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Spectral efficiency for selection combining RAKE receivers over Weibull fading channels

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Abstract

Novel closed-form expressions for the probability density function and the average output signal-to-noise ratio at the output of a selection combiner in Weibull fading are derived. Using these expressions, the spectral efficiency of a direct sequence code division multiple access system is analytically obtained and performance evaluation results are presented.

Keywords: Spectral efficiency; Diversity; Weibull fading; RAKE receivers; Average signal-to-noise ratio

1. Introduction

Spectral efficiency is of primary concern in the design of modern mobile radio systems and the utilization of the associated communication channels. Spectral efficiency (SE) is defined as the data rate per unit channel bandwidth for a specified average transmitted power and a fixed bit-error-rate (BER) value. With respect to this, the maximum spectral efficiency of a communication channel of bandwidth W would be equal to C/W, where C denotes the Shannon capacity [1] of this channel.

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For time-invariant channels, a variety of modulation and coding schemes have been investigated in order to further increase the already achieved spectral efficiency, while, for single-user channels, Shannon capacity can readily be used to estimate the upper bound of the achievable spectral efficiency, in bits/s/Hz. Commonly, in any communication system, physical channels can be shared among all users according to one of three basic accessing schemes: time division multiple access, frequency division multiple access, and code division multiple access (CDMA). In an ideal additive white Gaussian noise (AWGN) environment, the theoretically achieved spectral efficiency of a radio system utilizing any of those multiple access techniques will be the same, since it does not matter whether the available signal space (time and bandwidth) is divided into time-slots, frequencies, or codes. However, in mobile radio, physical channels exhibit randomly time-varying characteristics that result in signal fading and substantial capacity degradation. Hence, the theoretical equivalence of the three multiple-access schemes does not exist and the spectral efficiency achieved by each of them depends on the physical model of the fading channel, i.e. on what is known about the particular channel. For example, if nothing is known for the fading statistics of a channel then its countable capacity (in bits/s) will be dictated by the minimum (due to fading) signal-to-noise ratio (SNR), γ_{min}, and will thus tend to zero as $\gamma_{min} \to 0$. On the other hand, if fading statistics is known, then an "average" capacity formula can be derived after the distribution of the fading SNR γ for a fixed transmission rate [2].

One of the simplest and yet most efficient techniques to overcome the destructive effects of fading in wireless communication systems is diversity. For all diversity techniques the receiver has to process the obtained diversity signals in a fashion that maximizes the system's power efficiency. There are several diversity reception methods employed in communication receivers including equal gain combining (EGC), maximal ratio combining (MRC), selection combining (SC), and a combination of MRC and SC, called generalized selection combining (GSC). MRC is the optimal combining scheme but it comes at the expense of increased complexity since knowledge of all channel parameters, which usually affect the received signal, is required. EGC provides an intermediate solution for improved overall performance and low implementation complexity. For GSC(L, L_{c}), the L_{c} strongest signal(s) out of L available diversity branches are optimally combined. However, when L is large, GSC will give poorer error performance than MRC or EGC. SC is the least complicated since the processing is performed only on one of the diversity branches. Traditionally, in SC, the combiner will choose the branch with the highest SNR, which corresponds to the strongest signal, if equal noise power is assumed among the branches [3].

The performance of diversity receivers has been studied extensively in the past for several well-known fading channel models, such as Rayleigh, Rice, Nakagami-m and Nakagami-q, assuming independent or correlative fading [3]. However, another well-known fading channel model, namely the Weibull model, has not yet received as much attention, despite the fact that it provides an excellent fit to experimental fading channel measurements for both indoor [4] and outdoor environments [5]. Only very recently, the topic of communications in Weibull fading channels has begun to receive renewed interest. For example, Alouini and Simon have presented an analysis for the evaluation of GSC diversity receiver's performance [6].

In this paper, the probability density function (PDF) and the average SNR at the output of a SC diversity receiver in Weibull fading environment, is derived analytically in closed-form expressions, with arbitrary parameters for the channel's severity of fading as well as the number of diversity branches. Using these expressions and following the methodology presented in [7], the achievable SE of a non-cooperative direct sequence code division multiple access (DS-CDMA) system, assuming SC RAKE reception, is obtained.

