma(m+k)}{2^k\Gamma(m)m^k}\sum_{i=1}^{2}(\overline{\gamma}_i)^k + \frac{1}{2^k}\sum_{\substack{n_1 n_2=0 \\ n_1+n_2=2k}}^{2k}\frac{(2k)!}{n_1!n_2!}\frac{(\overline{\gamma}_1)^{n_1/2}(\overline{\gamma}_2)^{n_2/2}\Gamma\left(m+\frac{n_1}{2}\right)\Gamma\left(m+\frac{n_2}{2}\right)}{m^{(n_1+n_2/2)}[\Gamma(m)]^2} & \\ \quad\times {}_2F_1\left(-\frac{n_1}{2}, -\frac{n_2}{2}; m; \rho\right)\prod_{j=1}^{2}[1-\delta(n_j, 2k)], & \text{for } k>0 \end{cases} \tag{14}$$

[11]. Throughout this paper, Fisher kurtosis is assumed and it is defined as

$$\kappa(\gamma_{\text{out},L}) = \frac{E\left\langle\left(\gamma_{\text{out},L} - \overline{\gamma}_{\text{out},L}\right)^4\right\rangle}{\left(\text{var}\left\langle\gamma_{\text{out},L}\right\rangle\right)^2} - 3. \quad (17)$$

Another important statistical parameter that characterizes the distribution of the output SNR, is the skewness, which is defined as [11]

$$sk(\gamma_{\text{out},L}) = \frac{E\left\langle\left(\gamma_{\text{out},L} - \overline{\gamma}_{\text{out},L}\right)^3\right\rangle}{\sqrt{\text{var}\left\langle\gamma_{\text{out},L}\right\rangle}^3}. \quad (18)$$

Skewness measures the deviation of the distribution from symmetry. If the distribution has a longer tail less than the maximum, the function has negative skewness. Otherwise, it has positive skewness. Skewness and kurtosis can be evaluated using (15) and (16).

D. Amount of Fading (AoF) and Spectral Efficiency

AoF is a unified measure of the severity of a fading channel and is typically independent of the average fading power [1, p. 18]. It expresses the level of the sensitivity of a wireless system to fading and is defined as

$$AoF = \frac{\text{var}\langle\gamma_{\text{out},L}\rangle}{\overline{\gamma}^2_{\text{out},L}} = \frac{E\left\langle\gamma^2_{\text{out},L}\right\rangle}{\overline{\gamma}^2_{\text{out},L}} - 1. \quad (19)$$

The AoF can be evaluated using (4) and (11) and can be used to study the spectral efficiency of a flat fading channel in the very noise (low-power) region. In such a region the minimum bit energy per noise level, required for reliable communication is [19]

$$\left(\frac{E_b}{N_0}\right)_{\min} = -1.59175 \text{ dB} \quad (20)$$

and the slope of the spectral efficiency curve versus E_b/N_0 in b/s/Hz per 3 dB, at $(E_b/N_0)_{\min}$ is given by [19]

$$S_0 = \frac{2E^2\langle r^2\rangle}{E\langle r^4\rangle} \quad (21)$$

with r being the combiner's output envelope. Equation (21) can be written in terms of the instantaneous output SNR as

$$S_0 = \frac{2\overline{\gamma}^2_{\text{out},L}}{E\left\langle\gamma^2_{\text{out},L}\right\rangle} \quad (22)$$

and using(19), the slope of the spectral efficiency in the very noise (low-power) region can be expressed as

$$S_0 = \frac{2}{AoF+1}. \quad (23)$$

III. Numerical Results and Simulations

In this section, we provide several representative numerical curves illustrating EGC performance over correlated Nakagami-m and Rayleigh fading channels, using the analytical results derived in Section II. In order to check the accuracy of the derived formulae, simulations were performed and the results were compared with the corresponding ones from the mathematical analysis. The computer simulation was written in C++ programming language using the algorithm presented in [20]. For the generation of the Nakagami-m fading envelopes over a million samples were used.

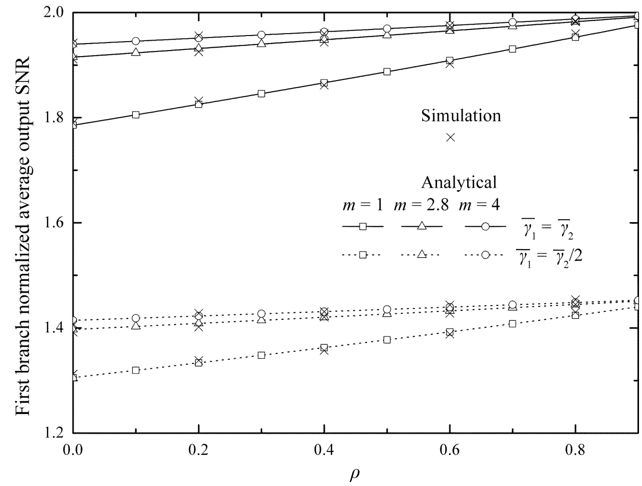

Fig. 1. First branch normalized average output SNR for dual EGC versus ρ, for $\overline{\gamma}_1 = \overline{\gamma}_2, \overline{\gamma}_1 = \overline{\gamma}_2/2$.

