Closed-Form Analysis for NOMA with Randomly Deployed Users in Generalized Fading

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***Abstract*—In the pioneering work [1], Ding et al. presented an analytical framework for the performance of non-orthogonal multiple access (NOMA) in downlink cellular networks with randomly deployed users. However, the involved integrals are approximated through quadrature methods, where the accuracy depends on the number of terms in the summation and –most importantly– these methods hide the impact of the parameters on the system design. In this paper, we present a novel general closed-form expression for the outage probability of NOMA downlink in cellular networks with randomly deployed users, which can be used for any fading distribution and can reveal useful insights for the system design. Finally, we utilize this formula to provide a closed-form analysis for the important case of Nakagami-m fading.**

I. INTRODUCTION

IN the recent years, non-orthogonal multiple access (NOMA) has attracted the research interest from both industry and academia, as a very promising technique which can address the spectral efficiency requirements of the next generation wireless networks. The reader can find details about the research on NOMA in the last years in [2]. Also, NOMA considered for various interesting scenarios and channel conditions, e.g., for hybrid RF and optical networks [3], [4].

One of the first and fundamental research papers on NOMA –and maybe the most cited– was that of Z. Ding et al. [1], where the performance of NOMA was investigated in a downlink cellular scenario with randomly deployed users, assuming Rayleigh fading. In that paper, it was revealed for first time that NOMA can achieve substantially superior performance in cellular networks compared to orthogonal schemes as well as that the outage performance depends critically on the choices of users' targeted data rates and allocated powers. However, numerical quadrature methods are used to approximate the outage probability and sum rate, where the accuracy depends on the number of terms in the summation, which is not known, and –most importantly– these methods hide the impact of the parameters on system design. Quadrature methods were also used in the analysis of a more recent work [5], which considers Nakagami-m fading and a scenario with only two users.

In this paper, we present a novel general closed-form analysis for the outage performance of NOMA downlink in cellular networks with randomly deployed users over generalized fading channels. Specifically, we derive an elegant closed-form expression for the outage probability, in terms of the Gauss hypergeometric function [6], which can be utilized with any fading distribution (e.g., Nakagami-m, Weibull, Rician, Generalized Gamma, etc.) as well as with randomly deployed users in the case of no fading. Furthermore, we investigate this expression in order to get insights for the system design. Finally, we utilize this formula to provide a closed-form outage analysis for the important case of Nakagami-m and its special case of Rayleigh fading, for arbitrary values of the path-loss factor. It is worth noting that in this analysis we present a quite simple closed-form expression for the system's diversity order, which agrees with that in [1] for the case of Rayleigh fading.

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II. SYSTEM MODEL

As in [1], we consider a cellular downlink NOMA scenario, where a base station (BS) broadcasts to a set of users $\mathcal{N}$, with $|\mathcal{N}| = N$, where $|\mathcal{A}| = A$ is the cardinality of the set $\mathcal{A}$. The users are located according to a uniform distribution in a cyclic disk $\mathcal{D}$ with radius R_D around the BS. The channel coefficient between the n-th user and the BS is given by

$$h_n = \frac{g_n}{\sqrt{1 + z_n^a}}, \tag{1}$$

where a is the path loss factor, z_n denotes the distance from the n-th user to the BS, and g_n denotes the fading channel coefficient.

Without loss of generality we assume that the channel gains are sorted as $|h_1|^2 \leq |h_2|^2 \leq \cdots \leq |h_N|^2$. According to the NOMA protocol, the BS transmits a superposition of the signals of the users weighted by their respective power coefficient, hence

$$s = \sum_{n=1}^{N} \sqrt{p_n P} s_n, \tag{2}$$

where s_n is the message for the n-th user, P is the transmission power, and p_n is the power allocation coefficient for the n-th user. User fairness dictates that $p_1 \geq p_2 \geq \cdots \geq p_N$. Therefore the received signal at user n can be written as

$$y_n = h_n \sum_{i=1}^{N} \sqrt{p_i P} s_i + w_n, \tag{3}$$

where w_n denotes the additive white Gaussian noise (AWGN) at the receiver of the n-th user. Successive interference cancellation (SIC) is carried out at the users – when possible. Therefore, the n-th user will detect the i-th user's message, if

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/LWC.2020.2987572, IEEE Wireless Communications Letters

$i < n$, and then remove the message from its observation, in a successive manner. If $i > n$, then the i-th message is treated as noise.

