Energy Efficient Cooperative Communications in Aggregated VLC/RF Networks with NOMA

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Abstract—Optimizing the energy efficiency (EE) of wireless networks is one of the key priorities in the design of beyond 5G mobile technologies. In this pursuit, the use of new frequency bands, in combination with advanced multiple access protocols and cooperative communications strategies, has recently shown promising results. To this end, this paper investigates an indoor wireless network that aggregates communication resources in visible light and radio-frequency (RF) bands, taking advantage of the complementary aspects of the two technologies. More specifically, a non-orthogonal multiple access (NOMA) scheme is introduced for the visible light communication (VLC) downlink, such that cell-edge users experiencing a weak VLC signal enhance their aggregated data rate with the aid of cooperative communications over RF sidelinks (i.e., device-to-device links). The optimal resource allocation strategy over both VLC and RF bands is derived aiming at EE maximization based on the Dinkelbach’s algorithm and successive convex approximation. Additionally, for the sake of flexibility, a weighted EE metric is proposed for the characterization of the aggregated VLC/RF network performance. Simulation results are provided to validate the proposed analysis, revealing the impact of various design and system parameters, such as the weighting factors, quality of service requirements, and channel conditions.

Index Terms—Visible light communications; Resource allocation; Energy efficiency; Aggregated VLC/RF networks; Cooperative NOMA; Sidelink communications.

I. INTRODUCTION

Conventional radio-frequency (RF) communications deal with the looming spectrum scarcity crisis due to the ever increasing number of devices requesting wireless connectivity, in particular in IoT applications, which also request for higher amounts of data traffic to be transported without a linear scaling on the energy that is consumed. More specifically, Ericsson forecasts that a smartphone’s monthly usage will be about 35 GB by the end of 2026, and that majority of this mobile traffic will be generated indoors [2]. Moreover, according to Cisco [3], video devices will considerably increase as well, creating a multiplier effect on the data traffic to be supported by future mobile networks. To tackle this issue, academia and industry have shifted their attention to unexploited parts of the electromagnetic spectrum, especially in situations where connectivity issues may hinder the development of novel applications that will shape the landscape beyond 5G (B5G) [4].

Within this context, optical wireless communication (OWC) has been recently considered as a complementary technology to RF-only communication systems. More specifically, visible light communication (VLC), which uses the abundant and unlicensed bandwidth that is available in the visible spectrum, has shown potential to support a significant part of the new indoor traffic that will be generated B5G. In this sense, VLC is considered as a promising technology that makes use of light-emitting diodes (LEDs), as high energy-efficient light sources, to offer both communication and illumination services simultaneously. Other advantages of the VLC technology include the high physical layer security and the possibility of using a high frequency reuse factor, since visible light signals do not penetrate opaque objects such as walls. Nevertheless, since line-of-sight (LoS) can be easily interrupted by the movement or rotation of the user device, VLC networks often experience link outages [5]. Since VLC does not interfere with RF signals [6], the study of wireless networks combining both technologies has become notably attractive due to their complementary benefits [7]. This constitutes the key idea behind hybrid cross-band VLC/RF wireless networks, which can be further categorized into aggregated and non-aggregated depending on how the communication resources are utilized [8]. In brief, in aggregated VLC/RF networks, information is received simultaneously over both bands, making decisions in a much faster scheduling time interval scale which resembles the carrier aggregation concept of 3GPP [9]. In contrast, non-aggregated VLC/RF networks use only one of these technologies in a much longer time scale, with the
aid of a specific time switching mechanism that resembles a vertical handover operation between the two networks [9].

The interplay between VLC/RF technologies and other promising enablers of 5G wireless access, such as the use of non-orthogonal multiple access (NOMA), is critical for the improvement of key performance indicators (KPIs) related to the massive connectivity of devices and the spectral efficiency of point-to-point wireless links [10]. The advantages of NOMA, when compared to the conventional orthogonal multiple access schemes, have been demonstrated in several recent works [11], [12] and experimental studies [13], validating its use as a promising multiple access method for VLC networks. Note that specific degrees of freedom of VLC networks, such as the reception angle, can enhance the performance of NOMA even further. Meanwhile in the design of such schemes, energy efficiency (EE) should be considered as a critical criterion [14], as the network/device energy consumption per transmitted bit needs to be reduced due to both environmental and economic reasons [15].

A. State-of-the-Art

Recent papers have extensively investigated different approaches to improve the EE of hybrid cross-band VLC/RF networks. In particular, for non-aggregated VLC/RF networks, a plethora of use cases and scenarios have been considered so far in [15]–[19]. For example, in [15], the power consumption of a hybrid VLC/RF network was minimized while fulfilling the data rate request of users and maintaining the illumination level requirements. Moreover, in [16], the use of power line communication (PLC) backhauling for a hybrid VLC/RF system was considered, studying the optimal resource allocation to maximize the sum-data-rate of users under the assumption of both perfect and imperfect channel state information (CSI). The authors in [17] minimized the power consumption of both VLC and RF APs while verifying a target link outage probability constraint. Similarly, the data rate maximization of a VLC AP that provides simultaneously energy harvesting and RF relaying services to users in a cooperative fashion was considered in [18]. Also, cooperative hybrid VLC/RF systems with simultaneous lightwave information and power transfer (SLIPT) were studied in [19], where the authors proposed a cognitive-based resource allocation policy and introduced bounds for the harvested energy.

Although a number of works have also considered the design of aggregated VLC/RF networks, to the best of the authors’ knowledge, they are not as extensively studied as the non-aggregated setups. For instance, the authors of [20] proved the non-surprising superiority of the aggregated over the non-aggregated approach in terms of the average system delay. Also, in a proof-of-concept experiment reported in [21], it was shown that the use of VLC/RF aggregation has potential to enable a three-fold gain on the average achievable throughput with respect to the client-server distance. When considering the use of a PLC backhaul for VLC APs, the authors of [22] optimized the resource allocation to maximize the EE of a PLC/VLC/RF network, whereas the authors in [23] defined a specific aggregated VLC/RF system based on cellular RF and VLC links to manage connectivity for outdoor and indoor users, respectively. Moreover, in [24], self-adaptive medium access control protocol was proposed to find a convenient trade-off between network delay, energy consumption, and probability of collision, while improving the overall data rate of the VLC/RF network. However, the considered solution was actually a non-aggregated VLC/RF network, since uplink and downlink were exclusively implemented over RF and VLC links respectively. On the other hand, [25], considered EE optimization for APs operating in both bands in the downlink, while in [26] a similar approach based on 802.11 MIMO was investigated for defining an aggregated VLC/RF network.

