On the Performance of Uplink Rate-Splitting Multiple Access

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***Abstract*— The use of new multiple access schemes emerges as a necessity in order to efficiently cope with the massive connectivity, reliability and high throughput requirements of the next generation Internet of Things. In this letter, we study the performance of an uplink rate-splitting multiple access (RSMA) network with two sources, in terms of outage probability and throughput. Specifically, we derive closed-form expressions for the outage probability of the transmitted messages and then, we utilize them to derive the throughput of the sources in a random access network with a medium access control protocol based on slotted ALOHA and RSMA.**

***Index Terms*— Rate-splitting multiple access, next generation multiple access, uplink network, outage probability, throughput.**

I. Introduction

NEXT generation Internet of Things (IoT) leads to the network integration of a huge amount of wireless devices, thus, raising several research and implementation challenges. Specifically, it necessitates the development of low-cost, reliable and scalable wireless networks, which will be able to support high spectral efficiency and massive connectivity [1]. The performance of these networks is greatly affected by the utilized multiple access schemes, thus next generation multiple access (NGMA) schemes are required. To this end, rate-splitting multiple access (RSMA) is considered as an effective approach, that enables the successive decoding to achieve the entire capacity region of the multiple access channel [2]. Furthermore, it provides a more general and robust transmission framework compared to non-orthogonal multiple access (NOMA). Especially, in IoT sensor networks, where random access (RA) schemes are used, RSMA is an effective method to reduce the number of collisions and increase connectivity.

Scanning the existing literature, most works focus on the downlink scenario. Specifically, in [3], a two-user multiple input single output broadcast downlink channel was considered and it was shown that RSMA generalizes the orthogonal multiple access, NOMA, space division multiple access and multicasting. Moreover, in [4], a rate allocation and power control problem is formulated as an optimization problem aiming to maximize the sum-rate under both rate and SIC constraints in an RSMA network. Regarding the uplink scenario, in [5], a framework for optimizing power allocation and message decoding was investigated in an RSMA network. Also, in [6], the rate-splitting (RS) principle was incorporated with uplink NOMA and its performance in terms of outage probability and achievable sum rate was investigated. However, the outage probability was derived for specific decoding order and relation of the channel coefficients. Finally, in [7], RS for an uplink NOMA network with two users was studied and it was shown that the fairness among users and the outage performance are improved. In this work, the outage probability was also derived by considering a specific decoding order. Concluding, the existing literature does not include closed-form expressions for the outage probability of an uplink RSMA network with arbitrary decoding order and general channel conditions.

Motivated by the above, we investigate the performance of an uplink RSMA network consisting of two sources, which is aligned with how NOMA is implemented in 3rd generation partnership project (3GPP) standards. Specifically, we present closed-form expressions for the outage probability of the transmitted messages as a function of the power allocation factor and the target rates, considering all possible decoding orders at the base station (BS). Utilizing these expressions, we consider a practical medium access control (MAC) layer protocol and derive the throughput of the sources in an RA network where slotted ALOHA (SA) and RSMA are utilized. In the numerical results, we validate the theoretical analysis with simulations and we illustrate the superiority of RSMA, in terms of the sources' throughput, over successive interference cancellation (SIC) and orthogonal multiple access (OMA).

II. System Model

We consider an uplink RSMA network consisting of a BS and two sources. All nodes are assumed to be equipped with a single antenna. For this network the message of one source needs to be split in order to achieve the capacity region [2]. Without loss of generality, it is assumed that the message of the first source is split. Thus, the received message at the BS can be written as

$$y = \sqrt{\alpha l_1 p_1} h_1 x_{11} + \sqrt{(1-\alpha) l_1 p_1} h_1 x_{12} + \sqrt{l_2 p_2} h_2 x_2 + n, \quad (1)$$

where l_i, p_i and h_i denote the path loss coefficient between the i-th source and the BS, the transmitted power of the i-th

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source and the complex channel coefficient of the i-th source, respectively. Moreover, $\alpha \in [0,1]$ and n denote the power allocation factor and the additive white Gaussian noise at the BS with zero mean and variance σ^2, respectively. Furthermore, x_{11}, x_{12} and x_2 denote the first message of the first source, the second message of the first source and the message of the second source, respectively.

For simplicity, we assume Rayleigh fading, thus $h_i \sim \mathcal{CN}(0,1)$ and $|h_i|^2$ follows the exponential distribution with rate equal to 1. We define the received signal-to-noise ratio (SNR) at the BS from the i-th source as $\gamma_i = \frac{l_i p_i |h_i|^2}{\sigma^2}$ which follows the exponential distribution with rate $\lambda_i = \frac{\sigma^2}{l_i p_i}$.

III. Performance Analysis

In this section, we investigate the outage performance of the considered network. The outage probability of the three messages is evaluated by considering the thresholds $\theta_{11} = 2^{\beta \hat{R}_1} - 1$, $\theta_{12} = 2^{(1-\beta)\hat{R}_1} - 1$ and $\theta_2 = 2^{\hat{R}_2} - 1$ for the three messages, where $\beta \in [0,1]$ and $\hat{R}_i$ denote the target rate factor and the target rate of the i-th source, respectively. It should be highlighted that a resource optimizer is assumed which is responsible for the selection of the decoding order, taking into account all possible decoding orders, which is reflected in the events in Tables I, II and III.

