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Gaussian Class Multivariate Weibull Distributions: Theory and Applications in Fading Channels

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***Abstract*—Ascertaining on the suitability of the Weibull distribution to model fading channels, a theoretical framework for a class of multivariate Weibull distributions, originated from Gaussian random processes, is introduced and analyzed. Novel analytical expressions for the joint probability density function (pdf), moment-generating function (mgf), and cumulative distribution function (cdf) are derived for the bivariate distribution of this class with not necessarily identical fading parameters and average powers. Two specific distributions with arbitrary number of correlated variates are considered and studied: with exponential and with constant correlation where their pdfs are introduced. Both cases assume equal average fading powers, but not necessarily identical fading parameters. For the multivariate Weibull distribution with exponential correlation, useful corresponding formulas, as for the bivariate case, are derived. The presented theoretical results are applied to analyze the performance of several diversity receivers employed with selection, equal-gain, and maximal-ratio combining (MRC) techniques operating over correlated Weibull fading channels. For these diversity receivers, several useful performance criteria such as the moments of the output signal-to-noise ratio (SNR) (including average output SNR and amount of fading) and outage probability are analytically derived. Moreover, the average symbol error probability for several coherent and noncoherent modulation schemes is studied using the mgf approach. The proposed mathematical analysis is complemented by various evaluation results, showing the effects of the fading severity as well as the fading correlation on the diversity receivers performance.**

***Index Terms*—Bit-error rate (BER), correlated fading, diversity, equal-gain combining (EGC), maximal-ratio combining (MRC), multichannel reception, multivariate analysis, outage probability, selection combining (SC), Weibull fading.**

I. Introduction

Multivariate statistics is a useful mathematical tool for modeling and analyzing realistic wireless channels with correlated fading. Such fading channels are usually met in digital contemporary communications systems employed with diversity receivers with not sufficiently separated antennas where space or polarization diversity is applied (e.g., hand-held mobile terminals and indoor base stations). In these applications, the correlation among the channels results in a degradation of the diversity gain obtained [1]–[3].

Reviewing the open technical literature, one can encounter several papers applying multivariate statistics for fading channel modeling, most of them concerning the Rayleigh and Nakagami-m distributions. In an early work, Nakagami has defined the m-bivariate probability density function (pdf) [4, p. 31], while many years later, an infinite series representation for the bivariate Rayleigh and Nakagami-m cumulative distribution functions (cdf)s have been presented by Tan and

Manuscript received October 5, 2004; revised June 12, 2005. The material in this correspondence was presented in part at the IEEE 62nd Semiannual Vehicular Technology Conference, Dallas, TX, September 2005.

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Communicated by R. R. Müller, Associate Editor for Communications.
Digital Object Identifier 10.1109/TIT.2005.855598

Beaulieu [5]. In a later work [6], Simon and Alouini have proposed an alternative cdf expression for the bivariate Rayleigh distribution, in the form of a single integral with finite limits and an integrand composed of elementary functions. Recently, Karagiannidis *et al.* [7] have introduced the multivariate Nakagami-m pdf with exponential correlation and identically distributed (i.d.) fading statistics. An infinite series approach for its corresponding cdf and a bound of the error resulting from truncation of the infinite series have been also included. By approximating the correlation matrix with a Green's matrix, the same authors have generalized [7] to the arbitrarily correlated Nakagami-m distribution [8]. Additionally, Mallik [9] has presented useful analytical pdf and cdf expressions for the multivariate Rayleigh distribution with exponential and constant correlation matrix which agree with those in [7] for the special case where the Nakagami-m reduces to the Rayleigh distribution.