Regarding the utilization of NOMA for VLC, it has been extensively studied in the context of non-aggregated VLC/RF networks [27]–[32]. Specifically, aiming at solving the optimal user grouping problem, the use of coalitional game theory was considered in [27]. Moreover, the performance of a cooperative hybrid OW/C/RF relay network with NOMA was examined in terms of outage probability in [29], utilizing for this purpose a cross-band selection diversity combining scheme to improve the overall system performance. In [30] and [31], power allocation and user pairing were further studied in a cooperative setup similar to the one in [29]. Also, in [32], cooperative diversity over RF links was utilized to improve the link reliability and to boost the outage performance of a NOMA-based VLC system. However, NOMA has not been investigated in the context of aggregated VLC/RF networks so far.

B. Motivation and Contribution

The aggregation of data rates in NOMA VLC systems with cooperative RF sidelink communications is crucial to capitalize the complementary advantages offered by these wireless technologies. When compared to non-aggregated VLC/RF systems, multi-link operation in a hybrid cross-band system using the aggregated approach can be utilized to provide more degrees of freedom to find a suitable trade-off between enhancing the data rate and improving the EE. It is important to highlight that the particularity of the proposed concept of aggregated VLC/RF networks here with respect to the literature, is that here the two wireless technologies are used simultaneously in every user’s terminal. To this end, the main contributions of this work can be summarized as follows:

- Firstly, an aggregated transmission protocol is proposed based on VLC NOMA and cooperative RF (sidelink) communications. Here, the cell-center users act as decode-and-forward relays, receiving the information over the VLC link and forwarding part of it over an RF sidelink. Then, a cell-edge user can aggregate the information received over both VLC and RF links, enhancing hence its performance.
- A resource allocation problem is formulated aiming at maximizing the EE of the considered VLC/RF network using the proposed cooperative NOMA protocol, while fulfilling the quality of service (QoS) requirements of the users and taking into account the network power consumption constraints. The formulated problem is solved
using Dinkelbach's algorithm, the difference of convex (DC) structure of the resulting optimization problem, and successive convex approximation (SCA).

- Simulation results are presented to prove the superiority of the proposed cooperative NOMA protocol versus benchmarks that do not use NOMA and/or cooperative sidelink communications. Moreover, the effect of the weights of the EE and the RF channel is investigated, while the utilization of the RF link is examined for different user requirements and RF channel quality.

C. Paper Structure

The rest of the paper is organized as follows: Section II presents the system model and the formulas that have been selected to model the achievable data rate on both VLC and RF links. Section III proposes the aggregated VLC/RF protocol based on VLC NOMA and RF sidelink. Section IV derives the algorithm to optimize the EE of the proposed cooperative NOMA protocol, whereas Section V presents the numerical results with their detailed interpretation. Finally, conclusions are drawn in Section VI.

II. SYSTEM MODEL

The system model for the hybrid VLC/RF downlink network is shown in Fig. 1 and consists of one VLC-AP and two users $U_1$ and $U_2$. Without loss of generality, the VLC-AP is placed at a height of $L$ with respect to the height of the users, and user $U_1$ (cell-center user) is closer to the VLC-AP when compared to $U_2$. The location of user with index $i$ is described by polar coordinates $(\rho_i, \omega_i)$, where $\rho_i$ is the distance from the reference point and $\omega_i$ is the angle from the reference direction. Without loss of generality, the reference point is in the horizontal plane that contains both $U_1$ and $U_2$, just below the VLC-AP, and the reference direction points north on the same horizontal plane.

The position of $U_1$ is uniformly distributed on a circular disk of radius $R_0$, whereas the position of $U_2$ is uniformly distributed on an annular area bounded by inner radius $R_0$ and outer radius $R_v$, respectively. The VLC-AP uses NOMA to handle simultaneous data transmission to both users. To improve the data rate of cell-edge user $U_2$, cell-center user $U_1$ further acts as a decode-and-forward (DF) cooperative relay, receiving information on the VLC downlink and forwarding it to the final destination over an RF sidelink.

It should be highlighted that the proposed system model can be extended to the case of more than two users. However, this may increase the computational complexity of perform successive interference cancellation (SIC) at the receiver, which is prone to error propagation in the signal detection stages. To address this, user pairing can be used instead where the network provides wireless to more than two users at the same time by grouping one cell-center and one cell-edge user using an orthogonal multiple access (OMA). Note that since the size of a VLC cell is relatively small (on the order of a few meters), only few users will participate in the pairing process described above. In addition, as the number of users participating in the scheme increases, the gain of NOMA in terms of sum data rate decreases more and more.

![Fig. 1. System model to implement the proposed cooperative VLC NOMA protocol. Cell-center user ($U_1$) performs SIC to recover its own message and the message to be forwarded over the RF sidelink. Cell-edge user ($U_2$) aggregates the data rate from its own VLC reception (no SIC) and the sidelink communication.](image-url)
where \( \Psi_{\text{FoV}} \) denotes the field-of-view (FoV) of the PD placed at the VLC receiver and \( \nu \) is the refractive index of its lens.

Without loss of generality, we assume that the PD of each user is pointing upwards, verifying \( \phi_i = \psi_i \). Then, using polar coordinates to represent distances, the VLC channel gains from the AP to the user \( U_i \) becomes

\[
h_i = \frac{(m + 1) C_i L_i^{m+1}}{(\rho_i^2 + L_i^2)^{m+1}}, \quad C_i = \frac{A_i T(\psi_i) g(\psi_i)}{2\pi}, \tag{7}
\]

According to the power domain NOMA principle, the cell-edge user \( U_2 \) decodes its own message with an achievable data rate \( R_2^V \) by treating the message for \( U_1 \) as noise. At the cell-center user \( U_1 \), where the received VLC signal is stronger, the message of \( U_2 \) is first decoded and removed via SIC. Here, the rate at which user \( U_1 \) can decode the message intended to user \( U_2 \) is denoted by \( R_{21}^V \). Following this, \( U_1 \) decodes its own message without interference with an achievable data rate equal to \( R_{12}^V \). Due to IM/DD is utilized in the downlink direction, the standard Shannon formula cannot be used to evaluate the channel capacity since there are additional constraints to be verified, such as the non-negativity of the transmitted signal and the constraint on the maximum optical transmit power emitted by the LED. Instead, using a lower bound of the corresponding capacity region, the corresponding maximum achievable rates are given by [33]

\[
R_{21}^V = B_v \log_2 \left( 1 + \frac{(\eta h_1 P_1)^2}{(\eta h_1 P_2)^2 + 9\sigma_\nu^2(1 + \epsilon_\nu)^2} \right) - \epsilon_\phi, \tag{8}
\]

\[
R_1^V = B_v \log_2 \left( 1 + \frac{(\eta h_1 P_1)^2}{9\sigma_\nu^2(1 + \epsilon_\nu)^2} \right) - \epsilon_\phi, \tag{9}
\]

\[
R_2^V = B_v \log_2 \left( 1 + \frac{(\eta h_2 P_1)^2}{(\eta h_2 P_2)^2 + g_\nu^2(1 + \epsilon_\nu)^2} \right) - \epsilon_\phi, \tag{10}
\]

where \( B_v \) is the electrical bandwidth of the VLC signal, \( \sigma_\nu^2 \) is the noise variance, and \( \epsilon_\phi = 0.016, \epsilon_\nu = 0.0015 \).