The remainder of this paper is organized as follows: in Section 2, the statistical analysis of the SC output SNR in Weibull fading is performed. Following this, in Section 3, the average SE in DS-CDMA systems is obtained. In Section 4, numerical results are presented and finally, concluding remarks are presented in Section 5.

2. Statistical analysis of SC in Weibull fading

We consider a SC diversity receiver with L branches operating in a Weibull fading environment. Assuming that the fading envelopes among the L branches are statistically independent, the cumulative density function (CDF) of the envelopes in the ith, $1 \leq i \leq L$, branch is given by [8]

$$F_{r_i}(r_i) = 1 - \exp\left[-\left(\frac{r_i}{\omega_i}\right)^{\beta}\right] \tag{1}$$

with $\omega_i = \sqrt{\overline{r_i^2}/\Gamma(d_2)}$, where $\Gamma(\bullet)$ is the Gamma function, $d_k = 1 + k/\beta, \overline{r_i^2}$ is the average signal power and β is the Weibull shape parameter $\beta > 0$. As β increases, the severity of fading decreases and for $\beta = 2$, Eq. (1) reduces to the Rayleigh CDF. The instantaneous SNRs are

$$\gamma_i = \frac{E_s}{N_o} r_i^2, \tag{2}$$

where E_s is the symbol energy and N_o is the noise power spectral density. The corresponding average SNRs are

$$\bar{\gamma}_i = \frac{E_s}{N_o}\overline{r_i^2} = \frac{E_s}{N_o}\omega_i^2\,\Gamma(d_2) \tag{3}$$

and thus using Eqs. (1–3), the CDF of the Weibull distributed SNRs can be rewritten as

$$F_{\gamma_i}(\gamma_i) = 1 - \exp\left[-\left(\frac{\gamma_i}{a\bar{\gamma}_i}\right)^{\beta/2}\right], \tag{4}$$

where $a = 1/\Gamma(d_2)$. Defining the instantaneous SNR at the combiner's output as $\gamma_{sc} = \max(\gamma_i)$, the CDF of γ_{sc} can be expressed as

$$F_{\gamma_{sc}}(\gamma_{sc}) = \prod_{i=1}^{L}\left\{1 - \exp\left[-\left(\frac{\gamma_{sc}}{a\bar{\gamma}_i}\right)^{\beta/2}\right]\right\} \tag{5}$$

and by differentiating Eq. (5), the corresponding PDF of the output SNR is obtained as

$$p_{\gamma_{sc}}(\gamma_{sc}) = \frac{\beta}{2}\sum_{i=1}^{L}\frac{\gamma_{sc}^{\frac{\beta}{2}-1}}{(a\bar{\gamma}_i)^{\frac{\beta}{2}}}\exp\left[-\left(\frac{\gamma_{sc}}{a\bar{\gamma}_i}\right)^{\frac{\beta}{2}}\right]\prod_{\substack{j=1\\ j\neq i}}^{L}\left\{1 - \exp\left[-\left(\frac{\gamma_{sc}}{a\bar{\gamma}_j}\right)^{\frac{\beta}{2}}\right]\right\}. \tag{6}$$

The first moment of Eq. (6) is the average output SNR, i.e.

$$\bar{\gamma}_{sc} = \int_0^{\infty} \gamma_{sc}\; p_{\gamma_{sc}}(\gamma_{sc})\; d\gamma_{sc}. \tag{7}$$

Assuming that the fading envelopes of the diversity input channels are independent and identical distributed (i.i.d.) random variables $\left(\text{i.e. } \bar{\gamma}_i = \bar{\gamma},\ \forall i\right)$, the above integral can be solved using (3.326/2) of [9]. Thus, after some straightforward simplifications, the gain of the combiner is derived in closed form as

$$G = \frac{\bar{\gamma}_{sc}}{\bar{\gamma}} = \sum_{m=0}^{L-1}(-1)^{L-1-m}\binom{L}{m}(L-m)^{-2/\beta}. \tag{8}$$

It is interesting to confirm that when $\beta \to \infty$, which is the no fading case, $G \to 1$.

