Outage Probability Analysis for a Rician Signal in L Nakagami Interferers with Arbitrary Parameters

George Karagiannidis, Stavros Kotsopoulos, and Chris Georgopoulos

Abstract: Effective techniques have been developed to determine outage probability in a Nakagami interference environment. However, there is no solution in the literature to the outage problem of a Rician desired signal in L Nakagami interferers with arbitrary parameters. In this paper we derive an alternative exact formulation of the Outage Probability for a mobile environment with a Rician signal in the presence of multiple L Nakagami interferers with arbitrary parameters. This result is particularly important since such a mobile environment seems to be the most realistic in a micro- or pico-cellular mobile radio communication system. Computer simulation results are also presented to illustrate the proposed formulation.

Index Terms: Rician Fading, Nakagami Fading, Outage Probability, Co-Channel Interference, Mobile Radio, Microcellular, Pico-Cellular.

I. INTRODUCTION

Outage probability is a measure to control the co-channel interference level helping the designers of cellular radio systems to re-adjust the system's operating parameters. The term outage is related to the criterion used for the assessment of satisfactory reception. In this paper, outage probability is defined as the probability that the undesired (sum of the interferers) signal power exceeds the desired signal power, by a protection ratio denoted as β.

The calculation of the outage probability in a Nakagami mobile environment is particularly important, since Nakagami fading is one of the most appropriate models in many practical mobile communication networks. Nakagami distribution (also called m-distribution) contains a set of other distributions as special cases and provides optimum fits to collected data in outdoor and indoor environments, in the frequency range from 800 MHz to 4 GHz [1], [2]. Several techniques have been developed to determine the outage probability for Nakagami channels [3], [4]. The latest and most general approach for a Nakagami desired signal in L Nakagami interferers was presented in [5] where the outage problem was solved in the framework of hypothesis testing and determined the outage probability from the characteristic function of the quadratic form through a special lemma on inverse Fourier transform.

On the other side, amplitude fading in a multi-path pico- or micro-cellular environment may follow different distributions depending on the area covered by measurements, presence or absence of a dominating strong component, and some other conditions. The motion of people within buildings causes Rician fading in Line of Sight (LOS) paths, while Rayleigh fading still dominates in non-LOS paths [1]. The Rician distribution contains the Rayleigh distribution as a special case and simultaneously is well approximated around its node by a Gaussian distribution.

In this paper, a new approach is proposed for the calculation of the outage probability of a Rician signal in the presence of L statistically independent Nakagami interferers. The outage problem was solved directly using the properties of the characteristic function and the outage probability is extracted in a closed form, which can be calculated with the desired accuracy, via the Gauss-Hermite method.

In Section II, the problem is formulated and the necessary mathematical analysis is presented, in order to derive the formula for the Outage probability. In Section III numerical results illustrate the proposed formulation. Finally, Section IV finishes the paper with concluding remarks.

II. PROBLEM FORMULATION AND MATHEMATICAL ANALYSIS OF THE PROPOSED METHOD

We consider a cellular mobile environment with a Rician desired signal corrupted by L Nakagami interferers of arbitrary parameters. For the purpose of the present analysis, the following assumptions are made:

1) The random variable $r_k(t)$ which represents the amplitude of the k^{th} co-channel interferer, follows Nakagami distribution with Probability Distribution Function (PDF), given by

$$f(r_k) = 2 \cdot \left(\frac{m_k}{\Omega_k}\right)^{m_k} \cdot \frac{{r_k}^{2\cdot m_k - 1}}{\Gamma(m_k)} \cdot \exp\left(-\frac{m_k}{\Omega_k} \cdot {r_k}^2\right), \quad r_k \geq 0, \qquad (1)$$

where m_k is an arbitrary fading parameter, Ω_k is the average power and $\Gamma(x)$ is the Gamma function.

2) All the $r_k(t)$ are statistically independent.

3) If the probability of interference for a given call is satisfactory at the Base station, it is also satisfactory at the mobile terminal.