III. A GENERAL CLOSED-FORM EXPRESSION FOR THE OUTAGE PROBABILITY

Since SIC is used in NOMA, the n-th user detects the i-th user's message, $i < n$, then removes it in a successive manner. Furthermore, the message for the i-th user, $i > n$, will be treated as noise at the n-th user's receiver. As a result, the achievable rate of user n is

$$R_n = \log\left(1 + \frac{\rho|h_n|^2 p_n}{\rho|h_n|^2 \sum_{i=n+1}^{N} p_i + 1}\right), \tag{4}$$

conditioned on $R_{j\to n} \geq \tilde{R}_j$, where ρ denotes the transmit signal-to-noise ratio (SNR) i.e., $\rho = P/\sigma_n$, where σ_n denotes the variance of the AWGN at the receiver. $\tilde{R}_j$ denotes the target rate of the j-th user, and $R_{j\to n}$ denotes the rate at which the n-th user detects the j-th user's message, $j \leq n$, i.e.,

$$R_{j\to n} = \log\left(1 + \frac{\rho|h_n|^2 p_j}{\rho|h_n|^2 \sum_{i=j+1}^{N} p_i + 1}\right). \tag{5}$$

Moreover, we define $E_{n,j} \triangleq \{R_{j\to n} < \tilde{R}_j\}$ as the event that the n-th user cannot detect the j-th user's message for $1 \leq j \leq n$, and $E_{n,j}^c$ is the complementary set of $E_{n,j}$. Then, the outage probability at the n-th user can be expressed as [1]

$$\mathrm{P}_n^{\text{out}} = 1 - \mathrm{P}(E_{n,1}^c \cap \cdots \cap E_{n,n}^c). \tag{6}$$

Following a similar procedure as in [1] the outage probability can be formulated as

$$\begin{aligned}\mathrm{P}_n^{\text{out}} &= 1 - \mathrm{P}(|h_n|^2 > \psi_n^*)\\ &= \int_0^{\psi_n^*} \frac{N!\left(F_{|\tilde{h}|^2}(x)\right)^{n-1}\left(1 - F_{|\tilde{h}|^2}(x)\right)^{N-n} f_{|\tilde{h}|^2}(x)}{(n-1)!(N-n)!}dx,\end{aligned} \tag{7}$$

where $F_{|\tilde{h}|^2}(x)$ denotes the cumulative probability function (CDF) of the unordered channel gain and $f_{|\tilde{h}|^2}(x)$ is the respected probability density function (PDF). Next, unless otherwise stated, the term channel gain will refer to the unordered channel gain . Also, ψ_j is defined as

$$\psi_j \triangleq \frac{\phi_j}{\rho\left(p_j - \phi_j \sum_{i=j+1}^{N} p_i\right)} \quad \text{for } j \leq N, \tag{8}$$

with $\phi_j = 2^{\tilde{R}_j} - 1$. To obtain (8), the following expression should be satisfied

$$p_j > \phi_j \sum_{i=j+1}^{N} p_i. \tag{9}$$

Then, ψ_n^* is defined as $\psi_n^* = \max\{\psi_1, \ldots, \psi_n\}$.

The authors in [1] used a quadrature method to approximate the integral in (7) and then to perform a high SNR analysis. However, as mentioned above, this is an approximated solution which, most importantly, withholds the information about the system design. Next, the outage probability at the n-th user is expressed in closed-form.

