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A Novel Two-Parametric ISI-Free Pulse Based on Inverse Hyperbolic Functions

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***Abstract*—In this article, a new two-parametric pulse that outperforms state-of-the-art pulses with the best performance in terms of BER is proposed, which is studied in terms of frequency and time domain characteristics. Specifically, the pulse is based on the inverse-hyperbolic functions acsch and asech which are used for the first time for an ISI-free pulse, and its design depends only on the roll-off factor and the timing jitter parameter (i.e. two-parametric design). Finally, the proposed pulse is shown to outperform most of the well-known pulses reported in the literature, since it presents lower error probability, smaller maximum distortion and wider eye-diagram.**

***Index Terms*—Nyquist pulses, intersymbol interference (ISI), pulse shaping methods, timing jitter.**

I. Introduction

THE emergence of promising applications such as virtual and augmented reality, industrial Internet of Things (IoT), digital twins etc., has imposed new demands for better bandwidth reuse and higher error-free data rates [1], [2]. Therefore, novel methods that can reduce the bit error rate (BER) of a digital communication system need to be proposed. In more detail, BER is an important metric of a digital communication system's performance, since it includes the effects of noise, synchronization and distortion. A method to guarantee favorable transmission with minimum number of errors according to the pioneering work of Nyquist [3], is by developing proper pulses that minimize the inter-symbol interference (ISI) effect, which is inherent on digital communication systems. Specifically, Nyquist proposed the well-known raised-cosine (RC) pulse, which is the most commonly used pulse that reduces the ISI effect and can be easily implemented in practical digital communication receivers [3]. Since then, a strong research activity in the field of pulse shaping has been going on, with many authors proposing better-than-RC pulses which are two-parametric and their shape depends only on the roll-off factor, α, and the timing jitter parameter, t/T, and that are able to further optimize the communication's BER by further reducing the ISI effect, and providing wider eye-diagrams. [4]–[10]. Additionally, in [11]–[14] by invoking extra parameters that have been obtained through various optimization techniques, better-than-RC pulses with more than two-parameters have been presented. However, the utilization of pulses with more parameters, leads to pulses with complex expressions and limited practicality, as it is more difficult for them to be implemented in typical digital communication receivers. Therefore, it is of high importance to propose better-than-RC pulses, whose shape will depend on the two most common parameters, the roll-off factor α and the timing jitter parameter $\frac{t}{T}$.

Towards the direction of two-parametric pulse shaping, the authors in [5] proposed the utilization of inverse hyperbolic functions, which offered enhanced performance in terms of error rate without increasing the number of parameters that determine the pulse's shape. By taking advantage of the inverse hyperbolic functions, the work in [6] have introduced the concept of *inner* and *outer* functions which led to the proposal of two better-than-RC pulses that provided enhanced BER. The same authors in [10] presented the pulse acos[log], among others, which is based on the composition of the functions *inverse cosine* (outer function) and *natural logarithm* (inner function), while in [6] the pulse acos[asinh] was presented, having the *inverse cosine* and the *inverse hyperbolic sine* function, as outer and inner function, respectively. To the best of the authors' knowledge, the aforementioned pulses present superior performance in terms of BER among two-parametric better-than-RC pulses of the existing literature. However, the design technique in [6], does not guarantee that the resulting pulse would always present a superior performance. Thus, the choice of the functions becomes a major issue during the design process of a pulse.

In this work, following the design process in [6], we propose a new two-parametric better-than-RC pulse, which outperforms the state-of-the-art pulses [6], [10]. In more detail, the inverse-hyperbolic functions acsch and asech are used in the design process for the first time, leading to a pulse that outperforms the state-of-the-art ones. Specifically, the frequency and time domain characteristics of the proposed pulse are illustrated, which indicate the ability of the proposed pulse to further suppress the amplitude of the side lobes than the pulses under comparison. Finally, we evaluate the performance of the proposed pulse in terms of BER and eye-diagram and we validate the supremacy of the new proposed pulse to the state-of-the-art two-parametric Nyquist pulses. It should be mentioned, that in modern digital communication

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systems the ISI pulses are software-implemented, and hence, there are no practical limitations on their application.