4) If r_k is a Nakagami variable, the corresponding power ξ_k

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is Gamma distributed with PDF given by

$$f(\xi_k) = \left(\frac{m_k}{\Omega_k}\right)^{m_k} \cdot \frac{{\xi_k}^{m_k-1}}{\Gamma(m_k)} \cdot \exp\left(-\frac{m_k}{\Omega_k} \cdot \xi_k\right), \xi_k \geq 0. \quad (2)$$

5) The desired signal power ψ follows the Rice distribution with PDF given by

$$\begin{aligned} f_{Rice}(\psi) =& \frac{1}{2 \cdot {\sigma_0}^2} \cdot \exp\left(-K_0\right) \cdot \exp\left(-\frac{\psi}{2 \cdot {\sigma_0}^2}\right) \\ & \cdot I_0\left(\frac{\sqrt{2K_0\psi}}{\sigma_0}\right), \quad \psi \geq 0, \end{aligned} \quad (3)$$

and Cumulative Distribution Function (CDF) which is expressed as [6], [7], [8]

$$F_{Rice}(\psi) = 1 - Q\left(\sqrt{2 \cdot K_0}, \frac{\sqrt{2\psi}}{\sigma_0}\right), \quad (4)$$

where $Q(a, b)$ is the Marcum's Q function [7] and K_0 is the Rice factor, defined as the ratio of the LOS power to diffused-scattered power $2\sigma_0^2$.

If $\sum_{i=1}^{L} \xi_i$ is the sum of the L statistically independent Nakagami interferers and ξ_0 the power of the desired signal, then the outage probability denoted as q_c is given by

$$q_c = \text{Prob}\left(\beta \cdot \sum_{i=1}^{L} \xi_i - \xi_0 > 0\right). \quad (5)$$

If we set $w = \xi_0 - \beta \cdot \sum_{i=1}^{L} \xi_i$, then (5) can be written as

$$q_c = \text{Prob}(w < 0). \quad (6)$$

The PDF of the $\beta \cdot \xi_i$ is given by

$$\begin{aligned} f_{\beta\cdot\xi_i}(x) &= \frac{1}{\beta} \cdot f_{\xi_i}\left(\frac{x}{\beta}\right) \\ &= \frac{1}{\beta} \cdot \left(\frac{m_k}{\Omega_k}\right)^{m_k} \cdot \frac{x^{m_k-1}}{\beta^{m_k-1} \cdot \Gamma(m_k)} \cdot \exp\left(-\frac{m_k}{\Omega_k} \cdot \frac{x}{\beta}\right). \end{aligned} \quad (7)$$

Let $\Phi_w(r)$, $\Phi_{\xi_0}(r)$, $\Phi_{\beta\cdot\xi_i}(r)$ be the characteristic functions of the variables w, s, and $\beta \cdot \xi_i$ respectively and r is an auxiliary variable used for definition purposes. The $\Phi_w(r)$ can be written as:

$$\Phi_w(r) = \Phi_{\xi_0}(r) \cdot \prod_{i=1,\cdots,L} \Phi_{\beta\cdot\xi_i}(-r). \quad (8)$$

Using the definition of the characteristic function, equation (8) can be written in the following form:

$$\Phi_w(r) = \Phi_{\xi_0}(r) \cdot \prod_{i=1,\cdots,L} \int_0^{\infty} \exp(-jrx_i) \cdot f_{\beta\cdot\xi_i}(x_i) dx_i. \quad (9)$$

Making the transformation $x_i = \frac{\beta\cdot\Omega_i}{m_i} \cdot r_i^2$, from (8) and (9) and after some simplifications we obtain:

$$\begin{aligned} &\Phi_w(r) \\ &= A \cdot \Phi_{\xi_0}(r) \cdot \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \exp\left(-jr \sum_{i=1,\cdots,L} \frac{\Omega_i \cdot \beta}{m_i} \cdot r_i^2\right) \\ &\cdot \left(\prod_{i=1,\cdots,L} r_i^{2\cdot m_i - 1}\right) \cdot \exp\left(-\sum_{i=1,\cdots,L} r_i^2\right) dr_1 \cdots dr_L, \end{aligned} \quad (10)$$

where A is a constant given by

$$A = \frac{2^L}{\prod_{i=1}^{L} \Gamma(m_i)}. \quad (11)$$

But, from (6) and by definition, we have:

$$q_c = \int_{-\infty}^{0} f_w(\tau) d\tau = \frac{1}{2\pi} \cdot \int_{-\infty}^{0} \int_{-\infty}^{\infty} \Phi_w(r) \cdot \exp(-jr\tau) dr d\tau, \quad (12)$$

where τ is another auxiliary variable.