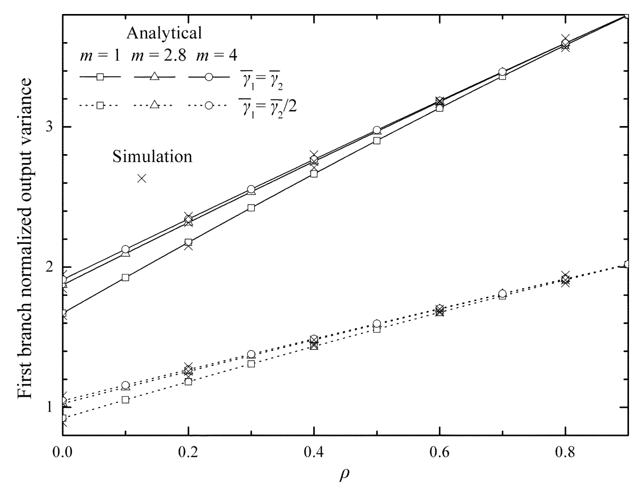

Fig. 2. First branch normalized variance of the output SNR for dual EGC versus ρ, for $\overline{\gamma}_1 = \overline{\gamma}_2, \overline{\gamma}_1 = \overline{\gamma}_2/2$.

Fig. 1 plots the first branch normalized average output SNR for a dual EGC receiver over correlated Nakagami-m channels, versus the correlation coefficient ρ, for equal ($\overline{\gamma}_1 = \overline{\gamma}_2$), unequal ($\overline{\gamma}_1 = \overline{\gamma}_2/2$) input SNRs and several values of the fading severity m. It is observed that the diversity SNR gain increases with an increase of the correlation coefficient and, as it was expected, the receiver performs better with an increase of the m parameter, while its performance deteriorates with an unbalance of the input SNRs. Also, it is obvious from Fig. 1 that while the fading severity decreases (m increases), the average output SNR becomes less sensitive to the correlation coefficient. In Fig. 2 the first branch normalized variance of the output SNR employing dual EGC is depicted versus ρ, for $\overline{\gamma}_1 = \overline{\gamma}_2$, $\overline{\gamma}_1 = \overline{\gamma}_2/2$ and

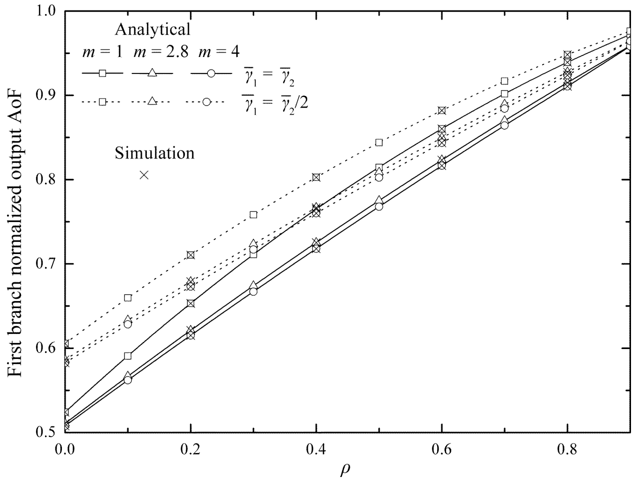

Fig. 3. First branch normalized AoF of the output SNR for dual EGC, versus ρ, for $\overline{\gamma}_1 = \overline{\gamma}_2$ and $\overline{\gamma}_1 = 5\overline{\gamma}_2$.

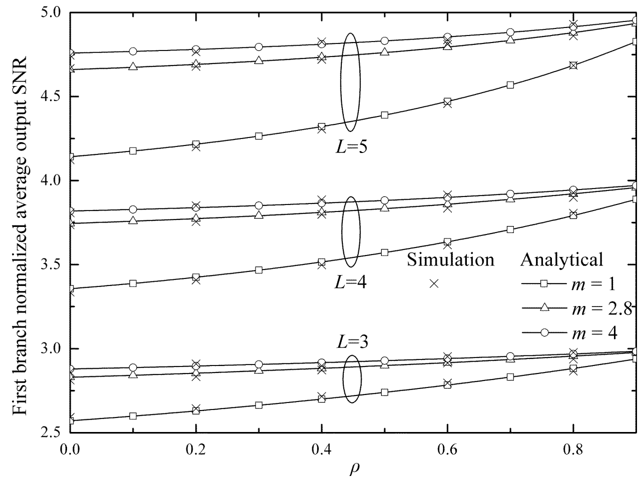

Fig. 5. First branch normalized average output SNR versus ρ, for $L = 3$, 4, 5, and exponential correlation.

