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Information Theoretic Analysis and Performance Gains of 3-Color Shift Keying

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***Abstract*—In this letter, we investigate the information theoretic performance of 3-Color Shift Keying (3-CSK), a promising modulation scheme for visible light communications that is based on the use of sequences of symbols. More specifically, considering its advantages, which include improved error performance and lower power consumption compared to 4-CSK and On-Off Keying, we evaluate its performance in terms of the maximum achievable rate, which is expressed as a function of the number of symbols per sequence, while a closed-form lower-bound and a tractable approximation are also obtained. Monte-Carlo simulations of the maximum achievable rate verify the analysis and illustrate that 3-CSK outperforms 4-CSK, when channel state information is not available at the receiver, with 3-CSK -in contrast to 4-CSK- reaching its maximum capacity in this scenario. Finally, to get further insights on the inferred 3-CSK gains, the bit-error-rate is also analyzed under different channel realizations, assuming no channel state information is available.**

***Index Terms*—color shift keying; visible light communications; error performance; mutual information;**

I. INTRODUCTION

VISIBLE light communications (VLC) have been proposed as an emerging technology for several scenarios such as indoor, outdoor, vehicle-to-vehicle (V2V) and underwater communications [1]. VLC use light emitting diodes (LEDs) in the visible region of the optical spectrum for simultaneous room illumination and data transfer. In addition to not interfering with radio frequency, the advantages of VLC include easy bandwidth reuse and inherent security, since light can be confined within a room [1]. Due to this potential, the IEEE Standard 802.15.7 was introduced in 2011, giving the definitions for the physical layer (PHY) and medium access control (MAC) layer for VLC [2]. Moreover, in 2018, the 802.11bb Task Group on Light Communications was created, focusing on introducing changes to the base IEEE 802.11 Standards to enable communications in the light medium [3].

The design of a VLC system is subject to requirements upon the visual perception of the light, especially for indoor applications, where the LEDs' intensity must remain constant to avoid flickering and to meet the safety requirements for the human eyes [4], while the average perceived illumination should be of a certain color, commonly white. These requirements make the design of efficient modulations challenging. Among the proposed alternatives, color shift keying (CSK) is particularly attractive, since, instead of using solely single-color LEDs, such as in On-Off Keying (OOK), it employs multi-color LEDs increasing the degrees-of-freedom. Thus, CSK is an intensity modulation scheme that transmits data imperceptibly through the variation of the light color emitted by RGB LEDs, sharing the basic concept of frequency shift keying (FSK), since the bit patterns are encoded into color (wavelength) bands [5]. Due to its appealing characteristics, CSK has been included in the IEEE 802.15.7 Standard. The most common CSK modulation schemes are 4-CSK, 8-CSK and 16-CSK that use 4, 8 and 16 symbols, respectively.

In [6], a novel CSK modulation scheme, named 3-Color Shift Keying (3-CSK), was proposed. This makes use of only the three peripheral symbols of the 4-CSK constellation, each of which corresponds to a different primary color, leading to information transmssion using sequences of symbols. The analysis of 3-CSK in [6] shows that it is an attractive alternative for a wide range of applications, since it achieves a balance between the data rate, bit-error-rate (BER), and complexity. However, the performance of 3-CSK has not been investigated yet from an information theoretic perspective, while its potential to perform without the need for channel state information remains unexplored in the existing literature, given that only the BER for the identity channel was found in [6]. Also, it is noted that the information theoretic analysis of 3-CSK is challenging, despite the prior corresponding analysis for 4-CSK [7], due to the use of sequences of symbols for transmission in 3-CSK.

