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Performance Analysis of Distributed Uplink NOMA

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***Abstract*—We propose an optimal joint user association and decoding order selection scheme for distributed uplink non-orthogonal multiple access (DU-NOMA) with fixed transmission rates. It is considered that the remote radio heads can either perform decoding independently from each other or exchange information via a dedicated feedback link with limited capacity. The outage probability is derived in closed-from and useful insights for the high signal-to-noise ratio regime are provided. Finally, it is illustrated that DU-NOMA with feedback together with the proposed coordination scheme achieves substantial gain compared to DU-NOMA without feedback. Also, in most of the practical cases it outperforms orthogonal multiple access schemes, even if the feedback link is not error-free.**

***Index Terms*—Non-orthogonal multiple access, remote radio heads, coordination, outage probability.**

I. Introduction

In order to meet the tremendous wireless traffic growth, fifth generation (5G) wireless networks and beyond have evolved as one of the hottest research topics both for industry and academia. Nevertheless, conveying an exponentially increasing amount of data to the end users within an acceptable latency will inevitably lead to severe congestion of the RF spectrum. The prime objectives or demands that need to be addressed in 5G wireless networks and beyond are: increased connectivity, improved data rate, decreased latency, reduced energy consumption, and better quality-of-service. To meet these demands, drastic improvements must be made in the current cellular network architecture.

Non-orthogonal multiple access (NOMA) has become a great paradigm for the design of radio access techniques for the wireless networks beyond 5G, since, compared to orthogonal multiple access (OMA), it can increase spectral efficiency and connectivity, with the latter being crucial in internet-of-things (IoT) applications [1]–[3]. More specifically, NOMA enables the users' multiplexing in the power domain, i.e., by using the same resource block in the time/frequency/code domain [4]. To this end, superposition coding and successive interference cancellation (SIC) can be employed (power-domain NOMA). Although NOMA has been extensively investigated, distributed NOMA -where distributed remote radio heads (RRHs) serve a number of users by utilizing the same resource block- as well as uplink NOMA have only be considered in a few works. However, both directions are particularly important and of high practical value. This becomes evident considering the importance of uplink for a vast number of wireless applications, such as IoT and teleconference, as well as the increasing interest for novel architectures, e.g., cloud-radio access networks (C-RANs), that are based on distributed deployments and the use of lightweight access points together with cloud computing resources.

The performance and practical challenges of distributed downlink NOMA have been considered in [5], where the corresponding capacity region is also discussed. In [6], the ergodic sum-rate gain of uplink NOMA over OMA was investigated. In [7], the use of uplink NOMA for random access-based applications was proposed, where among others the outage probability was derived assuming a two-user scenario with a single base station. Also, in [8], uplink NOMA with energy constraints was investigated, focusing on throughput maximization and fairness among users. In the pioneering work [9], the use of NOMA in coordinated muti-point (CoMP) networks has been identified as a promising scheme for beyond 5G networks, since it facilitates spectrum sharing between neighboring base stations that can collaborate to perform SIC exchanging decoded messages via a feedback link. CoMP becomes more flexible if a global medium access control (MAC) entity is used, which is enabled by the C-RAN technology [10]. The application of distributed NOMA (DU-NOMA) for the uplink of C-RANs has been proposed in [11], where the maximization of the capacity region under the assumption of adaptive transmission rates was considered. In more detail, in [11], it was assumed that the RRHs exchange digital information through high capacity error-free links. However, although several candidate technologies have been proposed to reduce the capacity cost for the fronthauling of future dense wireless networks, such as optical wireless communications and mmWave, where the probability of outage cannot be ignored [10], [12], [13], the impact of imperfect feedback link on DU-NOMA has not been investigated yet. Also, the performance of DU-NOMA has not been studied yet in terms of the outage probability, which is a significant metric for applications with fixed transmission rates.

To address the aforementioned issues, in this paper, we investigate the performance of DU-NOMA with fixed transmission rates, assuming that the RRHs can either perform decoding independently from each other or exchange information via a dedicated feedback link with limited capacity. More specifically, an optimal joint user association and decoding order selection scheme is proposed and the corresponding users' outage probability is derived in closed-form, focusing on the case of two RRHs. Moreover, the extension of the analysis to the general case regarding the number of RRHs is also considered. Furthermore, useful insights for the high signal-to-noise ratio (SNR) regime are provided. Finally, it is shown that the use of DU-NOMA with feedback together with the proposed coordination scheme achieves substantial gain compared to DU-NOMA without feedback and in most of the practical cases outperforms orthogonal multiple access schemes, even if the feedback link is not error-free.