Theorem 1: The outage probability of the n-th user in a NOMA system with randomly deployed users can be formulated as

$$\begin{aligned}\mathrm{P}_n^{\text{out}} = &\left[F_{|\tilde{h}|^2}(\psi_n^*)\right]^n \binom{N}{n} \times\\ &{}_2F_1(n, n-N; n+1; F_{|\tilde{h}|^2}(\psi_n^*)),\end{aligned} \tag{10}$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the Gauss Hypergeometric function [6] and $F_{|\tilde{h}|^2}(y)$ is the CDF of the channel gain.

Proof: The proof is given in Appendix A. ■

A. Insights from (10)

From (10), we can conclude the following insights:

1) For the practical case of $N = 2$, (10) can be simplified as [6]

$$\begin{aligned}\mathrm{P}_1^{\text{out}} &= 2F_{|\tilde{h}|^2}(\psi_n^*) - \left[F_{|\tilde{h}|^2}(\psi_n^*)\right]^2,\\ \mathrm{P}_2^{\text{out}} &= \left[F_{|\tilde{h}|^2}(\psi_n^*)\right]^2.\end{aligned} \tag{11}$$

From (11) it can be observed that the outage performance of the second user (with the best channel conditions) behaves similarly with that of a two branch selection diversity receiver with threshold ψ_n^*. For the first user (bad channel conditions) the first term dominates and thus, there is no essential change in the diversity order.

2) For the general case, the outage probability of the N-th user ($n = N$) can be simplified to

$$\mathrm{P}_N^{\text{out}} = \left[F_{|\tilde{h}|^2}(\psi_n^*)\right]^N, \tag{12}$$

since both the binomial coefficient and the hypergeometric function become unity, while for the rest users, i.e., $n < N$, the hypergeometric function is reduced to a polynomial of degree $N - n$. This is in line with the two user case above and means that users with better channel conditions perform better at the cost of increasing the outage probability at weaker users.

B. The scenario of no fading

Next, we investigate the scenario of *no fading*, where only path loss attributes for the channel gain. It is worth noting that this case can be treated as a lower bound of the outage probability in all cases, i.e., the best case scenario.

Corollary 1: For the case of randomly deployed users with no fading the outage probability is given by (13) at the top of the next page.

Proof: The proof is provided in Appendix B. ■

As it was expected, due to the absence of small-scale fading, the outage probability decreases rapidly when the SNR is high enough to accommodate even the weakest users in the system, e.g., those at the cell edge. This is evident from (13).

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$$\mathrm{P}_n^{\mathrm{out}} = \begin{cases} 0, & \psi_n^* < \frac{1}{1+R_D^a}, \\ \left[1 - \frac{\left[(\psi_n^*)^{-1}-1\right]^{\frac{2}{a}}}{R_D^2}\right]^n \binom{N}{n} {}_2F_1\left(n, n-N; n+1; 1 - \frac{\left[(\psi_n^*)^{-1}-1\right]^{\frac{2}{a}}}{R_D^2}\right), & \psi_n^* \in \left[\frac{1}{1+R_D^a}, 1\right], \\ 1, & \psi_n^* > 1. \end{cases} \quad (13)$$

IV. Performance analysis over Nakagami-m fading channels

In this section we utilize the general expression (10) to study the outage performance over Nakagami-m fading channels.

Let $\tilde{h}$ denote the channel gain coefficient. We assume that the users are uniformly distributed in a disc, $\mathcal{D}$, and the fading channel can be modeled through the Nakagami-m distribution with PDF given by [7]

$$f_N(x; m, \Omega) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m x^{2m-1} \exp\left(-\frac{m}{\Omega}x^2\right), \quad (14)$$

where $m \geq 1/2$ and $\Omega > 0$ are the shape and scale parameters of the distribution, respectively.

Theorem 2: The outage probability of the NOMA system with users uniformly distributed in a disk with radius R_D over Nakagami-m fading channels is given by (10), with

$$F_{|\tilde{h}|^2}(y) = 1 - \frac{2}{aR_D^2} \sum_{k=0}^{m-1} \sum_{i=0}^{k} \binom{k}{i} \frac{e^{-\frac{m}{\Omega}y}}{k!} \times \left(\frac{m}{\Omega}y\right)^{k-i-\frac{2}{a}} \gamma\left(\frac{2}{a}+i, \frac{m}{\Omega}R_D^a y\right), \quad (15)$$

where $\gamma(\alpha, x)$ is the incomplete gamma function [6].