II. New Two-Parametric Nyquist Pulse

A. Frequency Domain Characteristics

Based on the inner and outer functions' concept presented in [6], we proposed a new ISI-free pulse, whose frequency response reads as,

$$S(f) = \begin{cases} T, & |f| \leq B(1-\alpha) \\ T\left\{1 - \frac{1}{2\gamma} G\left(\frac{|f| - B(1-\alpha)}{2\alpha B}\right)\right\}, & B(1-\alpha) \leq |f| \leq B \\ T\left\{\frac{1}{2\gamma} G\left(\frac{-|f| + B(1+\alpha)}{2\alpha B}\right)\right\}, & B \leq |f| \leq B(1+\alpha) \\ 0, & |f| \geq B(1+\alpha) \end{cases} \quad (1)$$

where α represents the roll-off factor ($0 \leq \alpha \leq 1$), i.e., the excess bandwidth beyond the Nyquist bandwidth, and T denotes the symbol-period of the system, which defines the corresponding Nyquist frequency $B = 1/(2T)$. Also, $G(f) = g(h(f))$ is the composition function of the functions $h(f)$ and $g(f)$, where $h(f)$ is the inner function and $g(f)$ is the outer function, respectively. In particular, the function of the new proposed pulse is equal to $G(f) = \text{acsch}(\text{asech}(f))$, where, $\text{acsch}(\cdot)$ and $\text{asech}(\cdot)$ represent the *inverse hyperbolic cosecant*, and the *inverse hyperbolic secant* function, respectively. To ensure the continuity of (1) implies that γ ,as mentioned in [10], should meet the condition

$$\gamma = G(1/2), \quad (2)$$

Thus, for the proposed pulse γ constant is approximately equal to

$$\gamma_{\text{acsch[asech]}} = \text{acoth}(\text{acsch}(1/2)) \approx 0.7. \quad (3)$$

In Fig. 1 the frequency response of the new pulse as well as the frequency response of the pulses presented in [6], [10] are illustrated. It can be observed that all pulses' frequency response are concave when $B(1-\alpha) \leq |f| \leq B$, while convex for the frequency range $B \leq |f| \leq B(1+\alpha)$. Thus, they tend to transfer more power in the higher-frequency region, which leads to first side-lobes with lower amplitude in the time domain, as it will be shown, which consequently leads to improved robustness against the timing-jitter effects [10].

B. Time-Domain Characteristics

The impulse response of the new pulse is illustrated in Fig. 2, alongside with the acos[asinh] and acos[log] pulses for comparison sake. It should be noted that the new pulse presented in this work and defined in the frequency domain in (1) do not have closed-form expression in the time domain, and hence, they have been numerically evaluated, similarly to the pulses presented in [5], [6], [10]. As it can be seen, the first side lobe magnitude for the proposed pulse is greater than the compared pulses, thus demonstrating the side lobe suppression capabilities of the proposed pulse.

Another time-domain characteristic of the proposed pulse is its decay rate, which indicates how fast decay the tails of the pulse. In more detail, it can be estimated using the *Theorem 1* which is introduced previously in [7]. More specifically, we show that the following lemma is fulfilled.

Lemma 1: The decay rate of the pulse-family in (1) is $1/t^2$ when $\alpha \neq 0$ and $1/|t|$ when $\alpha = 0$.

Proof: For the shake of simplicity, we test in terms of time-domain properties only the pulse acsch[asech], while applying a similar procedure over the acsch[log] and acoth[acsch] will lead to exactly the same conclusion.

Assuming $\alpha \neq 0$ the first (i.e., $m = 1$) derivative of acsch[asech] is given by,

$$S'(f) = \begin{cases} 0, & |f| \leq B(1-\alpha) \\ T\frac{B\alpha}{\gamma c_1^2 \sqrt{c_1^2+1}\left(\text{asinh}\left(c_1^2\right)-1\right)}, & B(1-\alpha) \leq |f| \leq B \\ T\frac{B\alpha}{\gamma c_2^2 \sqrt{c_2^2+1}\left(\text{asinh}\left(c_2^2\right)-1\right)}, & B \leq |f| \leq B(1+\alpha) \\ 0, & |f| \geq B(1+\alpha) \end{cases} \quad (4)$$

where,

$$c_1 = \frac{2B\alpha}{f - B(1-\alpha)}, \quad c_2 = \frac{2B\alpha}{f - B(1+\alpha)}. \quad (5)$$