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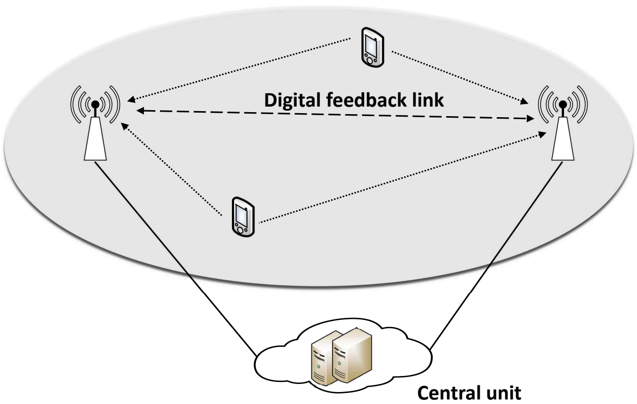

Fig. 1. System model of DU-NOMA with two users and two RRHs

II. System Model

We consider a wireless network that consists of two users, i and j, and two RRHs, a and b. It is assumed that all nodes use a single antenna for the communication between the users and the RRHs. We adopt a partially centralized cloud architecture [14], [15], where the RRHs have decoding capabilities, and the central unit is able to perform complex calculations or centralized decisions. In partially centralized CRAN architecture, distributed RRHs decode the received signals, not allowing their observations to be jointly processed through beamforming. It is further assumed that the RRHs can either perform decoding independently from each other or cooperate by exchanging decoded messages during SIC, through a dedicated digital feedback link, as it is shown in Fig. 1. Thus, three DU-NOMA schemes are identified and investigated, namely:

- *DU-NOMA without feedback.*
- *DU-NOMA with ideal feedback link*: The feedback link is assumed to be error-free. This can be implemented by using high capacity fiber optics.
- *DU-NOMA with non-ideal feedback link*: The users' transmission rates may exceed the capacity of the feedback link and errors can occur, which -among others- corresponds to the case that a wireless link is used.

Let γ_{kw} denote the SNR of the link between the user k and the RRH w, the avarage value of which is given by $\bar{\gamma}_{kw} = \frac{\Omega_{kw} p_k}{N_0}$, with Ω_{kw} being the mean-square value of the fading amplitude α_{kw}, that corresponds to the channel between user k and RRH w, i.e., $\Omega_{kw} = \mathbb{E}[\alpha_{kw}^2]$, where $\mathbb{E}[\cdot]$ denotes expectation. Also, N_0 is the power spectral density of the additive white Gaussian noise and p_k is the transmit power by the user k. Furthermore, Rayleigh fading is assumed, according to which γ_{kw} follows the exponential distribution with parameter $\lambda_{kw} = \frac{1}{\bar{\gamma}_{kw}}$.

Interestingly, DU-NOMA increases the degrees of freedom and creates new types of scheduling problems. Except of the decoding order, i.e., which of the users' messages is decoded first, the selection of RRH that decodes each user's message also affects the achievable rate. Let the policy $z = \{z_1, z_2\} = \{(k,w),(l,x)\}$ correspond to the case where user k is decoded by the RRH w, while its message is decoded prior to message of user l, with the latter being decoded by RRH x. Imperfect SIC is considered, according to which if decoding the message of a user is unsuccessful, its effect on the signal of other users with lower decoding orders is taken into account as interference [16]. Thus, by also considering the impact of the non-ideal feedback link, the achievable rate for user k with $k,l \in \{i,j\}$, $k \neq l$ and $w,x \in \{a,b\}$ is given by

$$C_{k,z} = \log_2(1+\Gamma_{k,z}) \quad \text{bits/s/Hz}, \tag{1}$$

where

$$\Gamma_{k,z} = \begin{cases} \frac{\gamma_{kw}}{\gamma_{lw}+1}, \text{if } z = \{(k,w),(l,x)\}, w,x \in \{a,b\}, \\ \frac{\gamma_{kw}}{(1-s_z^{\mathrm{SIC}} s_z^{\mathrm{f}})\gamma_{lw}+1}, \text{if } z = \{(l,x),(k,w)\}, w,x \in \{a,b\}. \end{cases} \tag{2}$$

Also, $s_z^{\mathrm{SIC}} \in \{0,1\}$ and $s_z^{\mathrm{f}} \in \{0,1\}$ are binary variables which are equal to one if the decoding of the first message is successful and if the decoded message is successfully received by the RRH that decodes the second signal, respectively. It is noted that when $x = w$, which implies that no message exchange between the RRHs is needed, s_z^{f} is always equal to one.

