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Letter

Signal Processing

Average channel capacity for generalized-selection combining RAKE receivers

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SUMMARY

A novel closed-form expression for the achievable average channel capacity of a generalized-selection combining RAKE receiver in Rayleigh fading, is derived. Performance comparison for the capacity achieved with maximal-ratio combining and RAKE receivers is also presented. The expression derived, fully conforms to the upper bound of the Shannon–Hartley theorem. Copyright © 2004 AEI.

1. INTRODUCTION

RAKE reception has been considered as an effective way to anticipate multipath signal fading in direct sequence code division multiple access (DS-CDMA) communications systems. Common diversity techniques used in conventional fingered RAKE receivers are maximal-ratio combining (MRC) and equal-gain combining (EGC). Conventional fingered RAKE receivers offer increased complexity, since the number of taps is fixed and independent of the number of multipath signals. The complexity of MRC and EGC diversity receivers depends on the number of resolvable paths available, which can be quite high for wideband spread spectrum signals, while SC receivers utilize only one path out of L available and thus, do not fully exploit the amount of diversity offered by the channel. Generalized-selection combining (GSC) has been proposed to bridge the gap between the two extreme cases MRC/EGC and SC. While MRC RAKE receivers provide optimum performance and high complexity, the GSC RAKE receivers offer less complexity, but with comparable performance. In GSC(L, L_c), the strongest L_c resolvable paths among the L paths available are adaptively combined [1]. The GSC reception is equivalent to MRC reception, if all L branches are being combined ($L_c = L$), while it is equivalent to SC reception, if one of the L branches is being used $L_c = 1$. In this letter, we derive a closed-form expression for the achievable average Shannon's channel capacity per user, of a noncooperative DS-CDMA system operating in Rayleigh fading. We essentially extend the results described in References [2, 3], using an efficient GSC RAKE receiver. Numerical examples are presented to show that the GSC RAKE receiver offer similar capacity with optimum MRC RAKE receiver.

2. SYSTEM MODEL

For a user transmitting signal with bandwidth W and power P over an additive white Gaussian noise (AWGN) channel, the received signal-to-noise ratio (SNR) is

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$\gamma = P/(N_0 W)$, where N_0 is the double-sided noise power spectral density. When K users are simultaneously transmitting in a non-cooperative DS-CDMA system, the $(K-1)$ users' signals are appeared as multiple access interference (MAI) at the receiver of each user. If we assume that each user's pseudorandom signal waveform is Gaussian distributed, then the received spread to bandwidth W_{ss} signal-to-interference ratio (SIR), prior to despreading, will be

$$\gamma_{\text{ss}} = \frac{P}{N_0 W_{\text{ss}} + (K-1)P} = \frac{\gamma}{G_{\text{p}} + (K-1)\gamma} \tag{1}$$

with $G_{\text{p}} = W_{\text{ss}}/W$ being the processing gain. Then, the Shannon channel capacity per user, for the AWGN channel, is [4]

$$C_{\text{user}} = W_{\text{ss}} \log_2(1 + \gamma_{\text{ss}}) \tag{2}$$

3. CHANNEL CAPACITY IN RAYLEIGH FADING

We consider that each user's GSC RAKE receiver has L taps corresponding to L resolvable signal paths with $L = \lceil W_{\text{ss}} T_{\text{m}} \rfloor + 1$. With T_{m} the total multipath channel's delay spread is denoted, on the condition that W_{ss} is much greater than the coherence bandwidth of the channel and $\lceil x \rfloor$ is the maximum integer less than or equal to x. The channel assigned to each user is modeled as a time invariant multipath tapped delay line, so that the resolvable paths model can be assumed to have equal path strengths on average.

Assuming that all users are simultaneously transmitting in Rayleigh fading and all independent and identical distributed (i.i.d.) L branches are selected and properly combined, the probability density function (pdf) of the instantaneous received SIR, $\gamma_{l,\text{ss}}$ in the lth, $l = 1, 2, \ldots, L$, branch of each user's GSC RAKE receiver will follow a χ^2 distribution with two degrees of freedom, so that

$$p(\gamma_{l,\text{ss}}) = \frac{1}{\bar{\gamma}_{l,\text{ss}}} \exp\left(-\frac{\gamma_{l,\text{ss}}}{\bar{\gamma}_{l,\text{ss}}}\right) \tag{3}$$

where $\bar{\gamma}_{l,\text{ss}}$ is the corresponding average received SIR. Here, it must be pointed out that the statistic of each interfering signal in Equation (3) need not considered separately since, either the total interference power at the RAKE receiver output, or the MAI from the $(K-1)$ other users prior to despreading, tends to be Gaussian distributed [5].