### B. Sidelink RF Transmission

During RF transmission, the baseband equivalent received signal at cell-edge user \( U_2 \) is given by

\[
y_k = \sqrt{L_{\text{RF}} P_{\text{RF}}} h_{\text{RF}} x_R + n_R, \tag{11}
\]

where \( L_{\text{RF}}, P_{\text{RF}}, x_R \) and \( n_R \) denote the path loss attenuation, the available RF power for retransmission at user \( U_1 \), the transmitted signal intended to user \( U_2 \), and the instantaneous noise at the RF receiver of \( U_2 \), which is statistically modeled as AWGN with zero mean and variance \( \sigma_n^2 \), respectively. Without loss of generality, we consider a Rician fading model for the received RF signal at the VLC receiver and \( \Omega \), denoting the total sum received power from LoS and non-LoS components.

Given the considered scenario and using the polar coordinates for the positions of users \( U_1 \) and \( U_2 \), the Euclidean distance between the two users can be calculated as

\[
d_{\text{RF}} = \sqrt{\rho_1^2 + \rho_2^2 - 2\rho_1 \rho_2 \cos(\theta_2 - \theta_1)}, \tag{12}
\]

where \( \theta_1 \) and \( \theta_2 \) are the angular coordinates for the positions of users \( U_1 \) and \( U_2 \), respectively. Regarding the path loss attenuation, we consider the formula \( L_{\text{dB}}^{\text{RF}} = L_{d_0}^{\text{dB}} - 10 \zeta \log_{10} \left( \frac{d_{\text{RF}}}{d_0} \right) \) which gives the value of the attenuation in dB, where \( \zeta \) is the path loss exponent, and \( L_{d_0}^{\text{dB}} = -68 \) dB is the attenuation at the reference distance \( d_0 = 1 \) m [34]. Then, using the Shannon capacity formula, the achievable data rate for the RF sidelink from user \( U_1 \) to user \( U_2 \) is given by

\[
R_2^\text{RF} = B_{\text{RF}} \log_2 \left( 1 + \frac{L_{\text{RF}} P_{\text{RF}} |h_{\text{RF}}|^2}{\sigma_n^2} \right), \tag{13}
\]

where \( B_{\text{RF}} \) denotes the bandwidth of the RF signal.

### III. Proposed Protocol for VLC/RF Aggregation

In this section, we propose an aggregated VLC/RF protocol based on VLC NOMA and RF sidelink communications. As it can be seen from (1) and Fig. 1, the VLC AP transmits the signal \( x \), which contains \( x_1 \) and \( x_2 \). We assume that \( x_1 \) is the sum of \( U_1 \)'s desired information \( x_1^{(1)} \) and a part of \( U_2 \)'s desired information denoted by \( x_2^{(2)} \), whereas \( x_2 \) contains a different part of \( U_2 \)'s desired information denoted by \( x_2^{(2)} \). We note that the way that \( x_1^{(1)} \) and \( x_2^{(2)} \) are separated at the cell-center user \( U_1 \) can be predefined and signaled from VLC AP to \( U_1 \). For instance, \( x_1^{(1)} \) and \( x_2^{(2)} \) can be aggregated frames obtained from \( x_1 \). It should also be highlighted that \( x_2^{(2)} \) is independent from both \( x_1^{(1)} \) and \( x_2^{(2)} \), thus, cell-edge user \( U_2 \) does not utilize selection combining scheme as in \( U_1 \), but needs to receive both \( x_2^{(1)} \) and \( x_2^{(2)} \) in an aggregated VLC-RF signal. To this end, the messages in the proposed protocol are given as

\[
x_1 = x_1^{(1)} + x_2^{(2)}, \tag{14a}
\]

\[
x_2 = x_2^{(2)}. \tag{14b}
\]

With the assumption of point-to-point communication and the independence of messages \( x_1^{(1)} \) and \( x_2^{(2)} \), the inequality

\[
R_1^{(1)} + R_2^{(2)} \leq R_1^V \tag{15}
\]

on the data rate of the VLC link of user \( U_1 \) can be defined, where \( R_1^{(1)} \) and \( R_2^{(2)} \) denote the achievable data rates of \( x_1^{(1)} \) and \( x_2^{(2)} \), respectively. After decoding \( x_1 \), \( U_1 \) separates \( x_1^{(1)} \) and \( x_2^{(2)} \), forwarding the latter part over the RF sidelink to \( U_2 \). Therefore, \( x_2^{(2)} \) is relayed using the decode-and-forward protocol and it should stand that

\[
R_2^{(2)} \leq R_{2\text{RF}}, \tag{16}
\]

where \( R_{2\text{RF}} \) is the channel capacity of the RF sidelink. Finally, cell-edge user \( U_2 \) receives its desired information, represented by \( x_2^{(2)} \) and \( x_2^{(2)} \), via the VLC NOMA link and the RF sidelink, respectively. As stated in [35], the channel capacity of an aggregated VLC/RF system is unknown, but we can obtain a lower bound for it. Considering that (10) is a lower bound for the VLC channel capacity when the input signal values follow the truncated Gaussian distribution, and that (13) is the standard Shannon capacity formula when the values of...
the input signal follows the normalized complex Gaussian distribution, denoted by $CN(0, 1)$, we can write
\[ R_2 = R_2^{RF} + R_2^V. \] (17)

Then, when the aim is to verify the QoS requirement for cell-edge user $U_2$, denoted by $R_2^{thr}$, the following inequality to be verified arises when combining (16) with (17):
\[ R_1^{(2)} + R_2^V \geq R_2^{thr}. \] (18)

Finally, the QoS requirement for cell-center user $U_1$, which is denoted by $R_1^{thr}$, the following inequality must be hold
\[ R_1^{(1)} \geq R_1^{thr}. \] (19)