Furthermore, it is assumed that the channels are perfectly estimated by the RRHs, while the users have no available channel state information (CSI) and they transmit with fixed rate $R_k, k \in \{i,j\}$. Thus, the threshold received signal-to-interference-plus-noise ratio for successful decoding of the message of user k is $r_k = 2^{R_k} - 1$.

III. Performance Analysis

A. User Association and Decoding Order Optimization

For the operation of the distributed RRHs a unified resource optimizer is assumed, which operates in synergy with a global MAC entity, unifying all the resource management operation including allocation, interference management and signalling for different RRHs [10]. Also, for the system's performance optimization the following centralized three-step procedure is proposed:

- **Step 1**: The unified resource optimizer attempts to find a policy z that achieves the decoding of both users' messages, without using the feedback link. To this end, all policies $z = \{(k,w),(l,x)\}$ with $k,l \in \{i,j\}, k \neq l$ and $w,x \in \{a,b\}$ for which $\frac{\gamma_{kw}}{\gamma_{lw}+1} \geq r_k$ and ($\gamma_{lw} \geq r_l$ or $\frac{\gamma_{lx}}{\gamma_{lx}+1} \geq r_l$) are suitable.
- **Step 2**: If no suitable policy is identified in step 1, the optimizer attempts to achieve the decoding of both users' messages by also using the feedback link. In this case, it is noted that the successful decoding of user l message also depends on the value of s_{f}, with $s_{\mathrm{f}} = 1$ leading to successful decoding. To this end, the optimizer searches the policies $z = \{(k,w),(l,x)\}$, with $k \neq l$ and $w \neq x$ to identify the one for which $\frac{\gamma_{kw}}{\gamma_{lw}+1} \geq r_k$, $\gamma_{lx} \geq r_l$, and $s_{\mathrm{f}} = 1$. This step is omitted if a feedback link is not available.
- **Step 3**: If a suitable policy is not identified in step 2, the optimizer aims at decoding at least of one of the users' messages by selecting $z_1 = \{(k,w)\}$ of z, for which $\frac{\gamma_{kw}}{\gamma_{lw}+1} \geq r_k$.

It is noted that the aforementioned procedure is optimal for both users, since the attempt to decode both users' messages does not negatively affect the decoding of any of the other users' messages, since step 3 is also considered.

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B. Outage Probability

Based on the user association and decoding order optimization procedure, that has been presented in the previous subsection, regarding user $k \neq l$, it is considered that an outage occurs if none of the RRHs manages to decode its message, i,e., if at least on of the following events occurs: i) None of the RRHs can decode any of the received messages handling the other message as interference, ii) All RRHs can decode the message of user l, but none of them can decode the message of user k even without interference, iii) Exactly one of the two RRHs can decode the message of user l, the other RRH can also acquire the decoded message (via the feedback link), but none of them can decode the message of user k even without interference, iv) Exactly one of the two RRHs can decode the message of user l, but it cannot decode the message of user k even without interference, while the other RRH cannot acquire the decoded message and it cannot decode the message of user k handling the message of user l as interference. Accordingly, the outage probability of each user, when a feedback link is used, is provided in closed-form in the following theorem.

Theorem 1: The outage probability of user i is given by

$$\begin{aligned} P_i^{\mathrm{o}} =& F_1(i,j,a)F_1(i,j,b) + F_2(i,j,a)F_2(i,j,b) + \\ & \left(F_2(i,j,a)F_3(i,j,b) + F_3(i,j,a)F_2(i,j,b)\right)(1-P_j^{\mathrm{f}}) + \\ & \left(F_2(i,j,a)F_1(i,j,b) + F_1(i,j,a)F_2(i,j,b)\right)P_j^{\mathrm{f}}, \end{aligned} \tag{3}$$

while the outage probability of user j is given by the same expression by replacing i with j and vice-versa. Also, in (3), P_j^{f} denotes the outage probability of the feedback link and is defined by

$$P_j^{\mathrm{f}} = P(C_0 < R_j), \tag{4}$$

with C_0 being the capacity of the feedback link. Moreover, F_1, F_2, and F_3 are given by