Theorem 1: The outage probability of x_{11}, i.e., the first message of the first source, is given by

$$P_{11} = P_{11}^1 + P_{11}^2 + P_{11}^3 + P_{11}^4 + P_{11}^5 - P_{11}^{\cap}, \tag{2}$$

where the expressions for P_{11}^j with $j = \{1,2,3,4,5,\cap\}$ are obtained from Table IV,

$$c_1 = \begin{cases} \frac{\theta_{12}+\theta_{12}\theta_2}{1-\theta_{12}\theta_2-\alpha-\alpha\theta_{12}}, & \theta_{12}\theta_2 < 1 \text{ and } a < \frac{1-\theta_{12}\theta_2}{1+\theta_{12}} \\ \infty, & \text{otherwise} \end{cases} \tag{3}$$

and

$$c_2 = \begin{cases} \frac{\theta_{11}+\theta_{11}\theta_2}{\alpha+\alpha\theta_{11}-\theta_{11}-\theta_{11}\theta_2}, & \theta_{11}\theta_2 < 1 \text{ and } \alpha > \frac{\theta_{11}+\theta_{11}\theta_2}{1+\theta_{11}} \\ \infty, & \text{otherwise.} \end{cases} \tag{4}$$

Proof: To derive the outage probability of the message x_{11}, the union of the five events in Table I should be calculated.

It should be highlighted that the only events that are not mutually exclusive are the events 3 and 5 and their intersection is given in the last row of Table I. After algebraic manipulations and considering the region of α, the probability of each of the five events and the intersection is derived by calculating integrals of the form

$$\begin{aligned} \int_{k_1}^{k_2} \int_{k_3 z_1}^{k_4 z_2} & \lambda_1 e^{-\lambda_1 z_1} \lambda_2 e^{-\lambda_2 z_2} dz_2 dz_1 \\ &= \frac{\lambda_1}{\lambda_1 + \lambda_2 k_3} \left(e^{(\lambda_1+\lambda_2 k_3)k_1} - e^{(\lambda_1+\lambda_2 k_3)k_2} \right) \\ &\quad - \frac{\lambda_1}{\lambda_1 + \lambda_2 k_4} \left(e^{(\lambda_1+\lambda_2 k_4)k_1} - e^{(\lambda_1+\lambda_2 k_4)k_2} \right). \end{aligned} \tag{5}$$

■

Next, the outage probability of the message x_{12} is derived.

TABLE I

Events of P_{11}

P_{11}^1	$\frac{\alpha\gamma_1}{(1-\alpha)\gamma_1+\gamma_2+1} < \theta_{11}$,	$\frac{(1-\alpha)\gamma_1}{\alpha\gamma_1+\gamma_2+1} < \theta_{12}$,	$\frac{\gamma_2}{\gamma_1+1} < \theta_2$
P_{11}^2	$\frac{(1-\alpha)\gamma_1}{\alpha\gamma_1+\gamma_2+1} \geq \theta_{12}$,	$\frac{\alpha\gamma_1}{\gamma_2+1} < \theta_{11}$,	$\frac{\gamma_2}{\alpha\gamma_1+1} < \theta_2$
P_{11}^3	$\frac{(1-\alpha)\gamma_1}{\alpha\gamma_1+\gamma_2+1} \geq \theta_{12}$,	$\frac{\gamma_2}{\alpha\gamma_1+1} \geq \theta_2$,	$\alpha\gamma_1 < \theta_{11}$
P_{11}^4	$\frac{\gamma_2}{\gamma_1+1} \geq \theta_2$,	$\frac{\alpha\gamma_1}{\gamma_2+1} < \theta_{11}$,	$\frac{(1-\alpha)\gamma_1}{\alpha\gamma_1+1} < \theta_{12}$
P_{11}^5	$\frac{\gamma_2}{\gamma_1+1} \geq \theta_2$,	$\frac{(1-\alpha)\gamma_1}{\alpha\gamma_1+1} \geq \theta_{12}$,	$\alpha\gamma_1 < \theta_{11}$
$P_{11}^{\cap}$	$\frac{(1-\alpha)\gamma_1}{\alpha\gamma_1+\gamma_2+1} \geq \theta_{12}$,	$\frac{\gamma_2}{\gamma_1+1} \geq \theta_2$,	$\alpha\gamma_1 < \theta_{11}$

TABLE II

Events of P_{12}

P_{12}^1	$\frac{\alpha\gamma_1}{(1-\alpha)\gamma_1+\gamma_2+1} < \theta_{11}$,	$\frac{(1-\alpha)\gamma_1}{\alpha\gamma_1+\gamma_2+1} < \theta_{12}$,	$\frac{\gamma_2}{\gamma_1+1} < \theta_2$
P_{12}^2	$\frac{\alpha\gamma_1}{(1-\alpha)\gamma_1+\gamma_2+1} \geq \theta_{11}$,	$\frac{(1-\alpha)\gamma_1}{\gamma_2+1} < \theta_{12}$,	$\frac{\gamma_2}{(1-\alpha)\gamma_1+1} < \theta_2$
P_{12}^3	$\frac{\alpha\gamma_1}{(1-\alpha)\gamma_1+\gamma_2+1} \geq \theta_{11}$,	$\frac{\gamma_2}{(1-\alpha)\gamma_1+1} \geq \theta_2$,	$(1-\alpha)\gamma_1 < \theta_{12}$
P_{12}^4	$\frac{\gamma_2}{\gamma_1+1} \geq \theta_2$,	$\frac{\alpha\gamma_1}{\gamma_2+1} < \theta_{11}$,	$\frac{(1-\alpha)\gamma_1}{\alpha\gamma_1+1} < \theta_{12}$
P_{12}^5	$\frac{\gamma_2}{\gamma_1+1} \geq \theta_2$,	$\frac{\alpha\gamma_1}{(1-\alpha)\gamma_1+1} \geq \theta_{11}$,	$(1-\alpha)\gamma_1 < \theta_{12}$
$P_{12}^{\cap}$	$\frac{\alpha\gamma_1}{(1-\alpha)\gamma_1+\gamma_2+1} \geq \theta_{11}$,	$\frac{\gamma_2}{\gamma_1+1} \geq \theta_2$,	$(1-\alpha)\gamma_1 < \theta_{12}$