Taking into account the equations (10), (11) and (12) and in combination with the following known – from the statistical theory – definition

$$\begin{aligned} &\int_{-\infty}^{\infty} \Phi_{\xi_0}(r) \cdot \exp\left[-jr\left(\tau + \sum_{i=1,\cdots,L} \frac{\beta \cdot \Omega_i}{m_i} \cdot r_i^2\right)\right] dr \\ &= 2 \cdot \pi \cdot f_{Rice}\left(\tau + \sum_{i=1,\cdots,L} \frac{\beta \cdot \Omega_i}{m_i} \cdot r_i^2\right), \end{aligned} \quad (13)$$

outage probability q_c can be written as

$$\begin{aligned} q_c =& A \cdot \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \left(\prod_{i=1,\cdots,L} r_i^{2\cdot m_i-1}\right) \cdot \exp\left(-\sum_{i=1,\cdots,L} r_i^2\right) \\ & \cdot F_{Rice}\left(\sum_{i=1,\cdots,L} \frac{\beta \cdot \Omega_i}{m_i} \cdot r_i^2\right) dr_1 \cdots dr_L. \end{aligned} \quad (14)$$

Equation (14) involves L integrals for L Nakagami and it's second part can be calculated numerically up to the desired accuracy using the Gauss-Hermite method. According to this method,

$$\int_{-\infty}^{\infty} \exp[-x^2] \cdot g(x) dx = \sum_{i=0}^{\nu} \alpha_i \cdot g(x_i), \quad (15)$$

where α_i and x_i are constants given by special tables [8]; ν is a constant that denotes the accuracy at the ν^{th} decade digit.

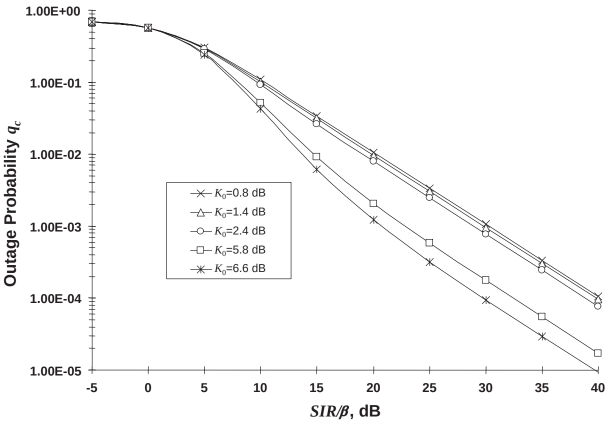

Fig. 1. The outage probability q_c versus the $\frac{SIR}{\beta}$ for three Nakagami interferers and propagation parameters shown on Table 1.

Table 1. Propagation parameters for the evaluation of q_c as depicted in Fig. 1.

m_k	1.5	1.2	0.85
Ω_k	6.1	6.4	5
β	18 dB		
K_0	0.8, 1.4, 2.4, 5.8, 6.6 dB		

Hence, the final formula for the outage probability is

$$q_c = A \cdot \sum_{i=1}^{\nu} \alpha_i \cdot \sum_{j=1}^{\nu} \alpha_j \cdots \sum_{n=1}^{\nu} \alpha_n \cdot \left(\prod_{t=i,j,\cdots,n} x_t^{2 \cdot m_t - 1} \right) \cdot F_{Rice} \left(\sum_{t=i,j,\cdots,n} \frac{\beta \cdot \Omega_t}{m_t} \cdot x_t^2 \right). \tag{16}$$

III. NUMERICAL RESULTS

First, we consider a mobile environment with a Rician signal in the presence of three Nakagami co-channel interferers with parameters m_k and Ω_k given in Table 1. The protection ratio is assumed to be β = 18 dB. Using equation (16) with a desired accuracy at 5^{th} decade digit, the outage probability q_c is calculated as a function of Signal-to-Interference Ratio (SIR) normalized by the protection ratio β, for several values of the Rice factor (shown in Table 1). These values are derived from narrow-band propagation measurements conducted at two floors of a multi-storied building at 1.9 GHz which are published in [9].