$$F_1(k,l,w) = \begin{cases} \lambda_{\mathrm{lw}} \left(\dfrac{e^{-r_k\lambda_{\mathrm{kw}}}\left(\exp\left(\frac{(r_k+1)r_l(r_k\lambda_{\mathrm{kw}}+\lambda_{\mathrm{lw}})}{r_kr_l-1}\right)-1\right)}{r_k\lambda_{\mathrm{kw}}+\lambda_{\mathrm{lw}}} \right. \\ \left. -\dfrac{r_le^{-r_l\lambda_{\mathrm{lw}}}\left(\exp\left(\frac{r_k(r_l+1)(\lambda_{\mathrm{kw}}+r_l\lambda_{\mathrm{lw}})}{r_kr_l-1}\right)-1\right)}{\lambda_{\mathrm{kw}}+r_l\lambda_{\mathrm{lw}}}\right) \\ +1-e^{-r_l\lambda_{\mathrm{lw}}}, \; r_kr_l<1, \\ 1-\frac{\lambda_{\mathrm{lw}}e^{-r_k\lambda_{\mathrm{kw}}}}{r_k\lambda_{\mathrm{kw}}+\lambda_{\mathrm{lw}}} - \frac{\lambda_{\mathrm{kw}}e^{-r_l\lambda_{\mathrm{lw}}}}{\lambda_{\mathrm{kw}}+r_l\lambda_{\mathrm{lw}}}, \; r_kr_l \geq 1, \end{cases} \tag{5}$$

$$F_2(k,l,w) = \frac{\lambda_{\mathrm{kw}}e^{-r_l\lambda_{\mathrm{lw}}}\left(1-e^{-r_k(\lambda_{\mathrm{kw}}+r_l\lambda_{\mathrm{lw}})}\right)}{\lambda_{\mathrm{kw}}+r_l\lambda_{\mathrm{lw}}}, \tag{6}$$

and

$$F_3(k,l,w) = 1-e^{-r_k\lambda_{\mathrm{kw}}} - \frac{\lambda_{\mathrm{kw}}e^{-r_l\lambda_{\mathrm{lw}}}\left(1-e^{-r_k(\lambda_{\mathrm{kw}}+r_l\lambda_{\mathrm{lw}})}\right)}{\lambda_{\mathrm{kw}}+r_l\lambda_{\mathrm{lw}}}, \tag{7}$$

respectively.

Proof: The proof is provided in Appendix A. ■

Remark 1: It is noted that the expression in (3) is general and provides the outage probability of each user for all three considered DU-NOMA schemes. More specifically, $P_j^{\mathrm{f}} = 0$ and $0 < P_j^{\mathrm{f}} < 1$ correspond to the utilization of an ideal and non-ideal feedback link, respectively, while $P_j^{\mathrm{f}} = 1$ implies that a feedback link is never used.

C. Useful Insights for the High SNR Region

In this section, the performance of the three considered DU-NOMA schemes in investigated for the high SNR region. Two different cases are considered, namely $r_ir_j < 1$ and $r_ir_j \geq 1$.

1) $r_ir_j < 1$: It is assumed that $\frac{p_i}{N_0} = \frac{p_j}{N_0} = \rho$. Defining the diversity order achieved by user k by

$$d_k = -\lim_{\rho\to\infty} \frac{\log(P_k^{\mathrm{o}})}{\log(\rho)}, \tag{8}$$

it can be shown that the diversity order achieved by both users is equal to 2 for all values of P_j^{f} and P_i^{f}, which corresponds to the diversity order of a single user system with two receiving antennas.

Thus, it is highlighted that similarly to some other forms of uplink NOMA [17], DU-NOMA does not always appear a floor, which is due to selection of the optimal decoding order. Taking into account (17), it holds that

$$\lim_{\rho\to\infty} P_i^{\mathrm{o}} = P\left(\frac{h_{ja}}{r_j} < h_{ia} < r_ih_{ja}\right) P\left(\frac{h_{jb}}{r_j} < h_{ib} < r_ih_{jb}\right), \tag{9}$$

where $h_{kw} = \alpha_{kw}^2$, from which it becomes apparent that $\lim_{\rho\to\infty} P_i^{\mathrm{o}} = 0$ if and only if ($r_ih_{ja} < \frac{h_{ja}}{r_j}$ or $r_ih_{jb} < \frac{h_{jb}}{r_j}$), i.e., $r_ir_j < 1$.