TABLE III

Events of P_2

P_2^1	$\frac{\alpha\gamma_1}{(1-\alpha)\gamma_1+\gamma_2+1} < \theta_{11}$,	$\frac{(1-\alpha)\gamma_1}{\alpha\gamma_1+\gamma_2+1} < \theta_{12}$,	$\frac{\gamma_2}{\gamma_1+1} < \theta_2$
P_2^2	$\frac{(1-\alpha)\gamma_1}{\alpha\gamma_1+\gamma_2+1} \geq \theta_{12}$,	$\frac{\alpha\gamma_1}{\gamma_2+1} < \theta_{11}$,	$\frac{\gamma_2}{\alpha\gamma_1+1} < \theta_2$
P_2^3	$\frac{(1-\alpha)\gamma_1}{\alpha\gamma_1+\gamma_2+1} \geq \theta_{12}$,	$\frac{\alpha\gamma_1}{\gamma_2+1} \geq \theta_{11}$,	$\gamma_2 < \theta_2$
P_2^4	$\frac{\alpha\gamma_1}{(1-\alpha)\gamma_1+\gamma_2+1} \geq \theta_{11}$,	$\frac{(1-\alpha)\gamma_1}{\gamma_2+1} < \theta_{12}$,	$\frac{\gamma_2}{(1-\alpha)\gamma_1+1} < \theta_2$
P_2^5	$\frac{\alpha\gamma_1}{(1-\alpha)\gamma_1+\gamma_2+1} \geq \theta_{11}$,	$\frac{(1-\alpha)\gamma_1}{\gamma_2+1} \geq \theta_{12}$,	$\gamma_2 < \theta_2$
$P_2^{\cap}$	$\frac{\alpha\gamma_1}{(1-\alpha)\gamma_1+\gamma_2+1} \geq \theta_{11}$,	$\frac{(1-\alpha)\gamma_1}{\alpha\gamma_1+\gamma_2+1} \geq \theta_{12}$,	$\gamma_2 < \theta_2$

Theorem 2: The outage probability of x_{12}, i.e., the second message of the first source, is given by

$$P_{12} = P_{12}^1 + P_{12}^2 + P_{12}^3 + P_{12}^4 + P_{12}^5 - P_{12}^{\cap}, \tag{6}$$

where the expressions for P_{12}^j with $j = \{2,3,5,\cap\}$ are obtained from Table V, $P_{12}^1 = P_{11}^1$ and $P_{12}^4 = P_{11}^4$.

Proof: To derive the outage probability of the message x_{12}, the union of the five events in Table II should be calculated.

Similarly with the message x_{11}, the only events that are not mutually exclusive are the events 3 and 5 and their intersection is given in the last row of Table II. Following similar procedure with the message x_{11}, the probability of each of the five events and the intersection is calculated and (6) is derived. ■

Next, the outage probability of the message x_2 is derived.

Theorem 3: The outage probability of x_2, i.e., the message of the second source, is given by

$$P_2 = P_2^1 + P_2^2 + P_2^3 + P_2^4 + P_2^5 - P_2^{\cap}, \tag{7}$$

where the expressions for P_2^j with $j = \{3,5,\cap\}$ are obtained from Table VI, $P_2^1 = P_{11}^1$, $P_2^2 = P_{11}^2$ and $P_2^4 = P_{12}^2$.

Proof: To derive the outage probability of the message x_2, the union of the five events in Table III should be calculated.

Similarly with the messages x_{11} and x_{12}, the only events that are not mutually exclusive are the events 3 and 5 and their intersection is given in the last row of Table III. Following similar procedure with the message x_{11}, the probability of each of the five events and the intersection is calculated and (7) is derived. ■

Next, we provide the expression for the joint outage probability, i.e, the probability that outage occurs in all three messages.