SIR is defined here as

$$SIR(\text{dB}) = 10 \cdot \log_{10} \frac{2\sigma_0{}^2 \cdot (1 + K_0)}{\sum_{i=1}^{L} \Omega_i}. \tag{17}$$

The results are depicted in Fig. 1. We note here that the outage probability decreases for high values of the Rice factor K_0. A slight change in K_0 leads to a significant change in q_c, especially for large $\frac{SIR}{\beta}$. This happens because an increase of the Rice factor means that the desired signal contains a large LOS component and a small diffuse-scattered component. Hence, the desired signal does not suffer from severe fading which degrades the outage performance.

In Figs. 2 and 3 the outage probability is plotted when there are four and six Nakagami interferers respectively. The relevant

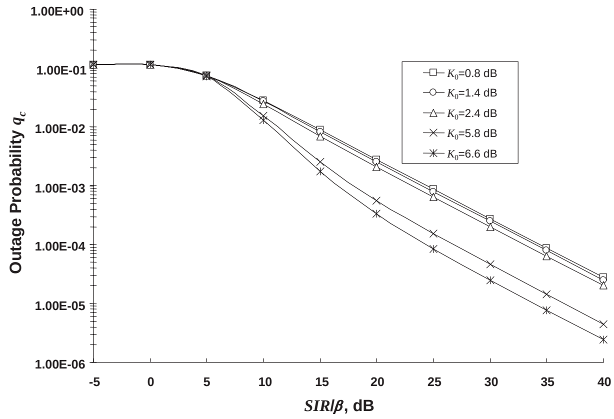

Fig. 2. The outage probability q_c versus the $\frac{SIR}{\beta}$ for four Nakagami interferers and propagation parameters shown on Table 2.

Table 2. Propagation parameters for the evaluation of q_c as depicted in Fig. 2.

m_k	2.3	2.1	2	1.8
Ω_k	6	5.7	5.2	3
β	18 dB			
K_0	0.8, 1.4, 2.4, 5.8, 6.6 dB			

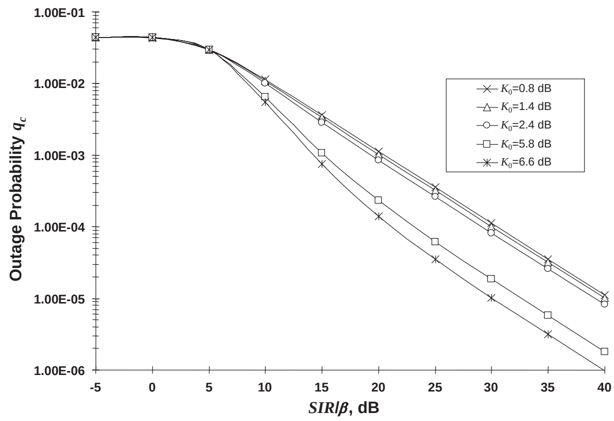

Fig. 3. The outage probability q_c versus the $\frac{SIR}{\beta}$ for six Nakagami interferers and propagation parameters shown on Table 3.

Table 3. Propagation parameters for the evaluation of q_c as depicted in Fig. 3.

m_k	1.8	1.6	1.5	1.2	1	0.85
Ω_k	5	5.4	5.6	5.9	6.1	6.7
β	18 dB					
K_0	0.8, 1.4, 2.4, 5.8, 6.6 dB					

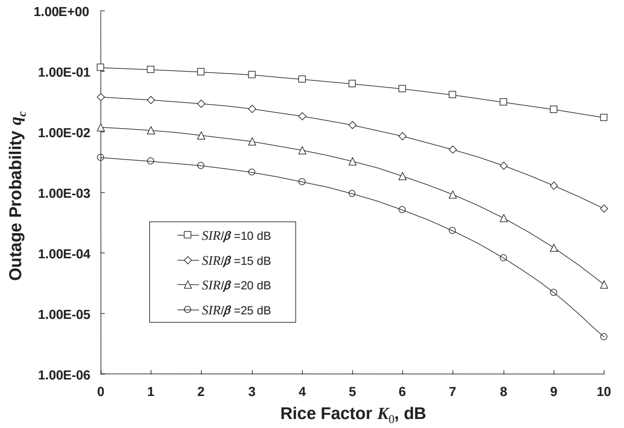

Fig. 4. The outage probability q_c versus the Rice factor K_0 for three Nakagami interferers and propagation parameters shown on Table 4.