2) $r_ir_j \geq 1$: In this case the diversity order achieved by both users is equal to zero for all the considered DU-NOMA schemes. Also, it can easily be shown that

$$\lim_{\rho\to\infty} P_i^{\mathrm{o}} = \frac{\Omega_{\mathrm{ia}}\Omega_{\mathrm{ib}}\Omega_{\mathrm{ja}}\Omega_{\mathrm{jb}}\left(r_ir_j-1\right)^2}{\left(r_i\Omega_{\mathrm{ja}}+\Omega_{\mathrm{ia}}\right)\left(r_i\Omega_{\mathrm{jb}}+\Omega_{\mathrm{ib}}\right)\left(\Omega_{\mathrm{ia}}r_j+\Omega_{\mathrm{ja}}\right)\left(\Omega_{\mathrm{ib}}r_j+\Omega_{\mathrm{jb}}\right)}, \tag{10}$$

while the asymptotic value of the outage probability of user j is also given by (10), by replacing i with j and vice-versa.

D. Extension to the general case of N RRHs

The proposed protocol can be extended to the general case of N RRHs, with $\mathcal{N}$ being the set of all RRHs. Similarly to Algorithm 1, the coordinator attempts to find the policy z that leads to the successful decoding of both messages or at least one of them if decoding both messages is not feasible. Let $\mathcal{M}_1$ denote the subset of RRHs that can successfully decode the message of user l and $\mathcal{M}_2$ the subset of RRHs that can successfully acquire the message of user l from an RRH that belongs to $\mathcal{M}_1$ via the feedback link. An outage for user $k \neq l$ occurs when either i) none of the RRHs can decode any of the messages or ii) the message of user k cannot be decoded (even without interference) by any of the RRHs in $\mathcal{M}_2$, while the RRHs that belong in $\mathcal{N}\backslash\mathcal{M}_2$ cannot decode the message of user k, handling the message of user l as interference. Thus, the outage probability for user i can be expressed as

$$P_i^{\mathrm{o}} = P(B_1) + \sum_{\mathcal{M}_1\subseteq\mathcal{N},\mathcal{M}_1\neq\emptyset} \; \sum_{\mathcal{M}_2\supseteq\mathcal{M}_1} P(B_{2,\mathcal{M}_1,\mathcal{M}_2}), \tag{11}$$

where

$$B_1 = \frac{\gamma_{kw}}{\gamma_{lw}+1} < r_k, \forall k,l \in \{i,j\}, k\neq l, \forall w \in \mathcal{N}, \tag{12}$$

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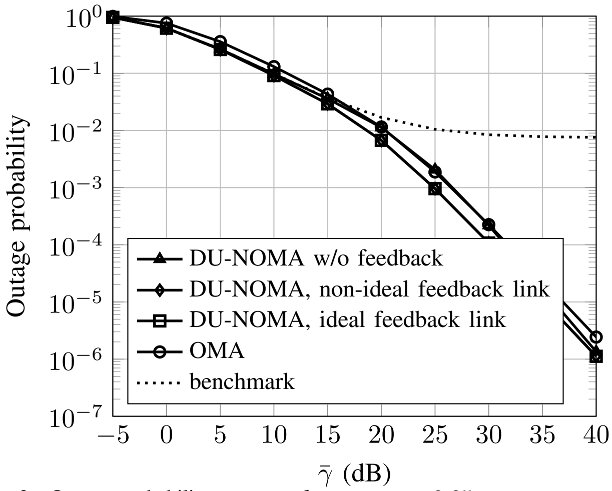