TABLE IV
OUTAGE PROBABILITY P_{11}

	Conditions		Expressions
P_{11}^1	$\theta_{11}\theta_{12}\geq 1$	$0<\alpha<\frac{1}{1+\theta_{12}}$	$\frac{\lambda_1}{\lambda_1+\lambda_2\frac{1-\alpha-\alpha\theta_{12}}{\theta_{12}}}\left(e^{-\lambda_1\frac{\theta_{12}}{1-\alpha-\alpha\theta_{12}}}-e^{-\left(\lambda_1+\lambda_2\frac{1-\alpha-\alpha\theta_{12}}{\theta_{12}}\right)c_1+\lambda_2}\right)-\frac{\lambda_1 e^{-\lambda_2\theta_2}}{\lambda_1+\lambda_2\theta_2}\left(e^{-(\lambda_1+\lambda_2\theta_2)\frac{\theta_{12}}{1-\alpha-\alpha\theta_{12}}}-e^{-(\lambda_1+\lambda_2\theta_2)c_1}\right)$
		$\frac{1}{1+\theta_{12}}\leq\alpha<\frac{\theta_{11}}{1+\theta_{11}}$	$1-\frac{\lambda_1 e^{-\lambda_2\theta_2}}{\lambda_1+\lambda_2\theta_2}$
		$\frac{\theta_{11}}{1+\theta_{11}}\leq\alpha<1$	$\frac{\lambda_1}{\lambda_1+\lambda_2\frac{\alpha+\alpha\theta_{11}-\theta_{11}}{\theta_{11}}}\left(e^{-\lambda_1\frac{\theta_{11}}{\alpha+\alpha\theta_{11}-\theta_{11}}}-e^{-\left(\lambda_1+\lambda_2\frac{\alpha+\alpha\theta_{11}-\theta_{11}}{\theta_{11}}\right)c_2+\lambda_2}\right)-\frac{\lambda_1 e^{-\lambda_2\theta_2}}{\lambda_1+\lambda_2\theta_2}\left(e^{-(\lambda_1+\lambda_2\theta_2)\frac{\theta_{11}}{\alpha+\alpha\theta_{11}-\theta_{11}}}-e^{-(\lambda_1+\lambda_2\theta_2)c_2}\right)$
	$\theta_{11}\theta_{12}<1$	$0<\alpha<\frac{\theta_{11}+\theta_{11}\theta_{12}}{\theta_{11}+\theta_{12}+2\theta_{11}\theta_{12}}$	$\frac{\lambda_1}{\lambda_1+\lambda_2\frac{1-\alpha-\alpha\theta_{12}}{\theta_{12}}}\left(e^{-\lambda_1\frac{\theta_{12}}{1-\alpha-\alpha\theta_{12}}}-e^{-\left(\lambda_1+\lambda_2\frac{1-\alpha-\alpha\theta_{12}}{\theta_{12}}\right)c_1+\lambda_2}\right)-\frac{\lambda_1 e^{-\lambda_2\theta_2}}{\lambda_1+\lambda_2\theta_2}\left(e^{-(\lambda_1+\lambda_2\theta_2)\frac{\theta_{12}}{1-\alpha-\alpha\theta_{12}}}-e^{-(\lambda_1+\lambda_2\theta_2)c_1}\right)$
		$\frac{\theta_{11}+\theta_{11}\theta_{12}}{\theta_{11}+\theta_{12}+2\theta_{11}\theta_{12}}\leq\alpha<1$	$\frac{\lambda_1}{\lambda_1+\lambda_2\frac{\alpha+\alpha\theta_{11}-\theta_{11}}{\theta_{11}}}\left(e^{-\lambda_1\frac{\theta_{11}}{\alpha+\alpha\theta_{11}-\theta_{11}}}-e^{-\left(\lambda_1+\lambda_2\frac{\alpha+\alpha\theta_{11}-\theta_{11}}{\theta_{11}}\right)c_2+\lambda_2}\right)-\frac{\lambda_1 e^{-\lambda_2\theta_2}}{\lambda_1+\lambda_2\theta_2}\left(e^{-(\lambda_1+\lambda_2\theta_2)\frac{\theta_{11}}{\alpha+\alpha\theta_{11}-\theta_{11}}}-e^{-(\lambda_1+\lambda_2\theta_2)c_2}\right)$
P_{11}^2	$\theta_{11}\theta_2\geq 1$	$0<\alpha<\frac{1}{1+\theta_{12}+\theta_{12}\theta_2}$	$\frac{\lambda_2 e^{-\lambda_1\frac{\theta_{12}}{1-\alpha-\alpha\theta_{12}}}}{\lambda_1\frac{\theta_{12}}{1-\alpha-\alpha\theta_{12}}+\lambda_2}\left(1-e^{-\left(\lambda_1\frac{\theta_{12}}{1-\alpha-\alpha\theta_{12}}+\lambda_2\right)\frac{\theta_2-\alpha\theta_2}{1-\alpha-\alpha\theta_{12}-\alpha\theta_{12}\theta_2}}\right)+\frac{\lambda_2 e^{\lambda_1\frac{1}{\alpha}}}{\lambda_1\frac{1}{\alpha\theta_2}+\lambda_2}e^{-\left(\lambda_1\frac{1}{\alpha\theta_2}+\lambda_2\right)\frac{\theta_2-\alpha\theta_2}{1-\alpha-\alpha\theta_{12}-\alpha\theta_{12}\theta_2}}-\frac{\lambda_2 e^{-\lambda_1\frac{\theta_{11}}{\alpha}}}{\lambda_1\frac{\theta_{11}}{\alpha}+\lambda_2}$
		$\frac{1}{1+\theta_{12}+\theta_{12}\theta_2}\leq\alpha<\frac{\theta_{11}}{\theta_{11}+\theta_{12}+\theta_{11}\theta_{12}}$	$\frac{\lambda_2 e^{-\lambda_1\frac{\theta_{12}}{1-\alpha-\alpha\theta_{12}}}}{\lambda_1\frac{\theta_{12}}{1-\alpha-\alpha\theta_{12}}+\lambda_2}-\frac{\lambda_2 e^{-\lambda_1\frac{\theta_{11}}{\alpha}}}{\lambda_1\frac{\theta_{11}}{\alpha}+\lambda_2}$