**Special Case: Non-aggregated VLC/RF Combining**

As a special case to the aforementioned protocol, a simple selection combining can be considered when either the pure VLC-NOMA mode is used without RF sidelink communication or mixed VLC/RF mode. In this special case, either $U_2'$s message is transmitted by the AP via the NOMA VLC method, or it is decoded and relayed by $U_1$ via the RF sidelink. This way, the message $x_1$ is not split and (18) becomes
\[ qR_2^{RF} + (1 - q)R_2^V \geq R_2^{thr} \] (20)

with
\[ q = \begin{cases} 0, & \text{for pure VLC-NOMA mode,} \\ 1, & \text{for mixed VLC/RF mode.} \end{cases} \] (21)

It is noted that in the mixed VLC/RF mode, $U_1$ decodes $U_2'$s message with achievable data rate $R_2^{V,1}$, since power domain NOMA is used, thus it should stand that
\[ R_2^{V,1} \geq R_2^{thr}. \] (22)

**IV. ENERGY EFFICIENCY OPTIMIZATION**

Here, we first define the EE of the target VLC/RF network and then investigate the resource allocation problem that maximizes it. In fact, EE expresses how effectively the available power is utilized to achieve the desired transmission data rates, and is measured in bps/Hz/W. Thus, EE can be defined as the ratio of the spectral efficiency to the total consumed power. We consider two weighting coefficients, $\alpha \in [0, 1]$ and $\beta \in [0, 1]$, which assist in adapting EE to different scenarios. This way, the weighted EE metric can be expressed as
\[ \mathcal{E} = \frac{\alpha R_1^{(1)} + (1 - \alpha)(R_2^{(2)} + R_2^V)}{\beta P_{LED} + (1 - \beta)P_{RF}}, \] (23)

where $P_{LED} = P_1 + P_2$, $\alpha$ is used to prioritize the users data rates, and $\beta$ is used to focus on a specific power source.

Taking into account the QoS requirements for both users, which are given by (18) and (19), and the expressions for the achievable data rates, as well as the hardware and illumination constraints described in the previous section, we can write the EE optimization problem as follows:

\[
\begin{aligned}
\max_{P_1, P_2, P_{LED}, R_1^{(1)}, R_1^{(2)}, R_2^{(1)}, R_2^{(2)}} & \quad \frac{\alpha R_1^{(1)} + (1 - \alpha)(R_2^{(2)} + R_2^V)}{\beta P_{LED} + (1 - \beta)P_{RF}} \\
\text{s.t.} & \quad C_1 : R_1^{(1)} \geq R_1^{thr}, \\
& \quad C_2 : R_1^{(2)} + R_2^V \geq R_2^{thr}, \\
& \quad C_3 : R_1^{(2)} \leq R_2^{RF}, \\
& \quad C_4 : R_1^{(1)} + R_2^{(2)} \leq V_1^C, \\
& \quad C_5 : P_1 + P_2 \leq P_{LED}^{max}, \\
& \quad C_6 : P_{RF} \leq P_{RF}^{max},
\end{aligned}
\] (24)

where $C_1$ and $C_2$ denote the QoS requirement constraints for the cell-center and cell-edge users, respectively, $C_3$ is the Shannon capacity constraint of the RF sidelink, $C_4$ is the data rate constraint for the VLC channel between AP and $U_1$, and $C_5$, $C_6$ are the power consumption constraints. Note that the SIC constraint $R_1^{(1)} + R_2^{(2)} \leq V_1^C$ is always fulfilled due to $C_2$ and the fact that $R_2^{V,2} > R_2^{thr}$ is observed when $h_1 > h_2$.

The optimization in (24) is a non-convex optimization problem. The reason of non-convexity is the existence of logarithmic functions with squared power terms in the expressions for the VLC achievable data rates in (9) and (10). Moreover, the objective function has a fractional form. To efficiently solve (24) in polynomial time, we need to transform it to an equivalent convex one. In line with this, we first introduce an auxiliary variable $r_2$ subject to:
\[ R_2^{thr} \leq r_2 \leq R_2^{V,2} + R_2^{(2)}. \] (25)

This definition affects the objective function by removing from its expression the non-convex term $R_2^{RF}$, thanks to its replacement by the new variable $r_2$. It also affects $C_2$ and introduces a new constraint, $C_7$. To this end, (24) becomes
\[
\begin{aligned}
\max_{P_1, P_2, P_{LED}, R_1^{(1)}, R_1^{(2)}, r_2} & \quad \frac{\alpha R_1^{(1)} + (1 - \alpha)r_2}{\beta P_{LED} + (1 - \beta)P_{RF}} \\
\text{s.t.} & \quad C_1 : R_1^{(1)} \geq R_1^{thr}, \\
& \quad C_2 : r_2 \geq R_2^{thr}, \\
& \quad C_3 : R_1^{(2)} \leq P_{RF}, \\
& \quad C_4 : R_1^{(1)} + R_2^{(2)} \leq V_1^C, \\
& \quad C_5 : P_1 + P_2 \leq P_{LED}^{max}, \\
& \quad C_6 : P_{RF} \leq P_{RF}^{max}, \\
& \quad C_7 : r_2 - R_2^{V,2} \leq R_2^{(2)}.
\end{aligned}
\] (26)

Due to the fractional form of the objective function, we utilize Dinkelbach’s algorithm for fractional programming, which converges superlinearly [36]. This iterative algorithm introduces a parameter $u$, which corresponds to the original fraction, and solves an equivalent parametric program to find the maximum $u$. Specifically, considering the fractional program \[ \max \{ U(z) = F(z) / G(z) \}, \] Dinkelbach’s algorithm solves the following equivalent parametric program:
\[ H(u) = \max \{ F(z) - uG(z) \}. \] (27)

In iteration $\lambda$, $u_{\lambda+1}$ must be renewed, such that $u_{\lambda+1} = U(z_\lambda) = F(z_\lambda) / G(z_\lambda)$ until $u_{\lambda+1} < \epsilon$, where $\epsilon$ denotes the convergence accuracy. $H(u_{\lambda})$ is continuous, convex and strictly decreasing in $\mathbb{R}$. Note that $z^+$ is optimal if and only
Algorithm 1: Dinkelbach’s Algorithm

**Initialization:** Set the initial point \( u_0 < u^+ \), e.g., \( u_0 = U(z_0) > 0 \) for some \( z_0 \). Also, set iteration index \( \lambda = 0 \) and the convergence accuracy \( \epsilon \); while \( H(u_\lambda) > \epsilon \) (for a given \( \epsilon \)) do

- Calculate an optimal solution \( z_1 \) of \( H(u_\lambda) \)
- s.t. \( C_1-C_7 \) in (28);
- Let \( u_{\lambda+1} = U(z_1) \).
- \( \lambda \leftarrow \lambda + 1; \)