Proof: The proof is given in Appendix C. ■

Note that the corresponding PDF of the channel gain can be derived after taking the first derivative of the CDF in (15) with regards to y and after some algebraic manipulations as

$$f_{|\tilde{h}|^2}(y) = \frac{2me^{-\frac{m}{\Omega}y}}{aR_D^2\Omega} \sum_{k=0}^{m-1} \sum_{i=0}^{k} \binom{k}{i} \frac{1}{k!} \left(\frac{my}{\Omega}\right)^{k-i-\frac{2}{a}} \times \left[-R_D^{ai+2}\left(\frac{my}{\Omega}\right)^{\frac{2}{a}+i-1} e^{-\frac{m}{\Omega}R_D^a y} - \left(k-i-\frac{2}{a}\right) \times \frac{\Omega}{my}\gamma\left(\frac{2}{a}+i, \frac{m}{\Omega}R_D^a y\right) + \gamma\left(\frac{2}{a}+i, \frac{m}{\Omega}R_D^a y\right)\right]. \quad (16)$$

Next, we present a closed-form expressions for the outage probability in the important special case of Rayleigh fading ($m = 1$). This is the case assumed in [1], where it was numerically approximated by a Gaussian quadrature method.

Corollary 2: The outage probability of the NOMA system when users are uniformly distributed in a disk with radius R_D and experience Rayleigh fading can be expressed as in (10) with

$$F_{|\tilde{h}|^2}(y) = 1 - \frac{2e^{-y}\gamma\left(\frac{2}{a}, yR_D^a\right)}{aR_D^2 y^{\frac{2}{a}}}. \quad (17)$$

Proof: Using $m = 1$ in (15) and after some algebraic manipulations (17) is derived. ■

A. Diversity Order

Lemma 1: The diversity order of the n-th user in a NOMA system with randomly deployed users over Nakagami-m fading channels is $d = \frac{m}{\Omega}n$.

Proof: When $\rho \to \infty$, it can be shown that $\psi_n^* \to 0$ [1]. In this case, the following approximation holds when $x \to 0$:

$$\frac{\gamma(a, x)}{x^a} \approx \frac{1}{a}. \quad (18)$$

Taking advantage of (18), it holds that

$$\frac{\gamma\left(\frac{2}{a}+i, \frac{m}{\Omega}R_D^a y\right)}{\left(\frac{m}{\Omega}R_D^a y\right)^{\frac{2}{a}+i}} \approx \frac{1}{\frac{2}{a}+i}, \quad (19)$$

and we can approximate (15) as

$$F_{|\tilde{h}|^2}(\psi_n^*) \approx 1 - \frac{2e^{-\frac{m}{\Omega}\psi_n^*}}{a} \sum_{k=0}^{m-1} \sum_{i=0}^{k} \binom{k}{i} \frac{R_D^{ai}}{k!} \left(\frac{m}{\Omega}\psi_n^*\right)^k \frac{1}{\frac{2}{a}+i}. \quad (20)$$

For $k > 0$, the double sum in (20) tends to zero as $\psi_n^* \to 0$ due to the term $\left(\frac{m}{\Omega}\psi_n^*\right)^k$. Therefore, we can approximate the CDF in the high SNR regime by taking only the term $k = 0$. Then, the CDF can be expressed as

$$F_{|\tilde{h}|^2}(\psi_n^*) \approx 1 - \frac{2e^{-\frac{m}{\Omega}\psi_n^*}}{a}\frac{a}{2} = 1 - e^{-\frac{m}{\Omega}\psi_n^*}. \quad (21)$$

Finally, by plugging (21) into (10), the latter can be approximated as

$$\mathrm{P}_n^{\mathrm{out}} \approx \left(1 - e^{-\frac{m}{\Omega}\psi_n^*}\right)^n \binom{N}{n} \times {}_2F_1\left(n, n-N; n+1; 1 - e^{-\frac{m}{\Omega}\psi_n^*}\right). \quad (22)$$