	$\theta_{11}\theta_2<1$	$0<\alpha<\frac{\theta_{11}}{\theta_{11}+\theta_{12}+\theta_{11}\theta_{12}}$	$\frac{\lambda_2 e^{-\lambda_1\frac{\theta_{12}}{1-\alpha-\alpha\theta_{12}}}}{\lambda_1\frac{\theta_{12}}{1-\alpha-\alpha\theta_{12}}+\lambda_2}\left(1-e^{-\left(\lambda_1\frac{\theta_{12}}{1-\alpha-\alpha\theta_{12}}+\lambda_2\right)\frac{\theta_2-\alpha\theta_2}{1-\alpha-\alpha\theta_{12}-\alpha\theta_{12}\theta_2}}\right)+\frac{\lambda_2 e^{\lambda_1\frac{1}{\alpha}}}{\lambda_1\frac{1}{\alpha\theta_2}+\lambda_2}\left(e^{-\left(\lambda_1\frac{1}{\alpha\theta_2}+\lambda_2\right)\frac{\theta_2-\alpha\theta_2}{1-\alpha-\alpha\theta_{12}-\alpha\theta_{12}\theta_2}}-e^{-\left(\lambda_1\frac{1}{\alpha\theta_2}+\lambda_2\right)\frac{\theta_2+\theta_{11}\theta_2}{1-\theta_{11}\theta_2}}\right)-\frac{\lambda_2 e^{-\lambda_1\frac{\theta_{11}}{\alpha}}}{\lambda_1\frac{\theta_{11}}{\alpha}+\lambda_2}\left(1-e^{-\left(\lambda_1\frac{\theta_{11}}{\alpha}+\lambda_2\right)\frac{\theta_2+\theta_{11}\theta_2}{1-\theta_{11}\theta_2}}\right)$
P_{11}^3		$0<\alpha<\frac{\theta_{11}}{\theta_{11}+\theta_{12}+\theta_{11}\theta_{12}+\theta_{12}\theta_2+\theta_{11}\theta_{12}\theta_2}$	$\frac{\lambda_1 e^{-\lambda_2\theta_2}}{\lambda_1+\lambda_2\alpha\theta_2}\left(e^{-(\lambda_1+\lambda_2\alpha\theta_2)\frac{\theta_{12}+\theta_{12}\theta_2}{1-\alpha-\alpha\theta_{12}-\alpha\theta_{12}\theta_2}}-e^{-(\lambda_1+\lambda_2\alpha\theta_2)\frac{\theta_{11}}{\alpha}}\right)-\frac{\lambda_1 e^{\lambda_2}}{\lambda_1+\lambda_2\frac{1-\alpha-\alpha\theta_{12}}{\theta_{12}}}\left(e^{-\left(\lambda_1+\lambda_2\frac{1-\alpha-\alpha\theta_{12}}{\theta_{12}}\right)\frac{\theta_{12}+\theta_{12}\theta_2}{1-\alpha-\alpha\theta_{12}-\alpha\theta_{12}\theta_2}}-e^{-\left(\lambda_1+\lambda_2\frac{1-\alpha-\alpha\theta_{12}}{\theta_{12}}\right)\frac{\theta_{11}}{\alpha}}\right)$
P_{11}^4	$\theta_{11}\theta_{12}\geq 1$	$0<\alpha<\frac{1}{1+\theta_{12}}$	$\frac{\lambda_1 e^{-\lambda_2\theta_2}}{\lambda_1+\lambda_2\theta_2}\left(1-e^{-(\lambda_1+\lambda_2\theta_2)\frac{\theta_{12}}{1-\alpha-\alpha\theta_{12}}}\right)$
		$\frac{1}{1+\theta_{12}}\leq\alpha<\frac{\theta_{11}}{1+\theta_{11}}$	$\frac{\lambda_1 e^{-\lambda_2\theta_2}}{\lambda_1+\lambda_2\theta_2}$
		$\frac{\theta_{11}}{1+\theta_{11}}\leq\alpha<1$	$\frac{\lambda_1 e^{-\lambda_2\theta_2}}{\lambda_1+\lambda_2\theta_2}\left(1-e^{-(\lambda_1+\lambda_2\theta_2)\frac{\theta_{11}}{\alpha+\alpha\theta_{11}-\theta_{11}}}\right)$
	$\theta_{11}\theta_{12}<1$	$0<\alpha<\frac{\theta_{11}+\theta_{11}\theta_{12}}{\theta_{11}+\theta_{12}+2\theta_{11}\theta_{12}}$	$\frac{\lambda_1 e^{-\lambda_2\theta_2}}{\lambda_1+\lambda_2\theta_2}\left(1-e^{-(\lambda_1+\lambda_2\theta_2)\frac{\theta_{12}}{1-\alpha-\alpha\theta_{12}}}\right)$
		$\frac{\theta_{11}+\theta_{11}\theta_{12}}{\theta_{11}+\theta_{12}+2\theta_{11}\theta_{12}}\leq\alpha<1$	$\frac{\lambda_1 e^{-\lambda_2\theta_2}}{\lambda_1+\lambda_2\theta_2}\left(1-e^{-(\lambda_1+\lambda_2\theta_2)\frac{\theta_{11}}{\alpha+\alpha\theta_{11}-\theta_{11}}}\right)$
P_{11}^5		$0<\alpha<\frac{\theta_{11}}{\theta_{11}+\theta_{12}+\theta_{11}\theta_{12}}$	$\frac{\lambda_1 e^{-\lambda_2\theta_2}}{\lambda_1+\lambda_2\theta_2}\left(e^{-(\lambda_1+\lambda_2\theta_2)\frac{\theta_{12}}{1-\alpha-\alpha\theta_{12}}}-e^{-(\lambda_1+\lambda_2\theta_2)\frac{\theta_{11}}{\alpha}}\right)$
$P_{11}^{\cap}$	$\theta_{12}\theta_2<1$	$0<\alpha<\frac{\theta_{11}-\theta_{11}\theta_{12}\theta_2}{\theta_{11}+\theta_{12}+\theta_{11}\theta_{12}+\theta_{12}\theta_2}$	$\frac{\lambda_1 e^{-\lambda_2\theta_2}}{\lambda_1+\lambda_2\theta_2}\left(e^{-(\lambda_1+\lambda_2\theta_2)\frac{\theta_{12}+\theta_{12}\theta_2}{1-\theta_{12}\theta_2-\alpha-\alpha\theta_{12}}}-e^{-(\lambda_1+\lambda_2\theta_2)\frac{\theta_{11}}{\alpha}}\right)-\frac{\lambda_1 e^{\lambda_2}}{\lambda_1+\lambda_2\frac{1-\alpha-\alpha\theta_{12}}{\theta_{12}}}\left(e^{-(\lambda_1+\lambda_2\frac{1-\alpha-\alpha\theta_{12}}{\theta_{12}})\frac{\theta_{12}+\theta_{12}\theta_2}{1-\theta_{12}\theta_2-\alpha-\alpha\theta_{12}}}-e^{-(\lambda_1+\lambda_2\frac{1-\alpha-\alpha\theta_{12}}{\theta_{12}})\frac{\theta_{11}}{\alpha}}\right)$