In this paper, we address the aforementioned points through the investigation of the performance of 3-CSK from an information theoretic perspective, and the exploration of its performance in comparison to 4-CSK, when channel state information is not available at the receiver. More specifically, an analytical expression for the maximum achievable rate of 3-CSK is derived with respect to the number of symbols per sequence in the presence of Additive White Gaussian Noise (AWGN) via a mutual information (MI) analysis. A closed-form lower-bound and a tractable approximation are also provided through Jensen's inequality, which give further insights on the system's performance and facilitate its optimization. Simulations for the maximum achievable rate of 3-CSK and 4-CSK are presented and, interestingly, show that 3-CSK -in contrast to 4-CSK- is able to reach its maximum capacity in channels characterized by cross-talk among different colors, even if the receiver has no knowledge of the channel and, thus, assumes an identity channel. To give further insight on this superiority of 3-CSK over 4-CSK, the BER of both schemes

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is simulated under different channel realizations. The results show that 3-CSK exhibits practical advantages over 4-CSK in all circumstances simulated.

II. System Model

Let's consider a typical K-CSK system with RGB LEDs [8], where K is the number of symbols in the CSK constellaion. At the transmitter, the vector $\mathbf{s} = [p_r, p_g, p_b]^T \in \Re^3$ is chosen equiprobably from the K symbols' set $\Lambda = \{\mathbf{s_1}, ..., \mathbf{s_K}\}$, where p_r, p_g, p_b represent the light intensities of the red, green and blue LED, respectively. The received signal vector is

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{H} = [h_{ij}]_{3\times 3}$ denotes the channel matrix and h_{ij} represents the channel gain between the i^{th} LED and j^{th} photo-detector PD. Finally, $\mathbf{n} = [n_r, n_g, n_b]^T \in \Re^3$ represents the zero-mean Gaussian noise vector $\mathbf{n} \backsim \mathcal{N}\left(0, \sigma^2 \mathbf{I_{3\times 3}}\right)$.

Since CSK is a visible light modulation scheme, there are both optic and electric constraints that should be satisfied in the constellation. First, the light intensity of each LED must be greater than or equal to 0 and is constrained by a maximum value, due to power limitations. In addition, the total operating intensity of each CSK symbol should be constant to avoid flickering and finally, an average constraint over the symbol set is required in order to fix a perceived color.

In 3-CSK, the constellation consists of only the three peripheral symbols of 4-CSK. Since, it is inefficient to map each possible bit string to a symbol, 3-CSK uses sequences of N successive symbols in order to transmit the information. Specifically, the number of the different sequences that can be constructed is 3^N, with each sequence containing a total of $M = \lfloor \log_2 3^N \rfloor$ bits, where $\lfloor \cdot \rfloor$ is the floor operator. In this context, the necessary number of sequences needed to transmit a bit string of M bits is 2^M. Furthermore, each received symbol is assigned to a symbol of the 3-CSK signal space by using the minimum euclidean distance criterion. Finally, the assigned sequence is mapped to an M-bit string. In order to maintain the same mean color as 4-CSK, the $3^N - 2^M$ unused combinations are transmitted once every 2^M sequences, which inevitably reduces the data-rate. It is important that 3-CSK can be easily implemented using the existing 4-CSK infrastructure, given that the only modification required is the replacement of the mapping process of symbols to bit strings in 4-CSK by the mapping of sequences of symbols to bit strings in 3-CSK.

In the 3D signal space of CSK, the 3-CSK modulation can be expressed as

$$Y = \mathbf{H}X + Z, \tag{2}$$

where $X = [X_1 \cdots X_N]\,,\ X_i \in \{\mathbf{s_1}, \mathbf{s_2}, \mathbf{s_3}\}, Y = [Y_1 \cdots Y_N]$, $Z = [n_1 \cdots n_N]$, $n_i \backsim \mathcal{N}\left(0, \sigma^2 \mathbf{I_{3x3}}\right)$, $N_0 = 3\sigma^2$, with $\mathbf{s_1}, \mathbf{s_2}, \mathbf{s_3}$ being the three constellation symbols of 3-CSK and N_0 the total noise power.

III. Maximum Achievable Rate of 3-CSK

In this section, an exact expression and a lower-bound for the maximum achievable rate of 3-CSK are derived $\forall N$, while a tractable approximation is also proposed.