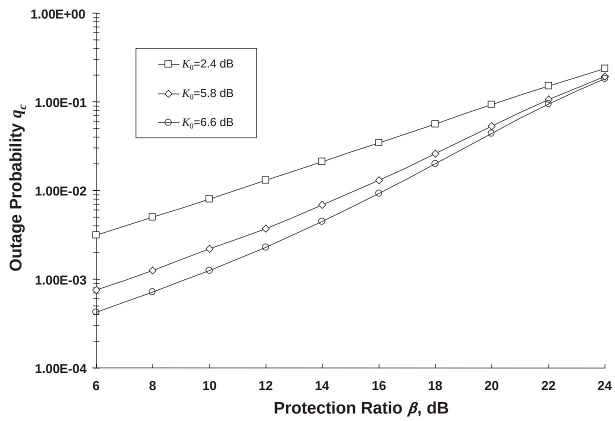

Fig. 5. The outage probability q_c versus the protection ratio β for three Nakagami interferers and propagation parameters shown on Table 5.

Table 4. Propagation parameters for the evaluation of q_c as depicted in Fig. 4.

m_k	1.5	1.2	0.85
Ω	6.1	6.4	5
β	18 dB		
$\frac{SIR}{\beta}$	10, 15, 20, 25 dB		

Table 5. Propagation parameters for the evaluation of q_c as depicted in Fig. 5.

m_k	1.5	1.2	0.85
Ω_k	6.1	6.4	5
SIR	30 dB		
K_0	2.4, 5.8, 6.6 dB		

parameters are shown on Tables 2 and 3. As shown in Figs. 1, 2, and 3, outage performance is improved when $\frac{SIR}{\beta}$ increases. Especially for $\frac{SIR}{\beta} < 10$ dB there is a slight decrease of the q_c. However, this decrease is not observed in the curves due to the scale used. For values of $\frac{SIR}{\beta} > 10$ dB, small changes in this parameter leads to a significant improvement of the outage performance and this can be seen in the respective curves. The reason for bad outage performance for small $\frac{SIR}{\beta}$, is the domination of the Nagakami Signals-Interferers. In Fig. 4, outage probability is depicted as a function of the Rice factor K_0 for several values of $\frac{SIR}{\beta}$. The number of Nakagami interferers is three with the parameters of Table 4. We observe here that an increase of the Rice factor K_0 leads to an improvement of the outage performance, but this improvement is not important especially for short values in $\frac{SIR}{\beta}$. In a real indoor pico-cellular environment, as described in [9], the Rice factor is within from 1 to 7 dB. In this range – as it is shown in Fig. 4 – a small increase in $\frac{SIR}{\beta}$ (5 dB) leads to a significant improvement of the outage performance (about one order), while an increase of K_0 does not improve correspondingly the outage probability.

Finally in Fig. 5, outage probability is shown related to the protection ratio β for three Nakagami interferers with the parameters given in Table 1 and several values of the Rice factor K_0. As we can see here, the influence of the protection ratio to the outage performance is particularly important and depends on K_0 for low values of β. On the contrary, for high values of β outage probability increases and tends to be independent from K_0. This is due to the fact that the high demands in the quality of communication (high values for β) dominate the improvement which is offered by the LOS communication (high values of K_0).

All the above numerical computations were performed on a 300 MHz Pentium Personal Computer and the average CPU times for the calculation of outage probability for each SIR are observed to be about 0.1 sec for three, 5 sec for four, and 10 sec for six Nakagami interferers. When the desired accuracy is at 4^{th} decade digit the required CPU times are $\ll$ 0.1 sec for three, 0.8 sec for four and about 3 sec for six interferers.

IV. CONCLUSIONS

An exact formulation is presented, for the evaluation of the outage probability in mobile cellular systems due to Rician signal and multiple Nakagami interferers with arbitrary parameters. The derived formula can be calculated numerically using Gauss-Hermite method with the desired accuracy. The proposed method provides reduced complexity, advantage in controlling the accuracy of computation and computational speed using the existing computer tools. The obtained accuracy of computation strengthens the interference minimization targets that Europe, Japan and North America have set for future mobile cellular systems in order to maintain the Quality of Service (QoS) at the required level and to control the spectrum efficiency.

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