**Result:** Optimal \( u^+, z^+ \)

if it is optimal for \( H(u^+_\lambda) \), where \( u^+_\lambda \) is the only zero of \( H \). Dinkelbach also noted that if \( F \) is concave and \( G \) is convex and positive, this algorithm leads to a convex program. In our case, \( F = \alpha R_1^{(1)} + (1-\alpha) r_2 \) and \( G = \beta (P_1 + P_2) + (1-\beta) P_{\text{RF}} \) are both affine, continuous and positive functions. Thus, (26) can be written as

\[
\begin{align*}
\max & \quad F \left( R_1^{(1)}, r_2 \right) - u G(P_1, P_2, P_{\text{RF}}) \\
\text{s.t.} & \quad C_1: R_1^{(1)} \geq R_1^{\text{thr}}, \\
& \quad C_2: r_2 \geq R_2^{\text{thr}}, \\
& \quad C_3: R_1^{(2)} \leq R_{\text{RF}}, \\
& \quad C_4: R_1^{(1)} + R_1^{(2)} \leq R_1^v, \\
& \quad C_5: P_1 + P_2 \leq P_{\text{VLC}}^{\text{max}}, \\
& \quad C_6: P_{\text{RF}} \leq P_{\text{RF}}^{\text{max}}, \\
& \quad C_7: r_2 - R_1^{(2)} \leq R_2^v,
\end{align*}
\]

(28)

where \( z = [P_1, P_2, P_{\text{RF}}, R_1^{(1)}, R_1^{(2)}, r_2] \) and \( u \) is fixed in each iteration. Dinkelbach’s algorithm is presented in Algorithm 1, where the outputs \( z^+ \) and \( u^+ \) denote the optimal resource allocation vector and the maximized EE, respectively.

Next, we should deal with the non-convex constraints \( C_4 \) and \( C_7 \). For this, we apply the geometric programming transformations \( P_1 = e^{p_1} \) and \( P_2 = e^{p_2} \), which affects \( G, C_4, C_5, C_7 \) and \( z \). It is possible to show that this transformation does not affect the convexity of \( G \) and \( C_5 \), but leads to a DC format for \( C_4 \) and \( C_7 \). As a result, unstable convexity issues of the last two aforementioned constraints can be overcome, and the problem can be solved in a tractable way. Thus, with the aid of (9), constraint \( C_4 \) can be rewritten as

\[
2 \left( R_1^{(1)} + R_1^{(2)} + \epsilon_\phi \right) / B_\phi - 1 - v_1(p_1) \leq 0,
\]

(29)

where \( v_1(p_1) = c_1 e^{2p_1} \) and \( c_1 = \frac{2 \sqrt{e_{\phi}^2 + 1}}{2 \sqrt{e_{\phi}^2 - 1}} \). Using (10) and performing similar algebraic manipulations, \( C_7 \) can be expressed as

\[
\log (e^{2p_1} + c_2) + \frac{\log(2)}{B_\phi} \left( r_2 - R_1^{(2)} + \epsilon_\phi \right) - v_2(p_1, p_2),
\]

(30)

where \( v_2(p_1, p_2) = \log (e^{2p_1} + c_2) + c_2 \), \( c_2 = \frac{2 \sqrt{e_{\phi}^2}}{\sqrt{e_{\phi}^2 - 1}} \) and \( c_3 = (1 + \epsilon_\mu) ^2 \). Therefore, \( C_4 \) and \( C_7 \) are now DC functions.

To this end, we can use SCA procedure to approximate the non-convex terms in each iteration by using first-order Taylor series approximation, which has been proven to have linear convergence [37]. In [38], this method is considered as an inner approximation algorithm for programs with convex objective functions and a finite number of both convex and non-convex constraints. It is noted that the approximating functions must fulfill three properties for the algorithm’s success, which are given by

\[
\begin{align*}
& g_{\text{app}}(x) \leq g_{\text{opt}}(x; x^k), \\
& g_{\text{app}}(x^k) = g_{\text{opt}}(x^k; x^k), \\
& \partial g_{\text{app}}(x^k) / \partial x_j = \partial g_{\text{opt}}(x^k; x^k) / \partial x_j,
\end{align*}
\]

(31)-(33)

where \( g_{\text{app}} \) represents any differentiable function and \( g_{\text{opt}} \) is any convex function that approximates \( g_{\text{opt}} \). Also, \( k \) is the iteration index for the SCA procedure. Moreover, first-order Taylor series expansion for a fixed point \( x^k \) can be written as

\[
T_{[1,2]}(x) \approx v_{[1,2]}(x^k) + \nabla v_{[1,2]}(x^k)^T (x - x^k),
\]

(34)

helping us to prove that (31)-(33) are valid. Then, replacing \( v_1 \) and \( v_2 \) with their closed-form expressions, we obtain

\[
v_1(p_1) \approx T_1 \left( p_1; p_1^k \right) = c_1 e^{2p_1k} \left( 1 + 2 e^{2p_1} - 2 e^{2p_1} \right)
\]

(35)

and

\[
v_2(p_1, p_2) \approx T_2 \left( p_1; p_2; p_1^k, p_2^k \right) = v_2 \left( p_1^k, p_2^k \right) + \frac{\partial v_2 \left( p_1^k, p_2^k \right)}{\partial p_1} (p_1 - p_1^k) + \frac{\partial v_2 \left( p_1^k, p_2^k \right)}{\partial p_2} (p_2 - p_2^k),
\]

(36)

where

\[
\begin{align*}
\frac{\partial v_2}{\partial p_1} &= \frac{2 e^{2p_1}}{e^{2p_1} + e^{2p_2} + c_2}, \\
\frac{\partial v_2}{\partial p_2} &= \frac{2 e^{2p_2}}{c_3 (e^{2p_1} + e^{2p_2} + c_2)} + c_2 \epsilon_\phi
\end{align*}
\]

(37)-(38)

are the first order derivatives of \( v_2 \) with respect to \( p_1 \) and \( p_2 \), respectively. To this end, the considered optimization problem can be rewritten as

\[
\begin{align*}
\max & \quad \alpha R_1^{(1)} + (1-\alpha) r_2 - u_\lambda \left( \beta (e^{p_1} + e^{p_2}) + (1-\beta) P_{\text{RF}} \right) \\
\text{s.t.} & \quad C_1: R_1^{(1)} \geq R_1^{\text{thr}}, \\
& \quad C_2: r_2 \geq R_2^{\text{thr}}, \\
& \quad C_3: R_1^{(2)} \leq R_{\text{RF}}, \\
& \quad C_4: 2 \left( R_1^{(1)} + R_1^{(2)} + \epsilon_\phi \right) / B_\phi - 1 - T_1 \left( p_1; p_1^k \right) \leq 0, \\
& \quad C_5: e^{p_1} + e^{p_2} \leq P_{\text{VLC}}^{\text{max}}, \\
& \quad C_6: P_{\text{RF}} \leq P_{\text{RF}}^{\text{max}}, \\
& \quad C_7: \log (e^{2p_1} + c_2) + \frac{\log(2)}{B_\phi} (r_2 - R_1^{(2)} + \epsilon_\phi) \\
& \quad - T_2 \left( p_1, p_2; p_1^k, p_2^k \right) \leq 0,
\end{align*}
\]

(39)

which is a convex problem. As stated in [38], if (39) satisfies Slater’s constraint qualification condition for convex programs, SCA stops at a Karush–Kuhn–Tucker (KKT) point of (28). Considering that (39) is convex, it can be easily proven than Slater’s condition is satisfied. Thus, in each iteration, we obtain the global optimal values of \( x_{\lambda+1} \) until they converge. The SCA algorithm is presented in Algorithm 2.