Proposition 1: The joint outage probability, P_j, of the considered network is given by P_{11}^1 in Table IV.

Remark 1: It should be highlighted that SIC can be considered as a special case of RSMA by setting either $\alpha=\beta=0$ or

TABLE V

OUTAGE PROBABILITY P_{12}

		Conditions	Expressions
P_{12}^2	$\theta_{12}\theta_2 \geq 1$	$\frac{\theta_{11}+\theta_{11}\theta_{12}}{\theta_{11}+\theta_{12}+\theta_{11}\theta_{12}} < \alpha < \frac{\theta_{11}+\theta_{11}\theta_2}{1+\theta_{11}+\theta_{11}\theta_2}$	$\frac{\lambda_2 e^{-\lambda_1 \frac{\theta_{11}}{\alpha+\alpha\theta_{11}-\theta_{11}}}}{\lambda_1 \frac{\theta_{11}}{\alpha+\alpha\theta_{11}-\theta_{11}}+\lambda_2} - \frac{\lambda_2 e^{-\lambda_1 \frac{\theta_{12}}{1-\alpha}}}{\lambda_1 \frac{\theta_{12}}{1-\alpha}+\lambda_2}$
		$\frac{\theta_{11}+\theta_{11}\theta_2}{1+\theta_{11}+\theta_{11}\theta_2} \leq \alpha \leq 1$	$\frac{\lambda_2 e^{-\lambda_1 \frac{\theta_{11}}{\alpha+\alpha\theta_{11}-\theta_{11}}}}{\lambda_1 \frac{\theta_{11}}{\alpha+\alpha\theta_{11}-\theta_{11}}+\lambda_2}\left(1-e^{-\left(\lambda_1 \frac{\theta_{11}}{\alpha+\alpha\theta_{11}-\theta_{11}}+\lambda_2\right)\frac{\alpha\theta_2}{\alpha+\alpha\theta_{11}+\alpha\theta_{11}\theta_2-\theta_{11}-\theta_{11}\theta_2}}\right) + \frac{\lambda_2 e^{\lambda_1 \frac{\theta_2}{\theta_2-\alpha\theta_2}}}{\lambda_1 \frac{1}{\theta_2-\alpha\theta_2}+\lambda_2}\left(e^{-\left(\lambda_1 \frac{1}{\theta_2-\alpha\theta_2}+\lambda_2\right)\frac{\alpha\theta_2}{\alpha+\alpha\theta_{11}+\alpha\theta_{11}\theta_2-\theta_{11}-\theta_{11}\theta_2}}\right) - \frac{\lambda_2 e^{-\lambda_1 \frac{\theta_{12}}{1-\alpha}}}{\lambda_1 \frac{\theta_{12}}{1-\alpha}+\lambda_2}$
	$\theta_{12}\theta_2 < 1$	$\frac{\theta_{11}+\theta_{11}\theta_{12}}{\theta_{11}+\theta_{12}+\theta_{11}\theta_{12}} < \alpha \leq 1$	$\frac{\lambda_2 e^{-\lambda_1 \frac{\theta_{11}}{\alpha+\alpha\theta_{11}-\theta_{11}}}}{\lambda_1 \frac{\theta_{11}}{\alpha+\alpha\theta_{11}-\theta_{11}}+\lambda_2}\left(1-e^{-\left(\lambda_1 \frac{\theta_{11}}{\alpha+\alpha\theta_{11}-\theta_{11}}+\lambda_2\right)\frac{\alpha\theta_2}{\alpha+\alpha\theta_{11}+\alpha\theta_{11}\theta_2-\theta_{11}-\theta_{11}\theta_2}}\right) - \frac{\lambda_2 e^{-\lambda_1 \frac{\theta_{12}}{1-\alpha}}}{\lambda_1 \frac{\theta_{12}}{1-\alpha}+\lambda_2}\left(1-e^{-\left(\lambda_1 \frac{\theta_{12}}{1-\alpha}+\lambda_2\right)\frac{\alpha\theta_2}{\alpha+\alpha\theta_{11}+\alpha\theta_{11}\theta_2-\theta_{11}-\theta_{11}\theta_2}}\right) + \frac{\lambda_2 e^{\lambda_1 \frac{\theta_2}{\theta_2-\alpha\theta_2}}}{\lambda_1 \frac{1}{\theta_2-\alpha\theta_2}+\lambda_2}\left(e^{-\left(\lambda_1 \frac{1}{\theta_2-\alpha\theta_2}+\lambda_2\right)\frac{\alpha\theta_2}{\alpha+\alpha\theta_{11}+\alpha\theta_{11}\theta_2-\theta_{11}-\theta_{11}\theta_2}} - e^{-\left(\lambda_1 \frac{1}{\theta_2-\alpha\theta_2}+\lambda_2\right)\frac{\theta_2+\theta_{12}\theta_2}{1-\theta_{12}\theta_2}}\right) + \frac{\lambda_2 e^{-\lambda_1 \frac{\theta_{12}}{1-\alpha}}}{\lambda_1 \frac{\theta_{12}}{1-\alpha}+\lambda_2}\left(e^{-\left(\lambda_1 \frac{\theta_{12}}{1-\alpha}+\lambda_2\right)\frac{\theta_2+\theta_{12}\theta_2}{1-\theta_{12}\theta_2}} - e^{-\left(\lambda_1 \frac{\theta_{12}}{1-\alpha}+\lambda_2\right)\frac{\alpha\theta_2}{\alpha+\alpha\theta_{11}+\alpha\theta_{11}\theta_2-\theta_{11}-\theta_{11}\theta_2}}\right)$
P_{12}^3		$\frac{\theta_{11}+\theta_{11}\theta_{12}+\theta_{11}\theta_2+\theta_{11}\theta_{12}\theta_2}{\theta_{11}+\theta_{12}+\theta_{11}\theta_{12}+\theta_{11}\theta_2+\theta_{11}\theta_{12}\theta_2} < \alpha \leq 1$	$\frac{\lambda_1 e^{\lambda_2}}{\lambda_1+\lambda_2 \frac{\alpha+\alpha\theta_{11}-\theta_{11}}{\theta_{11}}}\left(e^{-\left(\lambda_1+\lambda_2 \frac{\alpha+\alpha\theta_{11}-\theta_{11}}{\theta_{11}}\right)\frac{\theta_{12}}{1-\alpha}} - e^{-\left(\lambda_1+\lambda_2 \frac{\alpha+\alpha\theta_{11}-\theta_{11}}{\theta_{11}}\right)\frac{\theta_{11}+\theta_{11}\theta_2}{\alpha+\alpha\theta_{11}+\alpha\theta_{11}\theta_2-\theta_{11}-\theta_{11}\theta_2}}\right) - \frac{\lambda_1 e^{-\lambda_2\theta_2}}{\lambda_1+\lambda_2(\theta_2-\alpha\theta_2)}\left(e^{-(\lambda_1+\lambda_2(\theta_2-\alpha\theta_2))\frac{\theta_{12}}{1-\alpha}} - e^{-(\lambda_1+\lambda_2(\theta_2-\alpha\theta_2))\frac{\theta_{11}+\theta_{11}\theta_2}{\alpha+\alpha\theta_{11}+\alpha\theta_{11}\theta_2-\theta_{11}-\theta_{11}\theta_2}}\right)$
P_{12}^5		$\frac{\theta_{11}+\theta_{11}\theta_{12}}{\theta_{11}+\theta_{12}+\theta_{11}\theta_{12}} < \alpha \leq 1$	$\frac{\lambda_1 e^{-\lambda_2\theta_2}}{\lambda_1+\lambda_2\theta_2}\left(e^{-(\lambda_1+\lambda_2\theta_2)\frac{\theta_{11}}{\alpha+\alpha\theta_{11}-\theta_{11}}} - e^{-(\lambda_1+\lambda_2\theta_2)\frac{\theta_{12}}{1-\alpha}}\right)$
$P_{12}^{\cap}$	$\theta_{11}\theta_2 < 1$	$\frac{\theta_{11}+\theta_{11}\theta_{12}+\theta_{11}\theta_2+\theta_{11}\theta_{12}\theta_2}{\theta_{11}+\theta_{12}+\theta_{11}\theta_{12}+\theta_{11}\theta_2} < \alpha \leq 1$	$\frac{\lambda_1 e^{-\lambda_2\theta_2}}{\lambda_1+\lambda_2\theta_2}\left(e^{-(\lambda_1+\lambda_2\theta_2)\frac{\theta_{11}+\theta_{11}\theta_2}{\alpha+\alpha\theta_{11}-\theta_{11}-\theta_{11}\theta_2}} - e^{-(\lambda_1+\lambda_2\theta_2)\frac{\theta_{12}}{1-\alpha}}\right) - \frac{\lambda_1 e^{\lambda_2}}{\lambda_1+\lambda_2 \frac{\alpha+\alpha\theta_{11}-\theta_{11}}{\theta_{11}}}\left(e^{-\left(\lambda_1+\lambda_2 \frac{\alpha+\alpha\theta_{11}-\theta_{11}}{\theta_{11}}\right)\frac{\theta_{11}+\theta_{11}\theta_2}{\alpha+\alpha\theta_{11}-\theta_{11}-\theta_{11}\theta_2}} - e^{-\left(\lambda_1+\lambda_2 \frac{\alpha+\alpha\theta_{11}-\theta_{11}}{\theta_{11}}\right)\frac{\theta_{12}}{1-\alpha}}\right)$