Theorem 1. The maximum achievable rate of 3-CSK is

$$R_{3-\mathrm{CSK}} = \frac{1}{N}\frac{2^M}{3^N} I(Y;X) \ \left[\frac{\text{bits}}{\text{symbol}}\right], \tag{3}$$

where $I(Y;X)$ is the MI of 3-CSK, given by

$$\begin{aligned} I(Y;X) = -\int_y \left(\sum_{l=1}^{2^M}\prod_{i=1}^{N} p(y_i|x_{l,i})p(X_l)\right) \times \\ \log\left(\sum_{l=1}^{2^M}\prod_{i=1}^{N} p(y_i|x_{l,i})p(X_l)\right) dy - N\frac{3}{2}\log(2\pi e\sigma^2) \end{aligned} \tag{4}$$

and X_l denotes the sequence l, while $x_{l,i}$ is the i-th symbol of sequence l and y_i refers to the i-th received symbol of received sequence y.

Proof. The proof is provided in Appendix A. □

Eq. (3) is not in a closed form. Therefore, a closed-form approximation of the maximum achievable rate of 3-CSK would be helpful in gaining insights on the modulation and facilitating its optimization. Such an approximation of eq. (3) can be derived through the following theorem.

Theorem 2. A closed-form lower bound for the mutual information of 3-CSK is given by

$$\begin{aligned} I_{\mathrm{L}}(Y;X) = -\log\left(\sum_{l_1=1}^{2^M}\sum_{l_2=1}^{2^M}\left(\frac{1}{2^M}\right)^2\prod_{i=1}^{N}\frac{1}{8\pi^{3/2}\sigma^3}\times \right.\\ \left.\exp\left(-\frac{\sum\limits_{k\in\{r,j,b\}}\left(\widehat{x_{l_1,i}^{[k]}}-\widehat{x_{l_2,i}^{[k]}}\right)^2}{4\sigma^2}\right)\right) - N\frac{3}{2}\log(2\pi e\sigma^2), \end{aligned} \tag{5}$$

where $\widehat{x_{l,i}^{[k]}}$, refers to the k-th dimension (red, green or blue) of $\mathbf{H}x_{l,i}$, where $x_{l,i}$ is the i-th symbol of sequence l.

Proof. The proof is provided in Appendix B. □

It is noted that based on (5) an approximation for 3-CSK can be derived, considering that the gap between $I(Y;X)$ and $I_{\mathrm{L}}(Y;X)$ is the same for the low and high limit of the SNR regime. Thus, as in [7], we add this difference to (5) to get a valid approximation of the 3-CSK MI, which can be used to approximate the 3-CSK maximum achievable rate via (3).

IV. Simulations Results & Discussion

A. Evaluation of Maximum Achievable Rate

Simulations have been performed to find the maximum achievable rate of 3-CSK and compare it to that of 4-CSK. In order to ensure fairness, the graph is given with regard to E_{b}/N_0 (dB). In Fig. 1, in addition to the derived analytical expression of the maximum achievable rate of 3-CSK and the corresponding approximation, the maximum achievable rate for 4-CSK is also shown for the sake of comparison. Moreover, it is noted that the achievable rate for 4-CSK and 3-CSK with $N = 2$ are upper bounded by the capacity of the 3D and 6D AWGN signal space, respectively, which have also been included in Fig. 1. Taking into account that each 6D 3-CSK ($N = 2$) transmission requires double the time of a 4-CSK transmission, we can scale the rate to bits per 3D symbol. It is noted that the achievable rate of 4-CSK and 3-CSK present a gap to the respective channel capacity due to the positioning of the symbols of the constellation and the constraints of VLC.

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In Fig. 1 the maximum theoretical rate is depicted for two scenarios (A) and (B). In scenario (A) the channel is the identity matrix. It is evident in this case that 4-CSK has a greater maximum achievable rate than 3-CSK for the whole SNR regime. In addition, for high SNRs, 3-CSK with $N = 2$ achieves its theoretical maximum as found in [6]. The same behavior is expected for different values of N.