The Weibull distribution was first introduced by Waloddi Weibull back in 1937 for estimating machinery lifetime and became widely known in 1951 [10]. Nowadays, the Weibull distribution is used in several fields of science. For example, it is a very popular statistical model in reliability engineering and failure data analysis [11], [12]. It is also used in some other applications, such as weather forecasting and data fitting of all kinds, while it is widely applied in radar systems to model the dispersion of the received signals level produced by some types of clutters [13]. Concerning wireless communications, the Weibull distribution seems to exhibit good fit to experimental fading channel measurements, for both indoor [14]–[17], and outdoor [18]–[21] environments, with a reasonable physical justification to be given in [22]. However, only very recently the topic of digital communications over Weibull fading channels has begun to receive some interest. For example, by considering the performance of diversity receivers over Weibull fading channels, an analysis for the evaluation of the generalized-selection combining (GSC) performance over independent Weibull fading channels has been presented [23]. In that analysis, the first two moments of the signal-to-noise ratio (SNR) and the amount of fading (AoF) at the output of the GSC receiver have been derived. More recently, some other contributions dealing with switched and selection diversity have been presented by Sagias *et al.* in [24], [25] and [26], [27], respectively. In [24], [25], closed-form expressions for the average SNR, AoF, switching rate, and average symbol error probability (ASEP) at the output of the combiner have been obtained. In [26], an analytical study for dual-branch selection combining (SC) receivers operating over correlated fading channels has been performed, while in [27], important performance measures, such as the outage probability and average output SNR have been derived in closed form for L-branch SC receivers operating over independent Weibull fading channels. In another useful work by Cheng *et al.* [28], an analytical performance study for SC and maximal-ratio combing (MRC) receivers operating over independent and i.d. fading channels has been presented. In that paper, closed-form expressions for the moments of the combiner output SNR and the outage probability have been obtained, while the ASEP has been extracted in terms of the Meijer's G-function. Very recently, Sahu and Chaturvedi have studied the average bit-error probability (ABEP) of equal-gain combining (EGC) receivers for binary, coherent, and noncoherent modulation schemes [29]. However, it is well known that the assumption of interdependence among the input diversity channels, as in [23]–[25], [27]–[29], is not accurate for compact, hand-held, mobile terminals and indoor base stations with not sufficiently separated antennas. In order to analyze the performance of diversity receivers operating over more realistic correlated fading channels, multivariate Weibull statistical analysis must be utilized. Several classes of multivariate Weibull distributions have been proposed [12], [26], [30]–[36], but to the best of the authors' knowledge, no class of multivariate Weibull distributions generated from correlated Gaussian processes has ever been published.

In this correspondence, a class of Gaussian multivariate Weibull distributions is introduced and dealt with. More specifically, the bivariate Weibull pdf with not necessarily identical fading parameters as well as average powers is presented, while based on this pdf, the corresponding moments-generating function (mgf), cdf, and the Weibull correlation coefficient are obtained. Multivariate Weibull distributions with exponential and constant correlation matrixes are also introduced and for the former, useful analytical expressions for the joint pdf, cdf, mgf, and product moments are presented. These novel theoretical results are applied to the performance analysis of dual- and multibranch SC, EGC, and MRC receivers operating over correlated Weibull fading channels. For this kind of receivers, various important performance criteria such as the moments of the output SNR (including average output SNR and AoF) and the outage probability are analytically derived. Moreover, based on the well-known mgf approach, the ASEP for several coherent and noncoherent modulation schemes is obtained. The proposed mathematical analysis is complemented by various numerically evaluated results, including the effects of fading severity as well as fading correlation on the system performance.

The remainder of this correspondence is organized as following: In Section II, several formulas with different correlation models are presented. In Sections III and IV, the performance of dual- and multibranch diversity receivers is studied, respectively. Some numerical results are presented in Section V, while in Section VI, useful concluding remarks are provided.

II. A Class of Gaussian Multivariate Weibull Distributions

The fading model for the Weibull distribution considers a signal composed of clusters of one multipath wave, each propagating in a nonhomogeneous environment. Within any one cluster, the phases of the scattered waves are random and have similar delay times with delay-time spreads of different clusters being relatively large. The clusters of the multipath wave are assumed to have the scattered waves with identical powers. The resulting envelope is obtained as a nonlinear function of the modulus of the multipath component[1] h_ℓ. The nonlinearity is manifested in terms of a power parameter $\beta_\ell > 0$, such that the resulting signal intensity is obtained not simply as the modulus of the multipath component, but as this modulus to a certain given power $2/\beta_\ell > 0$ [22]. Hence, for the Weibull fading model, the complex envelope h_ℓ can be written as a function of the Gaussian in-phase X_ℓ and quadrature Y_ℓ elements of the multipath components

$$h_\ell = (X_\ell + \jmath Y_\ell)^{2/\beta_\ell} \tag{1}$$

where $\jmath = \sqrt{-1}$ is the imaginary operator.