3. SE in DS-CDMA systems

For a user transmitting a signal with bandwidth W and power P over an AWGN channel, the received SNR is given by $\gamma = P/(N_o W)$. When K such users are simultaneously transmitting in a non-cooperative DS-CDMA system, the (K–1) user's signals appeared as multiple access interference at the receiver of each user. Assuming that each user's pseudorandom signal waveform is Gaussian distributed [10], then the received spread to bandwidth W_{ss} signal-to-interference ratio (SIR) (prior despreading) is

$$\gamma_{ss} = \frac{P}{N_o W_{ss} + (K-1)P} = \frac{\gamma}{G_p + (K-1)\gamma} \tag{9}$$

with $G_p = W_{ss}/W$ being the processing gain. Then, the SE for the AWGN channel is given by [11]

$$S_e = \log_2(1 + \gamma_{ss}). \tag{10}$$

Considering SC RAKE receivers, we assume that each user's receiver has L taps corresponding to L resolvable signal paths with $L = \lceil W_{ss} T_m \rceil + 1$, where T_m is the total multipath channel's delay spread, on the condition that W_{SS} is much greater than the coherence bandwidth of the channel and $\lceil x \rceil$ is the maximum integer less than or equal to x. The channel assigned to each user is modelled as a time invariant multipath tapped delay line and thus the resolvable paths model can be assumed to have equal path strengths on average. For a fading environment, the average SE [2] is obtained averaging (10) over the PDF of the instantaneous output SIRs, γ_{ss}, of the SC RAKE receiver as

$$\langle S_e \rangle = \int_0^\infty \log_2\big(1+\gamma_{ss}\big) p_{\gamma_{ss}}(\gamma_{ss})\, d\gamma_{ss}. \tag{11}$$

Using Eqs. (6), (8) and (9) and the transformation $x = (L-m)\big(\gamma_{ss}/a\bar{\gamma}\big)^{\beta/2}$, (11) can be further expressed as

$$\langle S_e \rangle = \sum_{m=0}^{L-1}(-1)^{L-1-m}\binom{L}{m} \times \int_0^\infty \log_2\left[1+\frac{1}{G}\frac{\gamma}{G_p+(K-1)\gamma}\frac{a}{(L-m)^{2/\beta}}\chi^{2/\beta}\right]\exp(-x)\, dx. \tag{12}$$

The above integral is easily evaluated via numerical integration, using a well-known mathematical package, such as *Mathematica*. It is worth noticing that for a

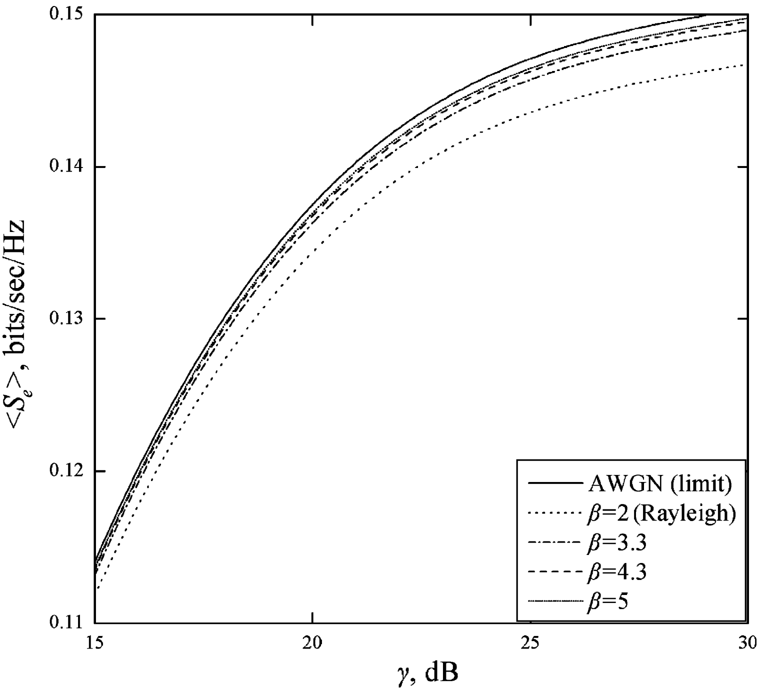

Fig. 1. Average spectral efficiency, $\langle S_e \rangle$, as a function of γ, for $L = 2$ taps and several values of β.

transmitted signal with $W_{ss} \to \infty$, i.e. $G_p \to \infty$, using the approximation $\lim_{x\to 0}[\log_2(1+x)] = x/\log_2(e)$, $\langle S_e \rangle$ in Eq. (12) simplifies to

$$\lim_{G_P\to\infty} \langle S_e \rangle = \frac{1}{\log_2(e)} \frac{\gamma}{G_P + (K-1)\gamma} \approx 1.44 \frac{\gamma}{G_P}. \tag{13}$$

Alternatively, the above expression can be obtained, if the same assumption for infinite spread bandwidth is used for Eq. (10). Hence, considering an infinite bandwidth DS-CDMA system, the upper limit for $\langle S_e \rangle$ will be the Shannon–Hartley channel capacity limit to W ratio.