Fig. 2. Outage probability versus $\bar{\gamma}$ for $r_i = r_j = 0.95$

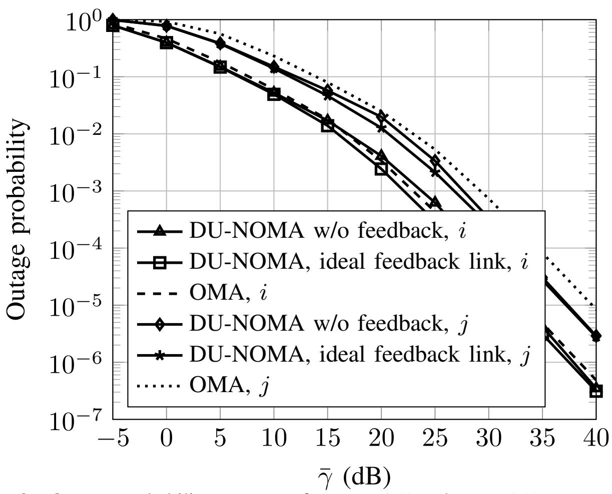

Fig. 3. Outage probability versus $\bar{\gamma}$ for $r_i = 0.5$ and $r_j = 1.5$

$$B_{2,\mathcal{M}_1,\mathcal{M}_2} = \frac{\gamma_{jw}}{\gamma_{iw}+1} \geq r_j, \forall w \in \mathcal{M}_1, \gamma_{ix} < r_i, \forall x \in \mathcal{M}_2,$$
$$\bar{\Delta}_{x,M_1}, \forall x \in \mathcal{M}_2, \frac{\gamma_{jw}}{\gamma_{iw}+1} < r_i, \forall w \in \mathcal{N}\backslash\mathcal{M}_1,$$
$$\frac{\gamma_{ix}}{\gamma_{jx}+1} < r_i, \forall x \in \mathcal{N}\backslash\mathcal{M}_2, \Delta_{x,M_2}, \forall x \in \mathcal{N}\backslash\mathcal{M}_2, \tag{13}$$

$$\Delta_{x,\mathcal{M}_1} = C_{wx} < R_j, \forall w \in \mathcal{M}_1, \tag{14}$$

C_{wx} is the capacity of the link between the RRHs w and x, $P(C_{ww} < R_j) = 0$, and $\bar{(\cdot)}$ denotes the complementary event of $(\cdot)$. Thus, since events that involve different RRHs are disjoint, the outage probability can be expressed in closed-form with the aid of functions F_1, F_2, F_3.

IV. Numerical Results & Discussion

In this section, we evaluate the outage performance of the considered three DU-NOMA schemes, namely without a feedback link, with ideal feedback link and with non-ideal feedback link. It is assumed that the users are symmetrically located to the RRHs, with each of them being closer to one of the two RRHs. Thus, both users have equal outage probability. More specifically, regarding the distance d_{kw} between user k and RRH w, it is assumed that $\frac{d_{ia}}{d_{ib}} = \frac{d_{ja}}{d_{jb}} = \frac{1}{5}$ and $d_{ia} = d_{jb}$. Also, it is further assumed that $\frac{\Omega_{kw}}{\Omega_{lw}} = (\frac{d_{lw}}{d_{kw}})^3$, which implies

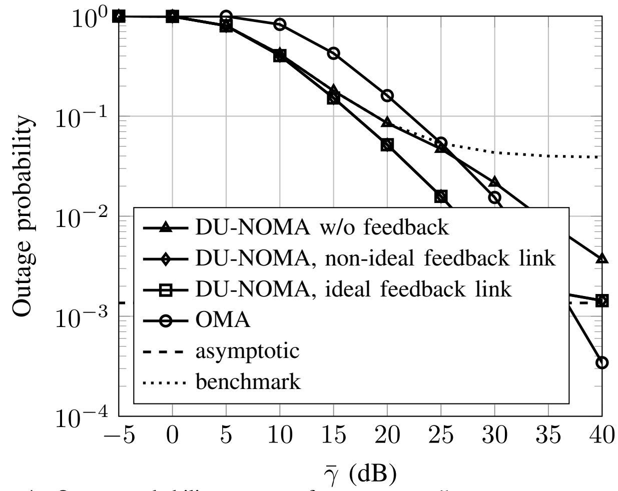

Fig. 4. Outage probability versus $\bar{\gamma}$ for $r_i = r_j = 5$

a path-loss exponent equal to 3, and $\bar{\gamma}_{ia} = \bar{\gamma}_{jb} = \bar{\gamma}$. For DU-NOMA with non-ideal feedback link, a relatively unreliable (e.g., wireless) link is assumed, with $P_i^{\mathrm{f}} = P_j^{\mathrm{f}} = 0.001$. Moreover, the outage of OMA is also plotted for comparison, for which it is assumed that each user is active half of the available time and remains silent when the other is active in the other half of the available time. In order to have a fair comparison with DU-NOMA, we scale the transmit power and the transmission rate of users, such as the the average data rates and consumed energy for both schemes are equal, i.e., with OMA the transmission rate and the transmit power of user k become $2R_k$ and $2p_k$, respectively. Also, in OMA, RRH selection is performed, i.e., each user is decoded by its best RRH. Furthermore, the outage probability for user i when its message is always decoded is first is also considered as benchmark.

In Fig. 2, the case with $r_i r_j < 1$ is considered, assuming that $r_i = r_j = 0.95$. It is observed that DU-NOMA with feedback outperforms OMA for the whole SNR region, even when a non-ideal feedback link with $P_i^{\mathrm{f}} = P_j^{\mathrm{f}} = 0.001$ is used, while for the low and high SNR region it achieves similar performance to DU-NOMA without feedback. Also, it deserves to be mentioned that DU-NOMA and OMA have the same diversity order, as it has been discussed in (III-C1). Moreover, in 3 the case in assuming that $r_i r_j < 1, r_i \neq r_j$ is considered, assuming $r_i = 0.5$ and $r_j = 1.5$, for which case it is observed that the gain of DU-NOMA with ideal feedback link compared to OMA is higher for the user with the higher transmission rate.