A. The Univariate Weibull Distribution

Let Z_ℓ be the magnitude of h_ℓ, i.e., $Z_\ell = |h_\ell|$. By taking into account the above physical justification for the Weibull fading model, Z_ℓ can be expressed as a power transformation of a Rayleigh distributed random variable (RV) $R_\ell = |X_\ell + \jmath Y_\ell|$ as

$$Z_\ell = R_\ell^{2/\beta_\ell}. \tag{2}$$

From the above equation, the pdf of Z_ℓ can be easily obtained as

$$f_{Z_\ell}(r) = \frac{\beta_\ell}{\Omega_\ell} r^{\beta_\ell - 1} \exp\left(-\frac{r^{\beta_\ell}}{\Omega_\ell}\right) \tag{3}$$

with $\Omega_\ell = \mathcal{E}\langle Z_\ell^{\beta_\ell}\rangle$ and $\mathcal{E}\langle \cdot \rangle$ denoting expectation. It is easily recognized, that the above pdf follows the Weibull distribution [37, Ch. 17] with the fading parameter β_ℓ expressing the fading severity $(\beta_\ell > 0)$

[1]In this paragraph and in Section II-A, ℓ is a dummy factor.

and Ω_ℓ being the average fading power. As β_ℓ increases, the fading severity decreases, while for the special case of $\beta_\ell = 2$, (3) reduces to the well-known Rayleigh pdf [1, eq. (2.6)]. Moreover, for the special case of $\beta_\ell = 1$, (3) reduces to the well-known negative exponential pdf. By defining a function $d_{\tau,\ell} = 1 + \tau/\beta_\ell$, where, in general, τ is a nonnegative value, the corresponding cdf and the nth-order moment of Z_ℓ can be expressed as

$$F_{Z_\ell}(r) = 1 - \exp\left(-\frac{r^{\beta_\ell}}{\Omega_\ell}\right) \tag{4}$$

and

$$\mathcal{E}\langle Z_\ell^n \rangle = \Omega_\ell^{n/\beta_\ell}\Gamma(d_{n,\ell}) \tag{5}$$

respectively, where $\Gamma(\cdot)$ is the Gamma function [38, eq. (8.310/1)] and n is a positive integer.

The mgf of Z_ℓ can be derived as

$$\mathcal{M}_{Z_\ell}(s) = \mathcal{E}\langle \exp(-sZ_\ell)\rangle \tag{6}$$

where by using the pdf expression given by (3), some integrals of the form

$$\Upsilon(\xi, u) = \int_0^\infty x^{u-1}\exp(-x - \xi x^{\beta_\ell})\,dx \tag{7}$$

are needed to be solved, with u and ξ being arbitrary positive values. The same kind of integrals has been already analytically solved in [26], under the constraint that β_ℓ is a rational number, as

$$\Upsilon(\xi, u) = \frac{\lambda^u\sqrt{\kappa/\lambda}}{(\sqrt{2\pi})^{\kappa+\lambda-2}} \times G^{\kappa,\lambda}_{\lambda,\kappa}\left[\xi^\kappa\frac{\lambda^\lambda}{\kappa^\kappa}\,\middle|\, \begin{matrix}(1-u)/\lambda, (2-u)/\lambda, \ldots, (\lambda-u)/\lambda \\ 0/\kappa, 1/\kappa, \ldots, (\kappa-1)/\kappa\end{matrix}\right] \tag{8}$$

where $G[\cdot]$ is the Meijer's G-function [38, eq. (9.301)]. Note that the Meijer's G-function is included as a built-in function in most popular mathematical software packages. Additionally, by using a method which is presented in the Appendix I, $G[\cdot]$ can be expressed in terms of more familiar generalized hypergeometric functions ${}_pF_q(\cdot;\cdot;\cdot)$ [38, Sec. 9.1] with p and q being positive integers. In (8), having assumed that β_ℓ belongs to rationals, κ and λ are positive integers so that

$$\frac{\lambda}{\kappa} = \beta_\ell \tag{9}$$

holds. Depending upon the specific value of β_ℓ, a set of minimum values of κ and λ can be properly chosen (e.g., for $\beta_\ell = 3.5$, we have to choose $\kappa = 2$ and $\lambda = 7$). Hence, by using (6) and (8), the mgf of the Weibull distribution can be obtained in closed form as

$$\mathcal{M}_{Z_\ell}(s) = \frac{1}{\Omega_\ell s^{\beta_\ell}}\frac{\lambda^{\beta_\ell}\sqrt{\kappa/\lambda}}{(\sqrt{2\pi})^{\kappa+\lambda-2}} \times G^{\kappa,\lambda}_{\lambda,\kappa}\left[\frac{\lambda^\lambda}{(\kappa\Omega_\ell s^{\beta_\ell})^\kappa}\,\middle|\, \begin{matrix}(1-\beta_\ell)/\lambda, (2-\beta_\ell)/\lambda, \ldots, (\lambda-\beta_\ell)/\lambda \\ 0/\kappa, 1/\kappa, \ldots, (\kappa-1)/\kappa\end{matrix}\right]. \tag{10}$$

For the special case where β_ℓ is an integer, $\kappa = 1$ and $\lambda = \beta_\ell$, while using [38, eq. (9.31/2)], (10) simplifies to an already known result [28, eq. (5)].