The selection of a feasible initial point is an important issue for the success of the SCA procedure [39]. Our goal is either to obtain the initial \( z \) that fulfills both (29) and (30), or to show that our problem is infeasible and stop the procedure.
Convex optimization problem.

We consider that the optimization problem is feasible and its solution can be used as initial point for the SCA procedure. To do so, we perform a complete linear search while $C_{\text{SCA}} > \epsilon$ (for a given $\epsilon$) do

- Calculate an optimal solution $z_{\lambda}^{k+1}$ of (39);
- Check if the power consumption constraint $C_5$ is satisfied;
- If the solution satisfies the constraints $C_5$ and $C_6$ of (24), we consider that the optimization problem is feasible and its solution can be used as initial point for the convex optimization methods, such as the interior-point method, and convergence can be reached in polynomial time.

Algorithm 2: SCA Algorithm

**Initialization:** Set the initial point $z_0^{\lambda}$. Also, set iteration index $k = 0$ and the convergence accuracy $\epsilon$.

**while** $C_{\text{SCA}} > \epsilon$ (for a given $\epsilon$) **do**

1. Calculate an optimal solution $z_{\lambda}^{k+1}$ of (39);
2. Check if the power consumption constraint $C_5$ is satisfied;
3. If the solution satisfies the constraints $C_5$ and $C_6$ of (24), we consider that the optimization problem is feasible and its solution can be used as initial point for the convex optimization methods, such as the interior-point method, and convergence can be reached in polynomial time.

**end**

**Result:** Optimal $z_{\lambda}^{k+1}$

For this purpose, we now formulate the power minimization problem of the VLC/RF system under consideration and study its feasibility. If the solution satisfies the constraints $C_5$ and $C_6$ of (24), we consider that the optimization problem is feasible and its solution can be used as initial point for the SCA procedure. The power minimization problem is formulated as

$$
\min_{p_1, p_2, p_{RF}, r_1, r_2} \beta(p_1^2 + p_2^2) + (1 - \beta) P_{\text{RF}}
$$

**s.t.**

1. $C_1: R_1^{(1)} + \beta R_2^{(2)} \leq R_1^{(2)}$
2. $C_2: r_2 - R_2^{(2)} \leq R_2^{(2)}$
3. $R_1^{(2)} \leq R_2^{(2)}$
4. $R_1^{(1)} \geq R_1^{(2)}$
5. $r_2 \geq R_2^{(2)}$

Using the coefficient $\theta \in [0, 1]$, such that for the RF sidelink data rate verifies $R_2^{(2)} = \theta R_2^{(2)}$, with $e^{-r_1} = R_1^{(1)} + R_2^{(2)}$ and $e^{-r_2} \geq R_2^{(2)}$, (40) can be equivalently transformed into

$$
\min_{p_1, p_2, p_{RF}, r_1, r_2, \theta} \beta(p_1^2 + p_2^2) + (1 - \beta) P_{\text{RF}}
$$

**s.t.**

1. $C_1: -2 p_1 - \log(c_1) + \log(2(c_1 + e^{\theta r_1})) / B_v - 1 \leq 0$, $c_1 = 2 p_1 - \log(c_1) + \log(2(c_1 + e^{\theta r_1})) / B_v - 1 \leq 0$, $c_2 = 2 p_2 - \log(c_2) + \log(2(c_2 + e^{\theta r_2})) / B_v - 1 \leq 0$, $c_3 = 2 p_{\text{RF}} - \log(c_3) + \log(2(c_3 + e^{\theta r_{\text{RF}}})) / B_v - 1 \leq 0$,
2. $C_1: \theta R_2^{(2)} \leq R_2^{(2)}$
3. $C_4: e^{-r_1} R_1^{(2)} + e^{-r_2} \theta R_2^{(2)} \leq 1$
4. $C_5: e^{-r_2} ((1 - \theta) R_2^{(2)} / \lambda) \leq 1$
5. $C_6: 0 \leq \theta \leq 1$.

**Proposition 1:** Considering a fixed value of $\theta$, (41) is a convex optimization problem.

**Proof:** The proof is provided in Appendix A.

Therefore, the most appropriate initial point is chosen from the value of $\theta$ and the corresponding solution of (41). To determine the value of $\theta$, we perform a complete linear search in the interval $[0, 1]$ with step $\theta_0$ and solve (41) at each step. Due to its convex form, (41) can be efficiently solved with convex optimization methods, such as the interior-point method, and convergence can be reached in polynomial time. After solving (41) and obtaining the optimal $z_{\theta}^* = [r_1^0, r_2^0, p_1^0, p_2^0, p_{\text{RF}}^0]$ for all $\theta$, we first obtain the combination $\theta^*, z_{\theta^*}^*$, which minimizes the objective function of (41). Next, we check if the power consumption constraints $C_5$ and $C_6$ of (39) are verified for $p_1^0$, $p_2^0$ and $P_{\text{RF}}^0$. If they are not verified, we consider that the optimization problem is infeasible and stop the process. Otherwise, we obtain the initial feasible point $z_{\text{in}} = [R_1^{(1)}, R_2^{(2)}, r_1, r_2, p_1, p_2, P_{\text{RF}}]$ as follows:

$$
R_1^{(1)} = e^{r_1^*} - \theta^* R_2^{(2)}, \quad R_2^{(2)} = \theta^* R_2^{(2)}, \\
r_1 = e^{r_1^*} + \theta^* R_2^{(2)}, \quad p_1 = p_1^0, \quad P_{\text{RF}} = P_{\text{RF}}^0.
$$

This algorithm is presented in Algorithm 3, which provides the optimal values for $z_{\theta^*}^*$. Regarding the computational complexity of the proposed algorithm, let $K_1$ denote the maximum iterations of Dinkelbach’s algorithm and $K_2$ the maximum iterations of SCA procedure to verify the convergence condition. The worst case in terms of computational complexity is given by $O(K_1 K_2 N^3)$, where $N$ is the number of optimization variables. Considering that a convex optimization is solved to determine the initial point of this procedure, the overall computational complexity becomes $O(N^3) + O(K_1 K_2 N^3)$, where $O(N^3)$ is the computational complexity of the interior-point method and $N$ denotes the number of optimization variables of the initial point search procedure. Note that in case a different convex optimization method is utilized with known computational complexity $F(N)$, the overall complexity of the proposed algorithm will become $F(N) + K_1 K_2 N$.