On the other hand, the case with $r_i r_j \geq 1$ is considered in Fig. 4, where it is seen that DU-NOMA with feedback link outperforms OMA for the practical values of received SNR $\bar{\gamma}$ and more specifically for $\bar{\gamma} \in [0, 35]$ dB, achieving up to a 5 dB gain. Furthermore, in contrast to OMA where a floor does not appear, the outage probability of all DU-NOMA schemes approaches the same floor as the SNR increases, which coincides with (10).

Appendix A
Proof of Theorem 1

The outage probability of user i is defined as

$$P_i^{\mathrm{o}} = P(\Gamma_i < r_i, \forall z). \tag{15}$$

Firstly, let's consider the following events:

$$\begin{aligned}
A_1 =& \tfrac{\gamma_{ia}}{\gamma_{ja}+1} < r_i, \tfrac{\gamma_{ib}}{\gamma_{jb}+1} < r_i, \tfrac{\gamma_{ja}}{\gamma_{ia}+1} < r_j, \tfrac{\gamma_{jb}}{\gamma_{ib}+1} < r_j,\\
A_2 =& \gamma_{ia} < r_i, \gamma_{ib} < r_i, \tfrac{\gamma_{ja}}{\gamma_{ia}+1} \geq r_j, \tfrac{\gamma_{jb}}{\gamma_{ib}+1} \geq r_j,\\
A_3 =& \gamma_{ia} < r_i, \gamma_{ib} < r_i, \tfrac{\gamma_{ja}}{\gamma_{ia}+1} \geq r_j, \tfrac{\gamma_{jb}}{\gamma_{ib}+1} < r_j, C_0 \geq R_j,\\
A_4 =& \gamma_{ia} < r_i, \gamma_{ib} < r_i, \tfrac{\gamma_{ja}}{\gamma_{ia}+1} < r_j, \tfrac{\gamma_{jb}}{\gamma_{ib}+1} \geq r_j, C_0 \geq R_j,\\
A_5 =& \gamma_{ia} < r_i, \tfrac{\gamma_{ib}}{\gamma_{jb}+1} < r_i, \tfrac{\gamma_{ja}}{\gamma_{ia}+1} \geq r_j, \tfrac{\gamma_{jb}}{\gamma_{ib}+1} < r_j, C_0 < R_j,\\
A_6 =& \tfrac{\gamma_{ia}}{\gamma_{ja}+1} < r_i, \gamma_{ib} < r_i, \tfrac{\gamma_{ja}}{\gamma_{ia}+1} < r_j, \tfrac{\gamma_{jb}}{\gamma_{ib}+1} \geq r_j, C_0 < R_j.
\end{aligned} \tag{16}$$

The message of user i can be successfully decoded by using the scheduling procedure that has been presented in III-A, except if any of the events $A_n, n \in \{1, ..., 6\}$ is true. By further considering that $A_n, n \in \{1, ..., 6\}$ are disjoint, (15) can be written as $P_i^{\mathrm{o}} = \sum_{n=1}^{6} P(A_n)$. Thus, taking into account the independence between some of the events that form each A_n, P_i^{o} can be expressed as