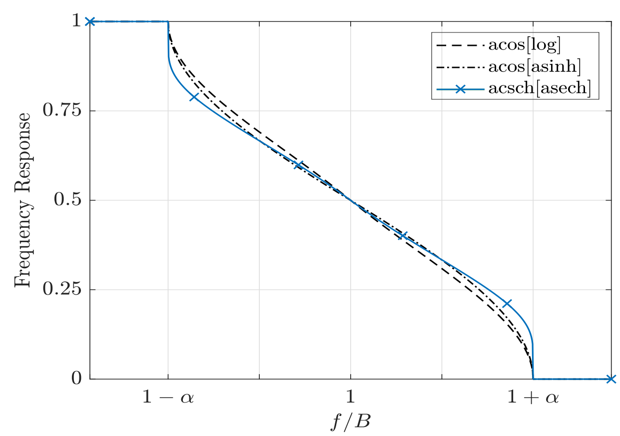

Fig. 1. Frequency response of the reference and new pulses with roll-off factor $\alpha = 0.35$.

Thus, based on (4), (5), we can readily observe that,

$$\lim_{f \to B(1-\alpha)} S'(f) = \lim_{f \to B(1+\alpha)} S'(f) \to \infty \quad (6)$$

and hence, discontinuity occurs at the transition points

$$f = B(1 \mp \alpha). \quad (7)$$

Assuming $\alpha = 0$, the frequency response $S(f)$ degenerates into a rectangular pulse and by invoking the inverse Fourier transform, it is converted into $\text{sinc}(\cdot)$ function which presents decay rate of $1/|t|$ [7], [10]. The latter statement concludes the proof. □

It is noted that despite the fact that the proposed pulse present lower decay rate than the well-known RC, which has

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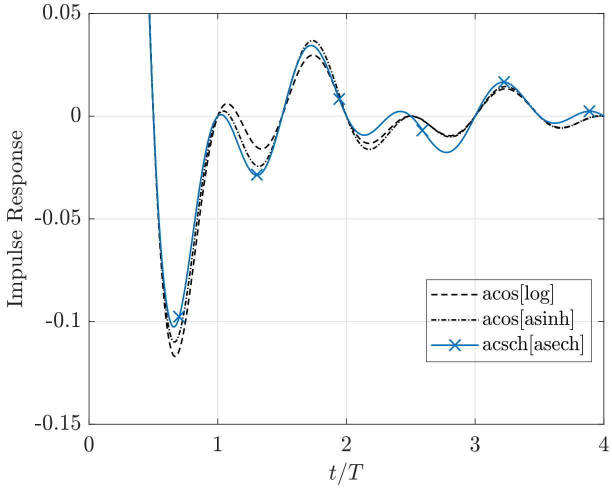

Fig. 2. Impulse response of the reference and new pulses with roll-off factor $\alpha = 0.35$.

TABLE I
EYE WIDTH AND MAX DISTORTION OF EYE PATTERNS

pulse	eye-width	max distortion
acos[log]	0.780	1.467
acos[asinh]	0.794	1.475
acsch[asech]	0.800	1.440

a decay rate of $1/t^3$, it presents superior performance in terms of BER, as it will be also verified next, since their first sidelobes amplitudes are lower than those of the RC pulse [4]–[6], [10].

C. Performance Evaluation

In this section, we evaluate the performance of the proposed Nyquist pulse in terms of eye-diagram and BER. In general, eye-diagram provides us with a great deal of visual information including the severity of ISI, sensitivity to timing jitter and the noise margin [15], [16]. In our case, from the eye-diagram of the new pulses depicted in Fig. 3, we can obtain the maximum distortion as well as the eye-width values, which are illustrated in Table I. As it can be observed, the proposed pulse presents wider eye-width than the two reference pulses, which in turn leads to longer sampling time interval, where the received signal can be sampled without the presence of ISI. Additionally, the new pulse has better performance in terms of max distortion which expresses the magnitude of the largest possible ISI sample at any given time instant.