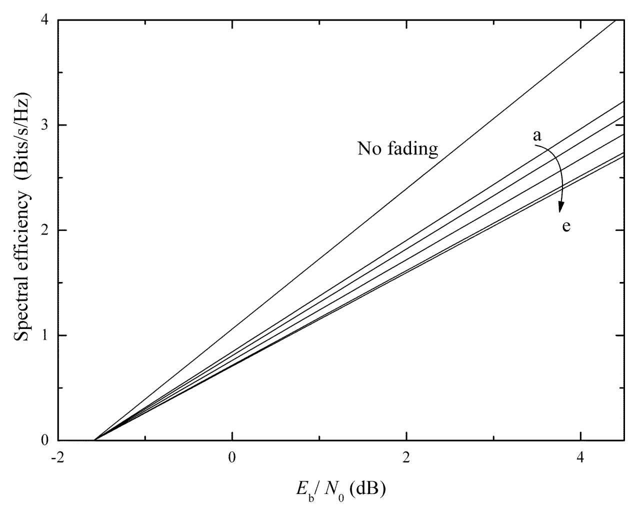

Fig. 4. Spectral efficiency of dual EGC for $m = 2$ and a:$\rho = 0$, b: $\rho = 0.2$, c: $\rho = 0.5$, d: $\rho = 0.9$ and e: No Diversity.

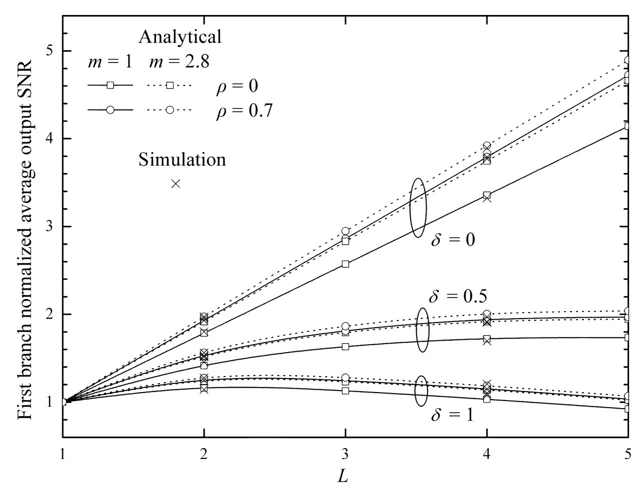

Fig. 6. First branch normalized average output SNR versus L for EGC with constant correlation and exponentially power delay profile.

several values of the fading severity m. With an increase of the correlation coefficient, the normalized variance of $\gamma_{out \cdot 2}$ increases, which is also observed when the channels condition improves. In Fig. 3, the first branch normalized output AoF is depicted versus ρ. As it was expected, the combiner mitigates the fading more effectively in low fading severity environments (higher values of m) and for SNR equal power input branches. Moreover, correlation affects negatively the fading performance, leading to the worst case (no diversity) when $\rho \rightarrow 1$. We observe from Figs. 1–3 that, although the normalized output SNR increases as ρ increases, the normalized output variance and the normalized output AoF also increase. Thus, an increase of the correlation does not imply an improvement of the receiver's performance.Fig. 4 plots the spectral efficiency of the dual EGC output in the high noise region of E_b/N_0 for several values of ρ. Note, that curves such as those in Figs. 3 and 4 are presented for first time in the literature.

In Fig. 5 the first branch normalized average output SNR is depicted for three, four and five branches EGC with exponential correlation ($\mathbf{\Sigma}_{i,j} \equiv \rho^{|i-j|}$), versus the correlation coefficient ρ, for several values of the fading severity m and for equal input SNRs. The conclusions from this figure are similar to that from Fig. 1. Regardless of the number of branches, the average output SNR is more sensitive to the correlation coefficient for low values of m. Furthermore, as the number of branches increases, the diversity gain increases with an improvement of the channels condition. Finally, it is assumed that the correlation is constant among the EGC branches ($\mathbf{\Sigma}_{i,j} \equiv \rho$) and that the receiver operates with an exponentially decaying power delay profile ($\Omega_i = \Omega_1 e^{-\delta(i-1)}, i = 1, 2, \dots L$). Under these assumptions, Fig. 6 plots the first branch normalized average output SNR as a function of the number of branches for various values of the Nakagami-m parameter, the correlation coefficient and the power decay factor δ. Note, that the combining loss of the receiver gets more accentuated as δ increases.

IV. CONCLUSION

In this paper, statistical properties of the predetection EGC output SNR over correlated Nakagami-m fading channels are presented. Closed-form expressions for important statistical parameters, such as the k-moment, the k-central moment, the kurtosis and the skewness are derived. Performance parameters, such as the average output SNR, the amount of fading and the spectral efficiency of the combiner in the high noise region are also investigated. Simulations showed the accuracy of the proposed mathematical analysis and numerical results depict clearly the effect of the fading correlation, the fading severity and input SNRs unbalancing on the EGC output performance. Another related contribution, dealing with the performance of EGC receivers over correlated Nakagami-m fading channels, was recently published [21].

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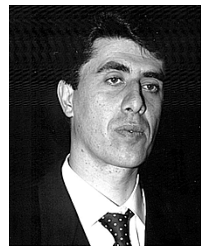

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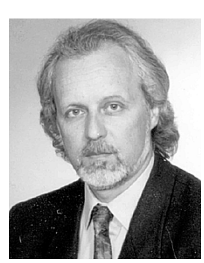

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