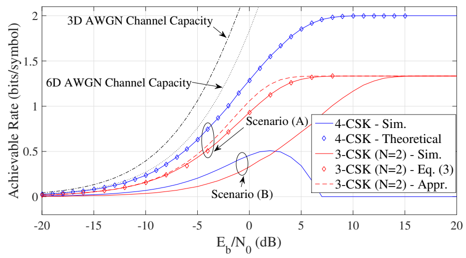
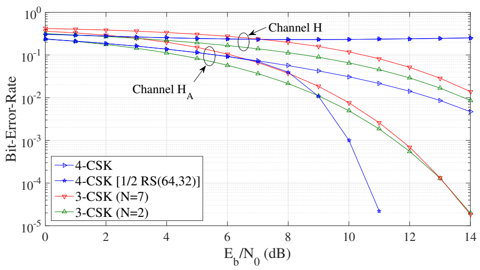
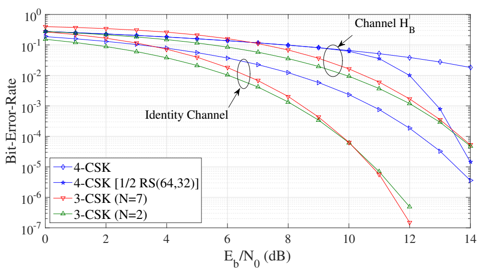


Fig. 1: Maximum Achievable Rate in bits per symbol.



(a) Error Performance under H $(\mathrm{a} = 1, \mathrm{b} = 0.5, \mathrm{c} = 0.1)$, H_{A} $(\mathrm{a} = 1, \mathrm{b} = 0.8, \mathrm{c} = 0.1)$.



(b) Error Performance under Identity Channel $(\mathrm{a} = \mathrm{b} = 1, \mathrm{c} = 0)$, H_{B} $(\mathrm{a} = \mathrm{b} = 0.7, \mathrm{c} = 0)$.

Fig. 2: Error Performance under Different Channels.

On the other hand, the superiority in terms of maximum theoretic rate of 3-CSK occurs in scenario (B), where in the generic channel form $\begin{bmatrix} a & 0 & 0 \\ 0 & b & c \\ 0 & c & b \end{bmatrix}$, $a = 1$, $b = 0.5$ and $c = 0.1$ for H, according to which the channel is characterized by cross-talk. In addition, in scenario (B), in contrast to scenario (A), it is assumed that neither scheme (for fairness) possesses channel state information and an identity channel matrix is assumed at the receiver. It is evident that 4-CSK is unable to send any number of bits with negligible probability of error with increasing SNR, while 3-CSK ($N = 2$) is able to reach its maximum theoretical rate of 1.333 bits/symbol.

B. Evaluation of Bit-Error-Rate over Non-Identity Channels

To give further insight on the 3-CSK and 4-CSK in terms of maximum achievable rate in cross-talk scenarios with no channel estimation, the error performance of 3-CSK is compared to that of 4-CSK under the influence of different channels in Fig. 2, assuming for fairness that neither system has channel state information, i.e., the receiver assumes an identity channel. It is observed that 3-CSK performs better in all three considered channels realization (H, H_A, H_B). The BER results for an identity channel are also included only for the sake of comparison. The superiority of 3-CSK validates that it mitigates the effects of channels characterized by cross-talk, i.e, where the photo-detectors are not ideal. This occurs because 3-CSK's symbols are strictly monochromatic and don't have any mix of primary colors, i.e, the RGB LEDs are never used simultaneously. In 4-CSK on the contrary, the central symbol, which is transmitted when all three primary colors are on, degrades performance, since it is accountable for the majority of the errors for all SNRs (for instance, over 70% for E_{b}/N_0 of 5 and 10 dB). The central symbol can be falsely detected as any of the three peripheral, while the three peripheral can also easily transition to the decision area of the central symbol, leading to error. This phenomenon is more grave when the channel is unknown at the receiver and not identity, due to the added distortion it poses on the symbols.