B. The Bivariate Weibull Distribution

Starting from the bivariate Rayleigh distribution given in Appendix II for the reader's convenience, we introduce the bivariate Weibull fading model with not necessarily i.d. both fading parameters and average powers.

1) Joint pdf: By applying the transformation of the RVs given by (2) in (II-1) and using [39, p. 143], the joint pdf of the Weibull distributed RVs Z_1 and Z_2 can be obtained as

$$f_{Z_1,Z_2}(r_1,r_2) = \frac{\beta_1\beta_2 r_1^{\beta_1-1}r_2^{\beta_2-1}}{\Omega_1\Omega_2(1-\rho)} \times \exp\left[-\frac{1}{1-\rho}\left(\frac{r_1^{\beta_1}}{\Omega_1}+\frac{r_2^{\beta_2}}{\Omega_2}\right)\right] I_0\left[\frac{2\sqrt{\rho}\,r_1^{\beta_1/2}r_2^{\beta_2/2}}{(1-\rho)\sqrt{\Omega_1\Omega_2}}\right] \tag{11}$$

where $\Omega_\ell = \mathcal{E}\langle Z_\ell^{\beta_\ell}\rangle$ and marginal pdfs given by (3) for $\ell = 1$ and 2.

2) Product Moments and Power Correlation Coefficient: By using (2), the product moments of the $(n+m)$th order of Z_1 and Z_2 can be derived as

$$\mathcal{E}\langle Z_1^n Z_2^m\rangle = \mathcal{E}\left\langle R_1^{2n/\beta_1}R_2^{2m/\beta_2}\right\rangle \tag{12}$$

which using (II-3), yields

$$\mathcal{E}\langle Z_1^n Z_2^m\rangle = (1-\rho)^{1+n/\beta_1+m/\beta_2}\Omega_1^{n/\beta_1}\Omega_2^{m/\beta_2} \times \Gamma\left(1+\frac{n}{\beta_1}\right)\Gamma\left(1+\frac{m}{\beta_2}\right){}_2F_1\left(1+\frac{n}{\beta_1}, 1+\frac{m}{\beta_2}; 1; \rho\right). \tag{13}$$

By definition, the (Weibull) power correlation coefficient of Z_1^2 and $Z_2^2 (0 \le \varrho < 1)$ can be expressed as

$$\varrho \triangleq \frac{\operatorname{cov}\left(Z_1^2, Z_2^2\right)}{\sqrt{\operatorname{var}\left(Z_1^2\right)}\sqrt{\operatorname{var}\left(Z_2^2\right)}} = \frac{\mathcal{E}\left\langle Z_1^2Z_2^2\right\rangle - \mathcal{E}\left\langle Z_1^2\right\rangle\mathcal{E}\left\langle Z_2^2\right\rangle}{\sqrt{\mathcal{E}\left\langle Z_1^4\right\rangle - \mathcal{E}^2\left\langle Z_1^2\right\rangle}\sqrt{\mathcal{E}\left\langle Z_2^4\right\rangle - \mathcal{E}^2\left\langle Z_2^2\right\rangle}} \tag{14}$$

where by using (5) and (13) and after some straightforward simplifications, ϱ can be obtained in closed form as

$$\varrho = \frac{(1-\rho)^{1+2/\beta_1+2/\beta_2}{}_2F_1(d_{2,1}, d_{2,2}; 1; \rho) - 1}{\sqrt{\Gamma(d_{4,1})/\Gamma^2(d_{2,1}) - 1}\sqrt{\Gamma(d_{4,2})/\Gamma^2(d_{2,2}) - 1}}. \tag{15}$$

For $\beta_1 = \beta_2 = \beta$, (15) reduces to

$$\varrho = \frac{(1-\rho)^{1+4/\beta}{}_2F_1(1+2/\beta, 1+2/\beta; 1; \rho) - 1}{\Gamma(1+4/\beta)/\Gamma^2(1+2/\beta) - 1}. \tag{16}$$

By numerically evaluating (16), in Fig. 1, ϱ is plotted as a function of ρ for several values of β. It is clear, that ϱ also ranges between zero and unity as ρ does, while for a fixed value of ρ, ϱ decreases as β increases. Moreover, for the special cases of $\rho = 0$ and $\rho \to 1$, $\varrho = 0$ and $\varrho \to 1$, respectively, independently of the value of β.