The pdf of the GSC's instantaneous output SNR, γ_{gsc}, assuming equal path strengths, $\bar{\gamma}_{l,\text{ss}} = \bar{\gamma}_{\text{ss}}, \forall l$, is given by [6]

$$\begin{aligned} p(\gamma_{\text{gsc}}) = \binom{L}{L_c} \exp\left(-\frac{\gamma_{\text{gsc}}}{\bar{\gamma}_{\text{ss}}}\right) \Bigg\{ & \frac{\gamma_{\text{gsc}}^{L_c-1}}{(L_c-1)!\bar{\gamma}_{\text{ss}}^{L_c}} \\ & + \frac{1}{\bar{\gamma}_{\text{ss}}} \sum_{l=1}^{L-L_c} (-1)^{L_c+l-1} \binom{L-L_c}{l} \left(\frac{L_c}{l}\right)^{L_c-1} \\ & \times \left[\exp\left(-\frac{l\gamma_{\text{gsc}}}{L_c \bar{\gamma}_{\text{ss}}}\right) - \sum_{m=0}^{L_c-2} \frac{(-1)m}{m!} \left(\frac{l\gamma_{\text{gsc}}}{L_c\bar{\gamma}_{\text{ss}}}\right)^m \right] \Bigg\} \end{aligned} \tag{4}$$

where $\binom{L}{L_c}$ is the binomial coefficient, defined as $\binom{L}{L_c} = L!/[L_c!(L-L_c)!]$ and $\bar{\gamma}_{\text{gsc}}$ is the corresponding average output SIR

$$\bar{\gamma}_{\text{gsc}} = L_c \left(1 + \sum_{l=L_c+1}^{L} \frac{1}{l}\right) \bar{\gamma}_{\text{ss}} \tag{5}$$

The average total channel capacity, $\bar{C}_{\text{t}}$, available to all K users, will be given by the total channel capacity, averaged over the pdf of the output SNR, γ_{gsc}, [7] as

$$\bar{C}_{\text{t}} = W_{\text{ss}} \int_0^\infty \log_2(1 + \gamma_{\text{gsc}})\, p(\gamma_{\text{gsc}})\, \mathrm{d}\gamma_{\text{gsc}} \tag{6}$$

and using Equation (4), the average channel capacity per user, $\bar{C}_{\text{user}}$, normalized over the system bandwidth, W, will be given in closed-form expression as

$$\begin{aligned} \frac{\bar{C}_{\text{user}}}{W} = \frac{G_p}{\ln(2)K} \binom{L}{L_c} \Bigg\{ & \frac{I_{L_c}\left(\frac{1}{\bar{\gamma}_{\text{ss}}}\right)}{(L_c-1)!\bar{\gamma}_{\text{SS}}^{L}} \\ & + \frac{1}{\bar{\gamma}_{\text{ss}}} \sum_{l=1}^{L-L_c} (-1)^{L_c+l-1} \binom{L-L_c}{l} \left(\frac{L_c}{l}\right)^{L_c-1} \\ & \times \left[I_1\left[\left(1 + \frac{l}{L_c}\right)\frac{1}{\bar{\gamma}_{\text{ss}}}\right] \right. \\ & \left. - \sum_{m=0}^{L_c-2} \frac{(-1)^m}{m!} \left(\frac{l}{\bar{\gamma}_{\text{ss}} L_c}\right)^m I_{m+1}\left(\frac{1}{\bar{\gamma}_{\text{ss}}}\right) \right] \Bigg\} \end{aligned} \tag{7}$$

where $I_n(\mu)$ is defined in [8, Equation (78)] as $I_n(\mu) = (n-1)!\exp(\mu)\sum_{k=1}^{n}\Gamma(-n+k,\ \mu)\,\mu^{-k}$, with $\Gamma(\cdot\,,\ \cdot)$ being the incomplete gamma function, defined as $\Gamma(a,\ x) = \int_x^\infty \exp(-t)t^{a-1}\,\mathrm{d}t$ and

$$\bar{\gamma}_{\text{ss}} = \frac{1}{L_c\left(1 + \sum_{l=L_c+1}^{L}\frac{1}{l}\right)} \frac{\gamma}{G_{\text{p}} + (K-1)\gamma} \tag{8}$$

At this point, its worthy to note that, for $K = 1$ and $G_p = 1$ ($L = L_c = 1$), Equation (7) directly leads to the expression of the average channel capacity of a single user transmitting, without spreading the signal bandwidth W in Rayleigh fading, as it first appeared in Reference [7]. In addition, considering a DS-CDMA system, with a transmitted signal bandwidth W_{ss} that tends to infinity, i.e. with $G_p \to \infty$, the average channel capacity per user $\bar{C}_{user}$ will tend to the capacity of a AWGN channel bandlimited to W

$$\lim_{G_p \to \infty} \bar{C}_{user} = W \log_2(1 + \gamma) \qquad (9)$$

since, in this case, the pdf of γ_{gsc} in Equation (4) will be given by the Delta-function [7]. The physical content behind this is that when $W_{ss} \to \infty$, using that $L = \lceil W_{ss} T_m \rfloor + 1$, yields $L \to \infty$. Note, that for an infinite number of input paths, at least one of them will not be in fading. Hence, even for the worst case of operation for the GSC, which is the SC, each time the combiner chooses the diversity path, which will be without fading. Equation (9) is the well-known Shannon–Hartley theorem for the continuous channel and provides the maximum information rate that the channel can provide for a given W and γ [7].