4. Numerical results

Eq. (12) was numerically evaluated for a typical urban area with $T_m = 3\,\mu s$, $K = 10$ users, $W = 30\,kHz$, $L = 2$ and 4 taps and the results are illustrated in Figs. 1 and 2, respectively. For comparison purposes, the Shannon-Hartley (i.e. AWGN channel) limit is also included in both figures. As clearly shown, for a fixed value of γ, $\langle S_e \rangle$ obtains significantly higher values when $\beta \geq 3.3$, than those for $\beta = 2$. In addition, as β increases, $\langle S_e \rangle$ tends to become identical to that for the AWGN

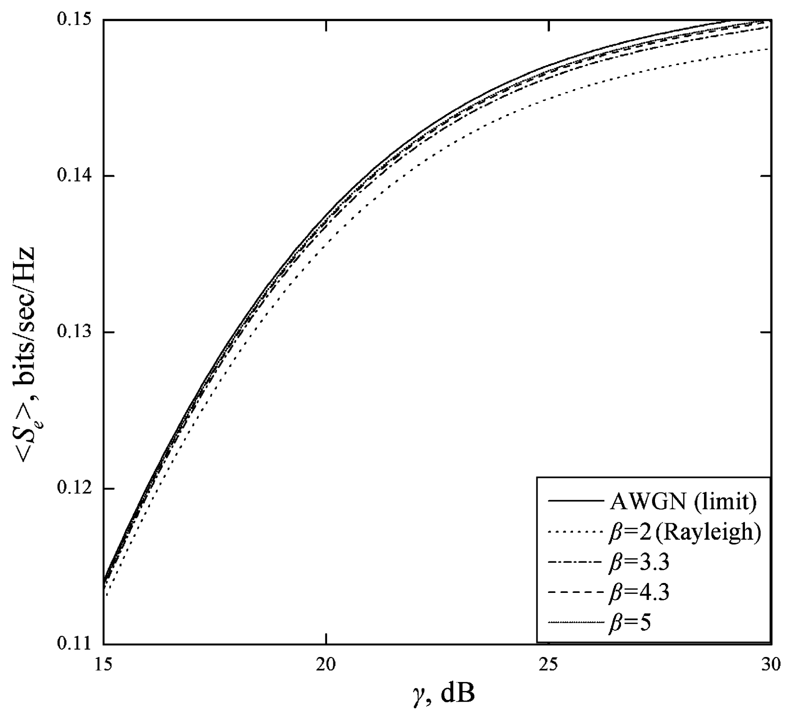

Fig. 2. Average spectral efficiency, $\langle S_e \rangle$, as a function of γ, for $L = 4$ taps and several values of β.

channel. This trend can be easily explained since for $\beta \to \infty$ the argument of the logarithm in Eq. (12) becomes equal to Eq. (9), which does not include the summation index m. Therefore, the sum in Eq. (12) becomes unity and Eq. (12) is further simplified to Eq. (10). As it is also observed in Fig. 1 ($L = 2$), for $\beta \geq 3.3$, the average SE tends to that of the AWGN channel. However, in Fig. 2 ($L = 4$), the fading severity has a minor effect on the channel capacity, since the gain of the SC has significantly increased and the SE, even with $\beta > 2$ is very close to the curve of the AWGN channel.

5. Conclusions

New closed-form expressions for the PDF and the average output SNR of a SC diversity receiver, in Weibull fading environment, have been obtained. One step further, we derive an expression for the average SE of a non-cooperative DS-CDMA system with SC RAKE reception. As shown, the analysis fully conforms to the Shannon-Hartley limit, while for a Weibull channel with, $\beta \geq 3.3$ the average SE tends to that of the AWGN channel.

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