3) Joint cdf: By using (2), the joint cdf of Z_1 and Z_2 can be easily obtained in closed form, replacing r_1 and r_2 with $r_1^{\beta_1/2}$ and $r_2^{\beta_2/2}$, in (II-2), respectively, i.e.,

$$F_{Z_1,Z_2}(r_1,r_2) = F_{R_1,R_2}\left(r_1^{\beta_1/2}, r_2^{\beta_2/2}\right). \tag{17}$$

4) Joint mgf: The form of the pdf in (11) is not mathematically tractable. Hence, by using an infinite series representation of the Bessel function [38, eq. (8.447/1)]

$$I_0(u) = \sum_{k=0}^{\infty}\frac{1}{(k!)^2}\left(\frac{u}{2}\right)^{2k} \tag{18}$$

the joint pdf of Z_1 and Z_2 in (11) can be written as

$$f_{Z_1,Z_2}(r_1,r_2) = \beta_1\beta_2\exp\left[-\frac{1}{1-\rho}\left(\frac{r_1^{\beta_1}}{\Omega_1}+\frac{r_2^{\beta_2}}{\Omega_2}\right)\right] \times \sum_{k=0}^{\infty}\frac{1}{(k!)^2}\frac{\rho^k}{(1-\rho)^{2k+1}}\frac{r_1^{-1+(k+1)\beta_1}r_2^{-1+(k+1)\beta_2}}{(\Omega_1\Omega_2)^{k+1}}. \tag{19}$$

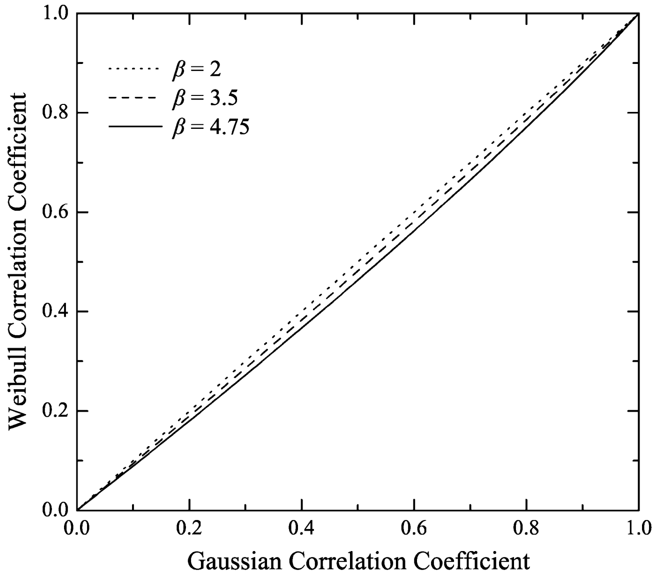

Fig. 1. Weibull correlation coefficient ϱ as a function of the Gaussian correlation coefficient ρ.

Based on the above pdf expression, the joint mgf of Z_1 and Z_2 can be derived as

$$\mathcal{M}_{Z_1,Z_2}(s_1,s_2)=\mathcal{E}\langle \exp(-s_1Z_1-s_2Z_2)\rangle \tag{20}$$

where some integrals of the form as in (7) appear. Thus, by using (8), the joint mgf of the bivariate Weibull distribution can be obtained as

$$\mathcal{M}_{Z_1,Z_2}(s_1,s_2)=\beta_1\beta_2\sum_{k=0}^{\infty}\frac{\rho^k}{(k!)^2(1-\rho)^{2k+1}} \times\prod_{i=1}^{2}\frac{1}{\left(s_i^{\beta_i}\Omega_i\right)^{k+1}}\Upsilon\left[\frac{1}{(1-\rho)s_i^{\beta_i}\Omega_i},(k+1)\beta_i\right]. \tag{21}$$

C. The Multivariate Weibull Distribution With Exponential Correlation

Several fading correlation models have been proposed and used for the performance analysis of various wireless systems, corresponding to specific modulation, detection, and diversity combining schemes. One of the most frequently used models is the exponential correlation one, which has been first addressed by Aalo in [2, Sec. II.B]. This model corresponds to the scenario of multichannel reception from equispaced diversity antennas, in which the correlation among the pairs of combined signals decays as the spacing between the antennas increases [1, p. 394]. The exponential model has been recently used by several researchers, who applied it to the performance analysis of space diversity techniques [3], [40], [41] or multiple-input multiple-output (MIMO) systems [42]. In those works, this model has been considered for a more accurate statistical description of fading providing more reasonable conclusions than independent ones.