The function ${}_2F_1(a, b; c; x)$ in (22) tends to 1 as $x \to 0$. Therefore, we examine only the first term, $\left(1 - e^{-\frac{m}{\Omega}\psi_n^*}\right)^n$, since it is the one that drives the outage probability to zero. By expressing the exponential term as a Taylor series with the first two terms only, since ψ_n^* is very small, we get ${\psi_n^*}^{\frac{mn}{\Omega}}$. Since ψ_n^* is a term related to $\frac{1}{\rho}$ [1] , it holds that

$$P_n^{\mathrm{out}} \to \rho^{-\frac{mn}{\Omega}}. \quad (23)$$

So, the diversity order is $d = \frac{mn}{\Omega}$ and the proof is completed. ■

Note that this result agrees with the expression for the diversity order given in [1] for Rayleigh fading channels (when $m = 1$ and $\Omega = 1$).

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V. Numerical Results and Discussion

In this section, the outage performance of NOMA is evaluated under different path loss and fading conditions for $N = 2$, assuming that the power allocation coefficients are $p_1 = 4/5$, $p_2 = 1 - p_1$. Moreover, Monte Carlo simulations are provided for 10^7 iterations, which match perfectly with the analytical results. Specifically, in Fig. 1, the outage probability of two users is plotted with regards to the transmit SNR ρ normalized to threshold SNR $\rho_{\text{thr}} = \phi_j$, $\forall j$. Moreover, it is assumed that $R_D = 5$, $\Omega = 1$, and $a = 3$. It can be easily observed that the diversity order follows (23). Therefore, as an example, a user with weak channel conditions ($n = 1$) that experiences fading with $m = 2$ has the same diversity order with a user with strong channel conditions ($n = 2$) that experiences Rayleigh fading ($m = 1$).

It is also remarkable that for the low SNR region, the outage probability of the weaker user is similar or even lower to the one of the stronger user for the same value of m, which is due to the utilized power allocation. However, as the SNR increases the stronger user always achieves lower outage probability than the weaker user, which is in line with the derived diversity order. In general, this figure can be efficiently used for the system design in different fading environments.

Finally, in Fig. 1 the case of no fading is presented, which acts as a lower bound for the outage probability. It can easily be observed that the outage probability in this case decreases rapidly with the increase of the SNR. This result is expected due to the form of (13), since after a certain SNR value the outage probability drops to zero.

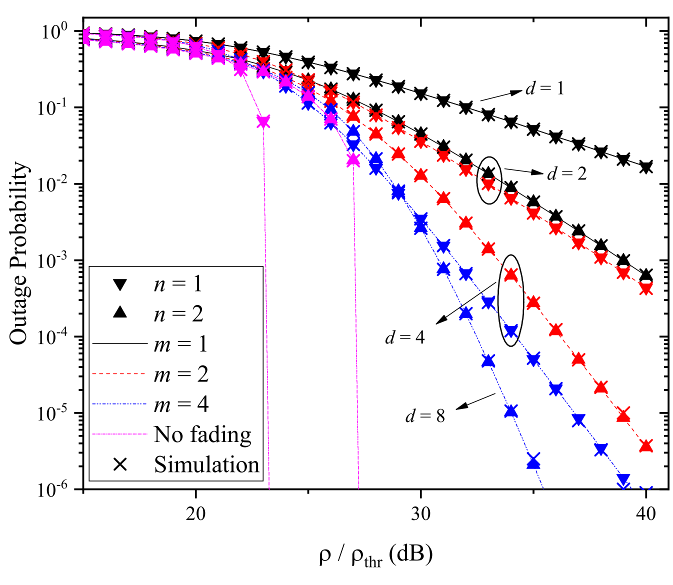

Fig. 1. Outage Probability versus normalized transmit SNR

VI. Conclusion

In this paper we have improved the results presented in [1], by providing an elegant closed-form expression for the outage probability in a NOMA system with randomly deployed users over generalized fading channels. This expression can be used for any fading distribution and revealed useful insights for the system design. Moreover, we have evaluated the important case of Nakagami-m fading and obtained a closed-form analysis as well a useful simple formula for the diversity order. In future research, the general closed-form expression could be used to evaluate the outage probability under different channel conditions (e.g., Rician, Weibull, Generalized Gamma or other composite fading channels).