It is remarkable in Fig. 2a that even for channel H where the performance of 4-CSK reaches a floor, 3-CSK continues to show a decreasing BER as the SNR increases. In this case, as mentioned above, the distortion imposed by the channel on 4-CSK due to the existence of the central symbol leads to mistaken detection no matter the magnitude of the noise. This means that if 4-CSK transmits its maximum two bits of information per symbol it cannot reach negligible probability of error with increasing SNR, as seen in Fig. 2a, showing that a rate of 2 is not achievable, as suggested in scenario (B) of Fig. 1. This does not occur for 3-CSK that does not use 4-CSK's central symbol. Another interesting observation is that in order for 4-CSK to outperform 3-CSK in terms of BER, the encoding suggested in [2] must be applied. It is evident in Fig. 2 that with $\frac{1}{2}$Reed-Solomon (64,32) encoding, 4-CSK is able to surpass the BER performance of uncoded 3-CSK in only two scenarios, namely H_A and H_B, while it cannot improve the performance of 4-CSK in channel H. It is also important to note that by using the encoding, the 4-CSK data-rate is reduced to 1 bit/symbol, which is lower than the data-rate of 3-CSK. If 3-CSK uses the same proposed encoding, its performance in terms of BER will further improve and the difference in data-rate between the encoded modulations will be smaller than the difference between their uncoded versions.

Appendix A
Proof of Theorem 1

The mutual information of 3-CSK can be expressed as:

$$I(Y;X) = h(Y) - h(Y|X). \tag{6}$$

Each term is analyzed separately. Considering the transmission of each symbol in the sequence independent (memoryless channel) and after performing some algebraic manipulation:

$$p(y|x) = p(y_1, ..., y_N|x_1, ..., x_N) = \prod_{i=1}^{N} p(y_i|x_i),\ y_i,\ x_i \in \Re^3, \tag{7}$$

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where y is a received sequence of symbols and x a transmitted sequence with their i-th symbol being y_i and x_i respectively.

Therefore, it holds that

$$\begin{aligned} h(Y|X) &= -\sum_x p(x)\int_y p(y|x)\log\left(p\left(y|x\right)\right)\ dy = \\ &-\sum_x p(x)\int_y \prod_{i=1}^{N} p(y_i|x_i)\log\left(\prod_{i=1}^{N} p(y_i|x_i)\right)\ dy = \\ &-\sum_x p(x)\int_y \prod_{i=1}^{N} p(y_i|x_i)\sum_{i=1}^{N}\log\left(p(y_i|x_i)\right)\ dy, \end{aligned} \tag{8}$$

which can be written as

$$\begin{aligned} &h(Y|X) = -\sum_x p(x)\times \\ &\int_y \prod_{i=1}^{N} p_{n_i}(y_i-\mathbf{H}x_i)\sum_{i=1}^{N}\log\left(p_{n_i}(y_i-\mathbf{H}x_i)\right)\ dy = \\ &-\sum_x p(x)\int_{n=[n_1,\dots,n_N]}\prod_{i=1}^{N} p_{n_i}(n_i)\sum_{i=1}^{N}\log\left(p_{n_i}(n_i)\right)dn = \\ &\sum_x p(x)h(Z) = h(Z) = Nh(n_i) = N\frac{3}{2}\log(2\pi e\sigma^2), \end{aligned} \tag{9}$$

since $h(Z)$ is actually the joint entropy of the random variables $n_1,\dots,n_N$ that are statistically independent and identically distributed.

In order to calculate $h(Y)$ we first note that

$$p(y) = \sum_{l=1}^{2^M} p(y|X_l)p(X_l)\ = \sum_{l=1}^{2^M}\prod_{i=1}^{N} p(y_i|x_{l,i})p(X_l), \tag{10}$$

where X_l is the l^{th} possible sequence of N symbols and $x_{l,i}$ is its i^{th} symbol. Of course in 3-CSK we consider all 2^M sequences with information equiprobable. Applying the above, we get:

$$\begin{aligned} &h(Y) = -\int_y p(y)\log(p(y))dy = \\ &-\int_y\left(\sum_{l=1}^{2^M}\prod_{i=1}^{N} p(y_i|x_{l,i})p(X_l)\right)\log\left(\sum_{l=1}^{2^M}\prod_{i=1}^{N} p(y_i|x_{l,i})p(X_l)\right)dy. \end{aligned} \tag{11}$$

Plugging (9), (11) into (6) we get the mutual information of the 3-CSK constellation in $\frac{\text{bits}}{\text{sequence}}$. Therefore, the maximum rate of 3-CSK is not the one shown in (6) and a scaling is necessary, due to the 3-CSK mean color constraint and the use of sequences of symbols.