**Special Case:** Non-aggregated VLC/RF Combining

As explained before, the special case of VLC/RF non-aggregated combining reduces naturally to a simpler optimization problem. More specifically, when plugging (25) into (20),

$$
P_{\text{RF}}^{\min} \leq r_2 \leq q R_{\text{RF}} + (1 - q) R_{\text{RF}}^v
$$

results, which falls into the category of mixed integer non-linear programming. Since the integer variable $q$ of the optimization problem can only take two distinct values, the problem can be simplified by first performing a search on $q$, then solving the resulting problem with the rest of the optimization variables, and finally choosing the optimal value of $q$. To further elaborate on this, the optimization problems that result for each possible value of $q$ are now presented.
For $q = 0$, optimization problem (39) is transformed into
\[
\max_z \alpha R_1^{(1)} + (1-\alpha)r_2 - u_\lambda(\beta(e^{p_1} + e^{p_2}) + (1-\beta)P_{RF})
\]
s.t.\[
C_1: R_1^{(1)} \geq R_{thr}^{1}, \\
C_2: r_2 \geq R_{thr}^{2}, \\
C_3: 2(R_1^{(1)} + e_{\phi})/B_v - 1 - T_1(p_1; p_1^{k}) \leq 0, \\
C_4: e^{p_1} + e^{p_2} \leq P_{VLC}^{\text{max}}, \\
C_5: \log(e^{p_1} + c_2) + \log(e^{p_2} + c_2) - T_2(p_1; p_2; p_1^{k}; p_2^{k}) \leq 0.
\]
Similarly, the consequent problem for $q = 1$ becomes
\[
\max_z \alpha R_1^{(1)} + (1-\alpha)r_2 - u_\lambda(\beta(e^{p_1} + e^{p_2}) + (1-\beta)P_{RF})
\]
s.t.\[
C_1: R_1^{(1)} \geq R_{thr}^{1}, \\
C_2: r_2 \geq R_{thr}^{2}, \\
C_3: 2(R_1^{(1)} + e_{\phi})/B_v - 1 - T_1(p_1; p_1^{k}) \leq 0, \\
C_4: e^{p_1} + e^{p_2} \leq P_{VLC}^{\text{max}}, \\
C_5: P_{RF} \leq P_{RF}^{\text{max}}, \\
C_6: \log(e^{p_1} + c_2) + \log(e^{p_2} + c_2) - T_2(p_1; p_2; p_1^{k}; p_2^{k}) \leq 0, \\
C_7: \log(e^{p_1} + \sigma_0^2/(\beta_0^2)) - 2p_2 \\
\quad + \log\left((1 + \epsilon_\mu)^2 - \frac{R_{thr}^{1} + e_{\phi}}{B_v} - 1\right) \leq 0.
\]
Problems (44) and (45) are both convex and can be solved with conventional convex optimization methods. Note that, in both special cases, Algorithm 3 is utilized, but the constraints of the resulting SCA procedure differ from the constraints of the general case.

V. SIMULATION RESULTS AND DISCUSSION

In this section, Monte Carlo simulation results are presented for the proposed algorithm for 104 snapshots in which the positions of the users change. The convergence accuracy $\epsilon$ for the two iterative algorithms are set equal to $10^{-6}$, while the selected step size is $\delta_{\mu} = 10^{-2}$. For the sake of simplicity, unless otherwise stated, we consider that users have common QoS requirements, which we denote as $P_{thr}^{1} = P_{thr}^{2} = P_{thr}^{2}$. The utilized parameters for the simulation results are presented in Table I. The bandwidth of the VLC subsystem is set $B_v = 4B_R$.

The performance of the proposed VLC/RF aggregated approach is compared with a conventional network setup, i.e., the non-aggregated VLC/RF approach considered in [1]. Since the superiority of NOMA has been established over standard orthogonal schemes, especially for two-user configurations, these cases can be considered as a benchmark scheme. As such, Fig. 2 presents the comparison between the proposed protocol and the benchmark, namely pure VLC, where the RF sidelink is not used, and the non-aggregated hybrid VLC/RF for the same fixed values of $\alpha$ and $\beta$. Based on this figure, the proposed protocol outperforms the two VLC/RF non-aggregated modes presented in [1]. For low QoS requirements, the proposed VLC/RF system exhibits better performance as $x_2(1)$ is decoded by cell-center user $U_1$ without interference. For high target data rates, the performance of the proposed protocol converges to the one of the VLC-NOMA scheme presented in [1].

In Fig. 3, the EE is plotted versus the $R_{thr}^{2}$ for different fixed values of the throughput tuning weight $\alpha$. It is noted that the different values of $\alpha$ highlight whether priority is given to one user over the other in terms of the achievable data rate. Setting $\beta = 0.5$ means that both power sources are equally accountable for the energy consumption of the aggregated VLC/RF network. It can be observed that, while $\alpha$ increases, higher EE can be achieved for low QoS requirements, however the value of $\alpha$ does not have any impact when higher data rates must be achieved. When priority can be given, it is easier for cell-center user $U_1$ to achieve a higher data rate with a lower energy consumption when compared to cell-edge user $U_2$. In the scenario that gives priority to $U_2$ (i.e., when $\alpha < 0.5$), a lower EE is inevitable due to the more demanding VLC link budget because of a longer distance from the VLC-AP when compared to user $U_1$, or due to the use of the RF sidelink to forward part of the information intended to user $U_2$.

In Fig. 4, we set $\alpha = 0.5$, so that both users have the same priority and investigate the impact of $\beta$. To provide further insights, low values of $\beta$ (i.e., $\beta < 0.5$) represent scenarios,
where the LED is also used for illumination and, thus, it is not necessary to focus on its power consumption. It can be observed that this scenario is the most energy efficient, regardless of the QoS requirements. On the other hand, when the indoor illumination is not needed, $\beta$ can be set higher than 0.5, to highlight the cost of VLC-AP. As expected, this turns out to be the least energy efficient scenario.