4. NUMERICAL RESULTS

We consider a DS-CDMA system with $K = 10$ users, $G_p = 100$, $W = 20\,\text{kHz}$, operating in a typical urban area with $T_m = 2\,\mu\text{s}$. Using Equation (7), $\bar{C}_{user}/W$ is plotted in Figures 1 and 2 as function of γ with $L = 4$ and 6 i.i.d. taps respectively for MRC, GSC and SC RAKE reception and $L_c = 2$. In both cases, the GSC RAKE's receiver performance is lower than that of MRC RAKE and higher than that of SC RAKE.

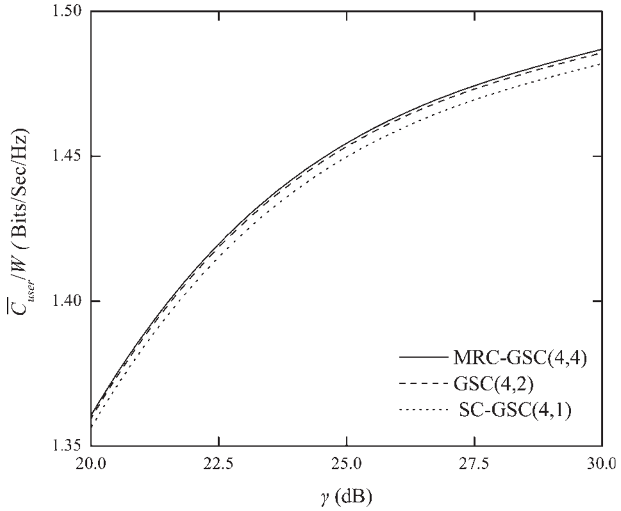

Figure 1. $\bar{C}_{user}/W$ as a function of γ, under Rayleigh fading, for a DS-CDMA system, $L = 4$ and $L_c = 2$ with $K = 10$ users, $W_{ss} = 2\,\text{MHz}$, $W = 20\,\text{kHz}$ and typical urban area with $T_m = 2\,\mu\text{s}$.

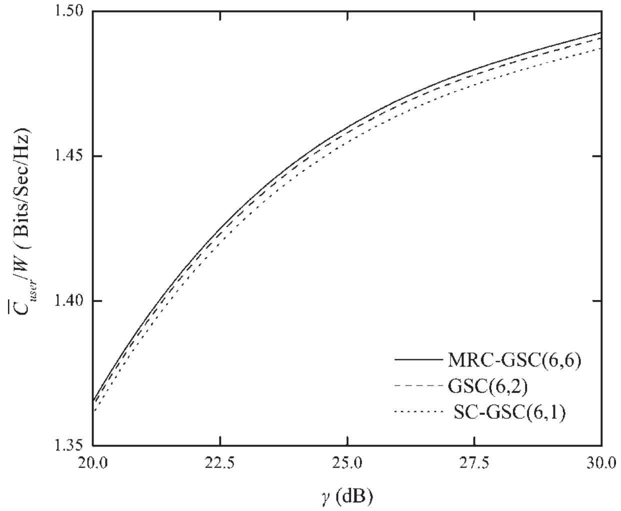

Figure 2. Same as Figure 1 with $L = 6$.

As shown in Figure 1, the average capacity per user offered by a GSC RAKE receiver is comparable to that achieved by an optimum MRC RAKE receiver, but as L increases the MRC RAKE receiver outperforms the GSC RAKE receiver, as it is clearly shown in Figure 2.

5. CONCLUSIONS

An analytical closed-form expression for the average Shannon channel capacity per user of a noncooperative DS-CDMA system, operating in a Rayleigh fading environment, with GSC RAKE reception has been derived. The derived expression allows a direct comparison to the capacity obtained using MRC RAKE and SC RAKE receivers. As shown, the GSC RAKE receiver offers similar capacity to the optimum MRC RAKE receiver. Furthermore, the derived expression fully conforms to the Shannon–Hartley channel capacity theorem.

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George K. Karagiannidis was born in Pithagorion, Samos Island, Greece. He received his university degree in 1987 and his Ph.D. in 1999, both in electrical and computer engineering, from the University of Patras, Greece. From 1990 to 1993, his research focused on the development of interfaces for diffuse IR communications links. From 1994 to 2000, he worked on developing of optimum channel assignment schemes and techniques for the improvement of QoS and GoS in wireless communications systems. From 2000 to 2004, he was a researcher at the Institute for Space Applications and Remote Sensing (ISARS), National Observatory of Athens (NOA), Greece. In June 2004, he joined the faculty of Aristotle University of Thessaloniki, Greece and he is currently an assistant professor at the Electrical and Computer Engineering Department. His major research interests include wireless digital communications, communications theory, satellite communications, mobile radio systems, interference problems and QoS in wireless networks. Dr. Karagiannidis has published and presented more than 70 technical papers in scientific journals and international conferences, he is a co-author in two chapters in books and also a co-author in a Greek edition book on *Mobile Communications*.