$$\begin{aligned}
P_i^{\mathrm{o}} =& P\left(\tfrac{\gamma_{ia}}{\gamma_{ja}+1} < r_i, \tfrac{\gamma_{ja}}{\gamma_{ia}+1} < r_j\right) P\left(\tfrac{\gamma_{ib}}{\gamma_{jb}+1} < r_i, \tfrac{\gamma_{jb}}{\gamma_{ib}+1} < r_j\right) +\\
& P\left(\gamma_{ia} < r_i, \tfrac{\gamma_{ja}}{\gamma_{ia}+1} \geq r_j\right) P\left(\gamma_{ib} < r_i, \tfrac{\gamma_{jb}}{\gamma_{ib}+1} \geq r_j\right) +\\
& P\left(\gamma_{ia} < r_i, \tfrac{\gamma_{ja}}{\gamma_{ia}+1} \geq r_j\right) P\left(\gamma_{ib} < r_i, \tfrac{\gamma_{jb}}{\gamma_{ib}+1} < r_j\right) (1 - P_j^{\mathrm{f}}) +\\
& P\left(\gamma_{ia} < r_i, \tfrac{\gamma_{ja}}{\gamma_{ia}+1} < r_j\right) P\left(\gamma_{ib} < r_i, \tfrac{\gamma_{jb}}{\gamma_{ib}+1} \geq r_j\right) (1 - P_j^{\mathrm{f}}) +\\
& P\left(\gamma_{ia} < r_i, \tfrac{\gamma_{ja}}{\gamma_{ia}+1} \geq r_j\right) P\left(\tfrac{\gamma_{ib}}{\gamma_{jb}+1} < r_i, \tfrac{\gamma_{jb}}{\gamma_{ib}+1} < r_j\right) P_j^{\mathrm{f}} +\\
& P\left(\tfrac{\gamma_{ia}}{\gamma_{ja}+1} < r_i, \tfrac{\gamma_{ja}}{\gamma_{ia}+1} < r_j\right) P\left(\gamma_{ib} < r_i, \tfrac{\gamma_{jb}}{\gamma_{ib}+1} \geq r_j\right) P_j^{\mathrm{f}}.
\end{aligned} \tag{17}$$

Next, F_1 will be calculated, which corresponds to

$$F_1(k,l,w) = P\left(\frac{\gamma_{kw}}{\gamma_{lw}+1} < r_k, \frac{\gamma_{lw}}{\gamma_{kw}+1} < r_l\right). \tag{18}$$

Using the random variables $X = \gamma_{kw}$ and $Y = \gamma_{lw}$, (18) can be rewritten as

$$F_1 = P\left(\frac{Y}{r_l} - 1 < X < r_k(Y+1)\right). \tag{19}$$

Considering that the expressions $\frac{X}{Y+1} = r_k$ and $\frac{Y}{X+1} = r_l$ intersect at the point $\frac{(r_k+1)r_l}{1-r_k r_l}$ if $r_k r_l < 1$, and they do not intersect if $r_k r_l \geq 1$, and that X and Y are independent, (19) can be rewritten as

$$F_1 = \int_0^d \mathrm{F_X}\left(r_k\left(y+1\right)\right) f_Y(y) dy - \int_{r_l}^d \mathrm{F_X}\left(\frac{y}{r_l} - 1\right) f_Y(y) dy, \tag{20}$$

where

$$d = \begin{cases} \frac{(r_k+1)r_l}{1-r_k r_l}, & r_k r_l < 1, \\ \infty, & r_k r_l \geq 1. \end{cases} \tag{21}$$

Considering that both X and Y follow the exponential distribution with parameter λ_{kw} and λ_{lw}, respectively, (20) becomes

$$\begin{aligned}
F_1 =& \int_0^d \lambda_{lw}(1 - e^{-\lambda_{kw} r_k (y+1)}) e^{-\lambda_{lw} y} dy - \\
& \int_{r_l}^d \lambda_{lw}(1 - e^{-\lambda_{kw}(\frac{y}{r_l} - 1)}) e^{-\lambda_{lw} y} dy.
\end{aligned} \tag{22}$$

Moreover, F_2 will be calculated, which corresponds to

$$F_2(k,l,w) = P\left(\frac{\gamma_{lw}}{\gamma_{kw}+1} \geq r_l, \gamma_{kw} < r_k\right). \tag{23}$$

Considering again that X and Y are independent and that they follow the exponential distribution, (23) can be rewritten as

$$\begin{aligned}
F_2 =& \int_0^{r_k} \left(1 - \mathrm{F_Y}\left(r_l(x+1)\right)\right) f_X(x) dx = \\
& \int_0^{r_k} \lambda_{kw} e^{-(\lambda_{kw} + \lambda_{lw} r_l)x - \lambda_{lw} r_l} dx.
\end{aligned} \tag{24}$$

Furthermore, F_3, which corresponds to

$$F_3(k,l,w) = \left(\frac{\gamma_{lw}}{\gamma_{kw}+1} < r_l, \gamma_{kw} < r_k\right), \tag{25}$$

similarly to the derivation of (24) can be rewritten as

$$\begin{aligned}
F_3 =& \int_0^{r_k} \mathrm{F_Y}\left(r_l(x+1)\right) f_X(x) dx = \\
& \int_0^{r_k} (1 - e^{-\lambda_{lw} r_l (x+1)}) \lambda_{kw} e^{-\lambda_{kw} x}.
\end{aligned} \tag{26}$$

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