TABLE VI

OUTAGE PROBABILITY P_2

		Conditions	Expressions
P_2^3		$0 < \alpha < \frac{\theta_{11}}{\theta_{11}+\theta_{12}+\theta_{11}\theta_{12}}$	$\frac{\lambda_2 e^{-\lambda_1 \frac{\theta_{11}}{\alpha}}}{\lambda_1 \frac{\theta_{11}}{\alpha}+\lambda_2}\left(1-e^{-\left(\lambda_1 \frac{\theta_{11}}{\alpha}+\lambda_2\right)\theta_2}\right)$
		$\frac{\theta_{11}}{\theta_{11}+\theta_{12}+\theta_{11}\theta_{12}} \leq \alpha < \frac{1}{1+\theta_{12}}$	$\frac{\lambda_2 e^{-\lambda_1 \frac{\theta_{12}}{1-\alpha-\alpha\theta_{12}}}}{\lambda_1 \frac{\theta_{12}}{1-\alpha-\alpha\theta_{12}}+\lambda_2}\left(1-e^{-\left(\lambda_1 \frac{\theta_{12}}{1-\alpha-\alpha\theta_{12}}+\lambda_2\right)\theta_2}\right)$
P_2^5		$\frac{\theta_{11}}{1+\theta_{11}} < \alpha < \frac{\theta_{11}+\theta_{11}\theta_{12}}{\theta_{11}+\theta_{12}+\theta_{11}\theta_{12}}$	$\frac{\lambda_2 e^{-\lambda_1 \frac{\theta_{11}}{\alpha+\alpha\theta_{11}-\theta_{11}}}}{\lambda_1 \frac{\theta_{11}}{\alpha+\alpha\theta_{11}-\theta_{11}}+\lambda_2}\left(1-e^{-\left(\lambda_1 \frac{\theta_{11}}{\alpha+\alpha\theta_{11}-\theta_{11}}+\lambda_2\right)\theta_2}\right)$
		$\frac{\theta_{11}+\theta_{11}\theta_{12}}{\theta_{11}+\theta_{12}+\theta_{11}\theta_{12}} \leq \alpha < 1$	$\frac{\lambda_2 e^{-\lambda_1 \frac{\theta_{12}}{1-\alpha}}}{\lambda_1 \frac{\theta_{12}}{1-\alpha}+\lambda_2}\left(1-e^{-\left(\lambda_1 \frac{\theta_{12}}{1-\alpha}+\lambda_2\right)\theta_2}\right)$
$P_2^{\cap}$	$\theta_{11}\theta_{12} < 1$	$\frac{\theta_{11}}{1+\theta_{11}} < \alpha < \frac{\theta_{11}+\theta_{11}\theta_{12}}{\theta_{11}+\theta_{12}+2\theta_{11}\theta_{12}}$	$\frac{\lambda_2 e^{-\lambda_1 \frac{\theta_{11}}{\alpha+\alpha\theta_{11}-\theta_{11}}}}{\lambda_1 \frac{\theta_{11}}{\alpha+\alpha\theta_{11}-\theta_{11}}+\lambda_2}\left(1-e^{-\left(\lambda_1 \frac{\theta_{11}}{\alpha+\alpha\theta_{11}-\theta_{11}}+\lambda_2\right)\theta_2}\right)$
		$\frac{\theta_{11}}{\theta_{11}+\theta_{12}+2\theta_{11}\theta_{12}} \leq \alpha < \frac{1}{1+\theta_{12}}$	$\frac{\lambda_2 e^{-\lambda_1 \frac{\theta_{12}}{1-\alpha-\alpha\theta_{12}}}}{\lambda_1 \frac{\theta_{12}}{1-\alpha-\alpha\theta_{12}}+\lambda_2}\left(1-e^{-\left(\lambda_1 \frac{\theta_{12}}{1-\alpha-\alpha\theta_{12}}+\lambda_2\right)\theta_2}\right)$