Appendix B
Proof of Theorem 2

Applying Jensen's inequality to $h(Y)$, it yields, as in [7]:

$$\begin{aligned} &h(Y) \geq -\log\left(\int_y p(y)^2 dy\right)\ = \\ &-\log\int_{\underbrace{\Re^3\times\cdots\times\Re^3}_{N}}\left[\sum_{l=1}^{2^M}\prod_{i=1}^{N} p(y_i|x_{l,i})p(X_l)\right]^2\ dy. \end{aligned} \tag{12}$$

Considering that all the sequences are equipobable:

$$\begin{aligned} &h(Y) \geq -\log\left(\int_{\Re^{3\times N}}\left(\left(\tfrac{1}{2^M}\right)\sum_{l=1}^{2^M}\prod_{i=1}^{N} p(y_i|x_{l,i})\right)^2\ dy\right) = \\ &-\log\left(\int_{\Re^{3\times N}}\left(\tfrac{1}{2^M}\right)^2\left(\sum_{l_1=1}^{2^M}\sum_{l_2=1}^{2^M}\prod_{i=1}^{N} p(y_i|x_{l_1,i})p(y_i|x_{l_2,i})\right)dy\right). \end{aligned} \tag{13}$$

It becomes evident that the integral will consist of a sum of terms in the form:

$$\begin{aligned} \prod_{i=1}^{N} p(y_i|x_{l_1,i})p(y_i|x_{l_2,i})\ &= \prod_{i=1}^{N}\frac{1}{\sqrt{(2\pi)^3\sigma^3}}\exp\left(-\frac{\|y_i-\mathbf{H}x_{l_1,i}\|^2}{2\sigma^2}\right) \\ &\times\frac{1}{\sqrt{(2\pi)^3\sigma^3}}\ \exp\left(-\frac{\|y_i-\mathbf{H}x_{l_2,i}\|^2}{2\sigma^2}\right). \end{aligned} \tag{14}$$

Therefore, the integral of sum can become a sum of integrals where each integral is in the form

$$\int_y\prod_{i=1}^{N} p(y_i|x_{l_1,i})p(y_i|x_{l_2,i})dy\ =\ \prod_{i=1}^{N} I_{i,l_1,l_2}, \tag{15}$$

where

$$\begin{aligned} &I_{i,l_1,l_2} = \int_{y_i} p(y_i|x_{l_1,i})p(y_i|x_{l_2,i})dy_i = \\ &\int_{y_i}\frac{1}{(2\pi)^3\sigma^6}\exp\left(-\frac{\|y_i-\overbrace{\mathbf{H}x_{l_1,i}}^{\widehat{x_{l_1,i}}}\|^2+\|y_i-\overbrace{\mathbf{H}x_{l_2,i}}^{\widehat{x_{l_2,i}}}\|^2}{2\sigma^2}\right)\ dy_i\ = \\ &\frac{1}{(2\pi)^3\sigma^6}\int_{y_i}\exp\left(-\frac{\sum_{k\in\{r,g,b\}}(y_i^{[k]}-\widehat{x_{l_1,i}^{[k]}})^2+(y_i^{[k]}-\widehat{x_{l_2,i}^{[k]}})^2}{2\sigma^2}\right)dy_i = \\ &\frac{1}{(2\pi)^3\sigma^6}\prod_{k\in\{r,g,b\}}\int_{y_i^{[k]}}\exp\left(-\frac{(y_i^{[k]}-\widehat{x_{l_1,i}^{[k]}})^2+(y_i^{[k]}-\widehat{x_{l_2,i}^{[k]}})^2}{2\sigma^2}\right)dy_i^{[k]}. \end{aligned} \tag{16}$$

The superscript $[\cdot]$ of a variable refers to a single dimension, $\{r,g,b\}$. Combining (13), (15) and (16) with (9) and (6) we get the low-bound for the mutual information of 3-CSK.

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