In Fig. 5, we set $\alpha = \beta = 0.5$ and we investigate the pure EE, without emphasizing either on achievable rate or on energy consumption. In this figure, the impact of higher QoS requirement of the cell-center user $U_1$ is illustrated. When $U_1$ requires higher data rate, the overall network becomes less energy efficient even for low target rate. Additionally, since the power consumption increases and tends to exceed permissible limits, i.e., $P_{\text{VLC}}^\text{max}$ and $P_{\text{RF}}^\text{max}$, our network cannot operate efficiently in the region above 10 bps/Hz.

To derive further insight to the proposed network’s operation, we denote as RF link’s usage $\Theta = \frac{R_{\text{thr}}^1}{R_{\text{thr}}^1 + R_{\text{thr}}^2}$ the ratio between the rate of decoded and retransmitted data by $U_1$ and the total achievable rate of $U_2$. In Fig. 6, $\Theta$ is plotted versus $R_{\text{thr}}^1$ with $\alpha = \beta = 0.5$. It can be observed that for low target rates the RF link is exclusively advantageous, while as the QoS requirements increase, VLC plays a dominant role in providing service to $U_2$ that has a weaker received signal.

Following that, we define $\gamma = \frac{L_{\text{thr}}}{\log_2(1+\gamma)}$ to investigate its impact on both EE and $\Theta$. In Fig. 7 the unweighted EE is studied by setting $\alpha = \beta = 0.5$, as illustrated in Fig. 6, and $R_{\text{thr}}^2 = 10$ bps/Hz is set. Fig. 7 highlights that, while $\gamma$ goes higher, EE increases by $2 \times 10^4$ bps/Hz/W, which
means that a high RF channel gain contributes significantly in the improvement of overall system’s performance. Finally, the red curve in 7 illustrates the RF link’s usage ratio $\Theta$ which increases as the RF channel gain improves, in a similar manner as the EE.

VI. CONCLUSIONS

In this work, we investigated a cross-band aggregated VLC/RF network with an RF relay link from a cell-center user to a cell-edge one, aiming to improve the latter’s performance with the assumption that the cell-edge user receives its message from both VLC and RF links in an aggregated manner. To maximize the EE of the network, a resource allocation optimization problem was proposed, which was subsequently solved by combining Dinkelbach’s and SCA algorithms. The presented numerical results validated proposed analysis and provided further insight into the impact of the involved parameters in the aggregated system’s performance. Future research will encompass the multi-cell extension of the network, investigating interference management solutions, as well as user scheduling methods.

APPENDIX A

PROOF OF PROPOSITION 1

In (40), we let $r_1 = R_1^{(1)} + R_2^{(2)}$ to denote the sum-rate decoded by $U_1$ which cannot exceed $R_1^V$ and $\bar{r}_2 = R_2 - R_1^{(2)}$ the rate that $U_2$ decodes $x_2^{(2)}$, which cannot exceed $R_2^V$. It should be highlighted that $r_1$ must be greater than $R_1^{thr} + R_2^{thr}$. Similarly, $\bar{r}_2$ should satisfy the QoS requirement for cell-edge’s user VLC channel, so $\bar{r}_2 \geq R_2^{thr} - R_2^{(2)} = (1 - \theta)R_2^{thr}$. Therefore, (40) can be written as

$$
\min_{P_1, P_2, P_{RF}, \bar{r}_1, \bar{r}_2, \theta} \beta \left( P_1 + P_2 \right) + (1 - \beta)P_{RF}
$$

s.t.

$$
C_1 : r_1 \leq R_1^V,
C_2 : \bar{r}_2 \leq \bar{R}_2^{thr},
C_3 : \theta P_{RF} \leq P_{RF},
C_4 : \bar{r}_1 \geq R_1^{thr} + \theta R_2^{thr},
C_5 : \bar{r}_2 \geq (1 - \theta)R_2^{thr},
C_6 : 0 \leq \theta \leq 1.
$$

Considering a fixed $\theta$, employing the geometric programming transformations $e^{\bar{r}_1} = r_1$ and $e^{\bar{r}_2} = \bar{r}_2$ and after some algebraic manipulations, (46) is transformed into a convex optimization problem as follows

$$
\min_{P_1, P_2, P_{RF}, \bar{r}_1, \bar{r}_2} \beta \left( e^{P_1} + e^{P_2} \right) + (1 - \beta)P_{RF}
$$

s.t.

$$
C_1 : -2P_1 - \log(C_1) + \log \left( \frac{e^{(e^{(P_1) + \epsilon)}}}{B_v} - 1 \right) \leq 0,
C_2 : \log \left( \frac{e^{2P_1} + C_2}{e^{P_1}} - 2P_2 + \log(C_3) \right) + \log \left( \frac{e^{(e^{(P_1) + \epsilon)}}}{B_v} - 1 \right) \leq 0,
C_3 : \theta P_{RF} \leq P_{RF},
C_4 : \bar{r}_1 \geq R_1^{thr} + \theta R_2^{thr},
C_5 : \bar{r}_2 \geq (1 - \theta)R_2^{thr}.
$$

To prove the convexity, we first rewrite $C_4$ and $C_5$ in (47), respectively, as

$$
C_4 : R_1^{thr} e^{-\bar{r}_1} + \theta R_2^{thr} e^{-\bar{r}_2} \leq 1
$$

(48)

It can be observed that $C_4$ and $C_5$ are convex, since $\theta$ and $(1 - \theta)$ are positive. Moreover, $C_1$ and $C_2$ consist of linear and convex terms, i.e., $\log \left( \frac{e^{2P_1} + C_2}{e^{P_1}} - 2P_2 + \log(C_3) \right)$ is convex as a log-sum-exp term and $\log \left( \frac{e^{(e^{(P_1) + \epsilon)}}}{B_v} - 1 \right)$ and $\log \left( \frac{e^{(e^{(P_1) + \epsilon)}}}{B_v} - 1 \right)$ are convex, because their second derivative with respect to $\bar{r}_1$ and $\bar{r}_2$, respectively, is positive, as presented below. The derivative is calculated as

$$
\frac{\log(2) e^{1.2 + x_v}}{B_v^2} - \frac{e^{1.2 + x_v}}{B_v} \left( 2 \left( \frac{e^{1.2 + x_v}}{B_v} - e^{1.2} \log(2) - 1 \right) \right).
$$

(50)

Considering that $\xi = 2 \frac{e^{1.2 + x_v}}{B_v} - e^{1.2} \log(2) - 1$ is an increasing function with respect to $\bar{r}_1$, and when $\bar{r}_1 \to -\infty$, $\xi \to 2 \frac{e^{1.2}}{B_v} - 1 > 0$, because $\frac{e^{1.2}}{B_v} > 0$. Also, $C_3$ is affine, $C_3$ is convex due to the concavity of $\bar{R}_2^{RF}$. Finally, the objective function is also convex as sum of exponentials, so the proof is completed.

REFERENCES


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