Appendix A
Proof of Theorem 1

To calculate the outage probability, (7) is rewritten as

$$\mathrm{P}_n^{\text{out}} = \int_0^{\psi_n^*} \frac{N! A(x) B(x) f_{|\tilde{h}|^2}(x)}{(n-1)!(N-n)!} dx, \tag{24}$$

with

$$A(x) = \left(F_{|\tilde{h}|^2}(x)\right)^{n-1}, \; B(x) = \left(1 - F_{|\tilde{h}|^2}(x)\right)^{N-n}. \tag{25}$$

$B(x)$ can be expressed with the help of the binomial theorem as

$$B(x) = \sum_{i=0}^{N-n} \binom{N-n}{i} \left(-F_{|\tilde{h}|^2}(x)\right)^i. \tag{26}$$

We can express the term $A(x)B(x)$ that appears in (24) as

$$A(x)B(x) = \left(F_{|\tilde{h}|^2}(x)\right)^{n-1} \left(1 - F_{|\tilde{h}|^2}(x)\right)^{N-n} = \sum_{i=0}^{N-n} \binom{N-n}{i} (-1)^i \left(F_{|\tilde{h}|^2}(x)\right)^{n+i-1}, \tag{27}$$

where $n + i - 1$ is a non negative integer number $\forall i \in \{0, 1, 2, ..., N - n\}$.

Also, by taking advantage of the binomial theorem

$$\frac{N!}{(n-1)!(N-n)!} = n\binom{N}{n} \tag{28}$$

and the fact that the PDF is the derivative of the CDF, after some algebraic manipulations the outage probability in (24) can be written as,

$$P_n^{\text{out}} = n\binom{N}{n} \sum_{i=0}^{N-n} (-1)^i \frac{\binom{N-n}{i} F_{|\tilde{h}|^2}(\psi_n^*)^{n+i}}{n+i}, \tag{29}$$

since it holds that $F_{|\tilde{h}|^2}(0) = 0$. Using the definition of the Gauss Hypergeometric function [6] we get (10) and this concludes the proof.

Appendix B
Proof of Corollary 1

Only the CDF of the path loss is required to derive (13) from (10). The CDF of the distance of uniformly distributed users in a disc is given by

$$F_Z(z) = \frac{z^2}{R_D^2}, \; z \in [0, R_D] \tag{30}$$

Following this, the CDF of the path loss can be easily derived from the CDF of distance as

$$F_{|\tilde{h}|^2}(y) = \mathrm{P}\left(\frac{1}{1+z^a} \leq y\right) = \mathrm{P}\left(z \geq (y^{-1}-1)^{\frac{1}{a}}\right) = 1 - F_Z\left((y^{-1}-1)^{\frac{1}{a}}\right) = 1 - \frac{(y^{-1}-1)^{\frac{2}{a}}}{R_D^2}, \quad (31)$$

when $z \in [0, R_D]$, i.e., when $y \in \left[\frac{1}{1+R_D^a}, 1\right]$. Also, $\forall z \geq R_D \Rightarrow y \leq \frac{1}{1+R_D^a}$, it holds that $F_Z(z) = 1 \Rightarrow F_{|\tilde{h}|^2}(y) = 0$. Similarly, $\forall z \leq 0 \Rightarrow y \geq 1$ it holds $F_{|\tilde{h}|^2}(y) = 0$. By plugging (31) into (10), (13) is derived and proof is completed.

Appendix C
Proof of Theorem 2

Following a similar procedure as in [8, Section III], the conditional CDF of the unordered channel gain given the distance z from the BS of a user is given by

$$F_{|\tilde{h}|^2|Z}(y) = \mathrm{P}(|\tilde{h}|^2 \leq y|z) = \mathrm{P}(|\tilde{g}|^2 \frac{1}{1+z^a} \leq y|z), \quad (32)$$

where $\mathrm{P}(\cdot)$ denotes probability.