Finally we calculate the BER value for a specific SNR value in the presence of time sampling errors. Specifically, we present the values of BER in Table II for the compared pulses for different values of the roll-off factor α and the timming jitter t/T parameter. It should be highlighted, that the parameters of the method in [17] are set as $N_1 = -256$, $N_2 = 256$, SNR = 15 dB, $N_M = 23$ and $N = 2^9$ interfering symbols, where $N_M = \frac{M+1}{2}$, M is the number of coefficients used in the truncated Fourier series presented in [17] for the BER calculation, and N_1, N_2 represent the number of interfering symbols before and after the transmitted symbol, respectively. As it can be observed, the proposed pulse outperforms all two-parametric state-of-the-art pulses in terms of BER, even for decreased roll-off factors, which are more preferable for future communication systems, since smaller α leads to lower bandwidth. The superior performance of the proposed pulse comes from the fact that its dominant sidelobes are controlled better than the compared pulses, a property, which in turn, comes from the unique characteristics of the mathematical functions (i.e., inverse-hyperbolic functions cash and asech), which have been used in design process of the filter.

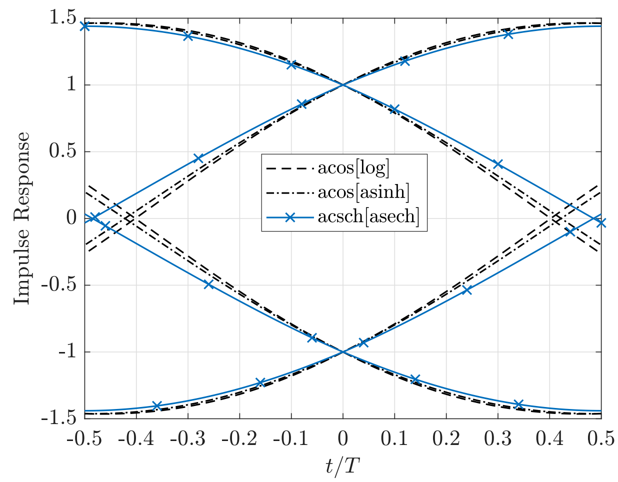

Fig. 3. Inner and outer boundaries of the eye-diagrams of the reference and the new pulses with roll-off factor $\alpha = 0.35$.

III. Conclusion

In this work, a new Nyquist ISI-free pulse, which outperforms the state-of-the-art pulses in terms of BER, maximum distortion and eye-diagram width, was presented. Specifically, it was studied in terms of its frequency and time domain characteristics. The new pulse is also characterized by low-complexity, since its shape depends only on the roll-off factor and the timing jitter parameter, indicating that it can be deployed easily in practical digital receivers.

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TABLE II
BIT ERROR RATE (BER) WITH $N = 2^9$ INTERFERING SYMBOLS AND SNR = 15dB .

α	pulse	$t/T = \pm 0.05$	$t/T = \pm 0.1$	$t/T = \pm 0.2$	$t/T = \pm 0.3$
0.2	acos[log]	6.7242e-8	1.8026e-6	5.5498e-4	1.9027e-2
	acos[asinh]	6.5145e-8	1.6861e-6	5.0950e-4	1.8015e-2
	acsch[asech]	**6.2568e-8**	**1.5439e-6**	**4.5393e-4**	**1.6764e-2**
0.25	acos[log]	5.3332e-8	1.0726e-6	2.7416e-4	1.2183e-2
	acos[asinh]	5.1488e-8	9.9816e-7	2.4946e-4	1.1349e-2
	acsch[asech]	**4.9210e-8**	**9.0533e-7**	**2.1816e-4**	**1.0306e-2**
0.35	acos[log]	3.5470e-8	4.3365e-7	7.3486e-5	4.5509e-3
	acos[asinh]	3.4124e-8	4.0410e-7	6.7653e-5	4.2252e-3
	acsch[asech]	**3.2414e-8**	**3.6393e-7**	**5.8702e-5**	**3.7520e-3**
0.5	acos[log]	2.1559e-8	1.4514e-7	1.4987e-5	1.2082e-3
	acos[asinh]	2.0758e-8	1.3617e-7	1.4609e-5	1.2202e-3
	acsch[asech]	**1.9693e-8**	**1.2137e-7**	**1.2947e-5**	**1.1507e-3**

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