$\alpha = \beta = 1$. However, since in these cases either θ_{11} or θ_{12} is equal to zero, the derived expressions are undefined.

The outage probability for an uplink network with two sources where SIC is used is derived in [1].

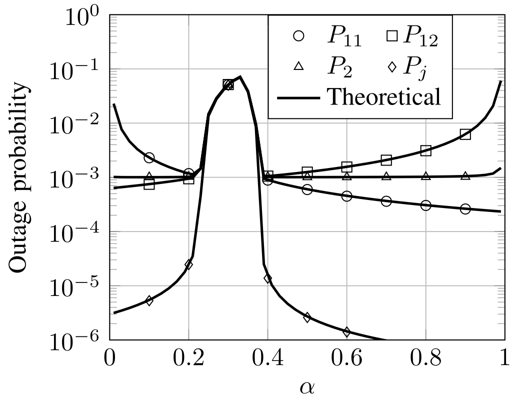

Fig. 1. Outage probability versus power allocation factor α.

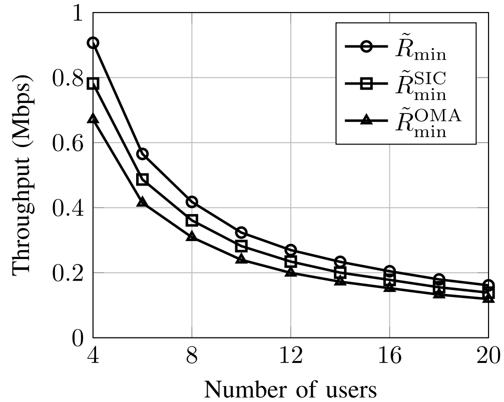

Fig. 2. Throughput versus number of users.

IV. A Practical MAC Layer Protocol With RSMA

It has been shown in the existing literature that MAC schemes based on non-orthogonal multiplexing can fundamentally improve the performance of uplink system, especially in terms of connectivity. It should be highlighted that increasing connectivity is the main motivation of using the power domain to multiplex the sources' messages [8]. For example, this framework can be used in IoT networks with RA to increase connectivity by reducing the number of collisions. Indicatively, in a slotted ALOHA (SA) network where the sources access the channel with access probability q, RSMA can increase the number of served users, while each user retains the same throughput. In the following proposition, we derive the throughput of the sources in an RA network with a MAC layer protocol based on SA and RSMA, consisting of a BS and N sources, similar to the one in [1] where a protocol based on SA and SIC is used.

Proposition 2: The throughput of the source s_1, i.e., the one performing RS, and the source s_2, the one that does not perform RS, are given, respectively, by

$$\tilde{R}_1 = \hat{R}_1 q \Bigg(\bar{q}^{N-1}(1-P_1^{\mathrm{o}}) + q\bar{q}^{N-2} \times \sum_{s_2 \in \mathcal{S}/s_1} (b(1-P_{11}) + (1-b)(1-P_{12})) \Bigg) \quad (8)$$

and

$$\tilde{R}_2 = \hat{R}_2 q \Bigg(\bar{q}^{N-1}(1-P_2^{\mathrm{o}}) + q\bar{q}^{N-2} \sum_{s_1 \in \mathcal{S}/s_2} (1-P_2) \Bigg), \quad (9)$$

where $\bar{q} = 1-q$ and P_i^{o} denotes the outage probability of the i-th source for the OMA case [1].

In (8) and (9), the throughput is obtained as the sum of the OMA case, i.e, SA, where exactly one source is served, and the case that exactly two sources are served utilizing RSMA.

V. Numerical Results and Simulations

In this section, we illustrate the performance of the considered network and validate the derived expressions with simulations. We set the average received SNR at the BS from all sources $\mathbb{E}[\gamma_i] = 30$ dB and the bandwidth equal to 1 MHz.

In Fig. 1, we set the target rates $\hat{R}_1 = \hat{R}_2 = 1$ Mbps and the target rate factor $\beta = 0.3$. The outage probability of the three messages and the joint outage probability are illustrated versus the power allocation factor α and the tradeoff between P_{11} and P_{12} is obvious as α increases. It should be highlighted that there exists a region of α that the outage performance of the considered network deteriorates, thus the selection of α is of paramount importance for the design of an RSMA network.

In Fig. 2, we illustrate the performance of the considered RA network in terms of throughput versus the number of users, assuming that $q = \frac{1}{N}$. Specifically, we maximize numerically the minimal throughput among all sources, i.e., $\tilde{R}_{\min} = \min(\tilde{R}_i)$ with $i = \{1, \ldots, N\}$, subject to α, β and $\hat{R}_i$. It should be highlighted that the optimization of the power splitting factor and the target rates exploits all the necessary degrees of freedom provided by RSMA to achieve the maximal throughput. Also, for the sake of comparison, the performance of two also optimized baseline schemes is illustrated, namely i) OMA-based SA network where at most one user can be served in each time slot and ii) SA network with SIC that was proposed in [1]. It becomes apparent that the considered network outperforms the two benchmarks, thus RSMA can be utilized in an RA network to increase connectivity.

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