Since $|\tilde{g}|$ follows a Nakagami-m distribution, its square follows a gamma distribution. Therefore (32) can be expressed via the CDF of the gamma distribution as

$$F_{|\tilde{h}|^2|Z}(y) = \frac{\gamma\left(m, \frac{m}{\Omega}(1+z^a)y\right)}{\Gamma(m)}, \quad (33)$$

where $\Gamma(\cdot)$ is the gamma function [6] and $\gamma(a,x)$ is the lower incomplete gamma function [6].

Following this, the CDF of the channel gain can be obtained with the help of the PDF of the location of a random user in the disk, which is given by

$$f_Z(z) = \frac{2z}{R_D^2},\ z \in (0, R_D). \quad (34)$$

Then, the CDF can be expressed as

$$F_{|\tilde{h}|^2}(y) = \int_{\mathcal{D}} F_{|\tilde{h}|^2|Z}(y) f_Z(z) dz = \int_{\mathcal{D}} \frac{\gamma(m, \frac{m}{\Omega}(1+z^a)y}{\Gamma(m)} \times \frac{2z}{R_D^2} dz = \frac{2}{R_D^2 \Gamma(m)} \int_0^{R_D} \gamma\left(m, \frac{m}{\Omega}(1+z^a)y\right) z dz. \quad (35)$$

In order to evaluate (35), we consider the case that m is finite and integer. Using the lower incomplete gamma function's series representation [6, 8.352],

$$\gamma(a,x) = (a-1)!\left(1 - e^{-x}\sum_{k=0}^{a-1}\frac{x^k}{k!}\right). \quad (36)$$

As such, (35) can be expressed as

$$F_{|\tilde{h}|^2}(y) = \frac{2(m-1)!}{R_D^2\Gamma(m)}\int_0^{R_D}\left(1 - e^{-\frac{m}{\Omega}(1+z^a)y} \times \sum_{k=0}^{m-1}\frac{\left(\frac{m}{\Omega}(1+z^a)y\right)^k}{k!}\right) z dz. \quad (37)$$

After some algebraic manipulations, (37) can be expressed as

$$F_{|\tilde{h}|^2}(y) = 1 - \frac{2e^{-\frac{m}{\Omega}y}}{R_D^2}\sum_{k=0}^{m-1}\frac{\left(\frac{m}{\Omega}y\right)^k}{k!}\int_0^{R_D} e^{-\frac{m}{\Omega}z^a y}(1+z^a)^k z dz. \quad (38)$$

In order to calculate the CDF of the channel gain it is evident that we just need to evaluate the integral

$$I(y) = \int_0^{R_D} e^{-\frac{m}{\Omega}z^a y}(1+z^a)^k z dz = \int_0^{R_D} e^{-\frac{m}{\Omega}z^a y} z \times \sum_{i=0}^{k}\binom{k}{i} z^{ai} dz = \sum_{i=0}^{k}\binom{k}{i}\int_0^{R_D} e^{-\frac{m}{\Omega}z^a y} z^{ai+1} dz, \quad (39)$$

which is derived by utilizing the binomial theorem. After substituting $z^a = t$ in (39) it can be written as

$$I(y) = \frac{1}{a}\int_0^{R_D^a} e^{-\frac{m}{\Omega}ty} t^{i-1+\frac{2}{a}} dt. \quad (40)$$

By using the definition of the lower incomplete gamma function,

$$\gamma(a,x) = \int_0^x e^{-x} t^{a-1} dt, \quad (41)$$

$I(y)$ can be written as

$$I(y) = \frac{1}{a}\left(\frac{m}{\Omega}y\right)^{-i-\frac{2}{a}} \gamma\left(\frac{2}{a}+i, \frac{m}{\Omega}R_D^a y\right). \quad (42)$$

By applying (42) in (38) we get the final expression for the CDF, (15), and the proof is completed.

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