The multivariate pdf of the i.d. Rayleigh distributed RVs with exponential correlation, $\{R_\ell\}_{\ell=1}^L$, is given by [9, eqs. (57) and (16)], [7] and let ρ be the Gaussian correlation coefficient between two successive squared RVs (e.g., between R_i^2 and R_{i+1}^2). Then, in general, the correlation coefficient between R_i^2 and R_j^2 is given by $\rho_{i,j}=\rho_{j,i}=\rho^{|i-j|}$, when $i\neq j$, while $\rho_{i,j}=1$, when $i=j$, with $i,j=1,2,\ldots,L$.

1) Joint pdf: By applying the transformation given by (2) in the multivariate Rayleigh pdf with exponential correlation and by using a standard method for the transformation of RVs described in [39, p. 183], the joint pdf of the Weibull RVs $\{Z_\ell\}_{\ell=1}^L$ can be obtained in closed form as

$$f_{\overrightarrow{Z}}(\overrightarrow{r})=\frac{1}{\Omega^L}\frac{\prod_{i=1}^L\beta_i r_i^{\beta_i-1}}{(1-\rho)^{L-1}} \times\exp\left\{-\frac{1}{(1-\rho)\Omega}\left[r_1^{\beta_1}+r_L^{\beta_L}+(1+\rho)\sum_{i=2}^{L-1}r_i^{\beta_i}\right]\right\} \times\prod_{i=1}^{L-1}I_0\left[\frac{2\sqrt{\rho}}{(1-\rho)\Omega}r_i^{\beta_i/2}r_{i+1}^{\beta_{i+1}/2}\right] \tag{22}$$

where $\Omega=\mathcal{E}\langle Z_\ell^{\beta_\ell}\rangle\ \forall\ell$, $\overrightarrow{Z}=[Z_1\ Z_2\ \cdots\ Z_L]$ and marginal pdfs given by (3) for $\ell=1,2,\ldots,L$. The (Weibull) power correlation coefficient between Z_i^2 and Z_j^2 is given by $\varrho_{i,j}=\varrho_{j,i}=\varrho^{|i-j|}$, when $i\neq j$, while $\varrho_{i,j}=1$, when $i=j$, with $i,j=1,2,\ldots,L$ and ϱ given by (15). By substituting the Bessel function in (22) with its infinite series representation given by (18), (22) can be rewritten in a mathematically tractable form as

$$f_{\overrightarrow{Z}}(\overrightarrow{r})=\frac{\prod_{i=1}^L\beta_i}{\Omega^L(1-\rho)^{L-1}} \times\exp\left\{-\frac{1}{\Omega(1-\rho)}\left[r_1^{\beta_1}+r_L^{\beta_L}+(1+\rho)\sum_{i=2}^{L-1}r_i^{\beta_i}\right]\right\} \times\sum_{k_1,k_2,\ldots,k_{L-1}=0}^{\infty}\left\{\prod_{i=1}^{L-1}\left[\frac{\sqrt{\rho}}{\Omega(1-\rho)}\right]^{2k_i}\right\} \times\frac{r_1^{(k_1+1)\beta_1-1}r_L^{(k_{L-1}+1)\beta_L-1}\prod_{i=2}^{L-1}r_i^{\beta_i(k_i+k_{i-1}+1)-1}}{\prod_{i=1}^{L-1}(k_i!)^2} \tag{23}$$

which consists only of standard functions such as powers and exponentials.

2) Product Moments: The $(\sum_{i=1}^L n_i)$th-order moment of the product of $\{Z_\ell\}$'s can be derived as

$$\mathcal{E}\left\langle\prod_{i=1}^L Z_i^{n_i}\right\rangle=\underbrace{\int_0^\infty\int_0^\infty\cdots\int_0^\infty}_{L\text{-fold}}\left(\prod_{i=1}^L r_i^{n_i}\right) \times f_{\overrightarrow{Z}}(\overrightarrow{r})\,dr_1\,dr_2\cdots dr_L \tag{24}$$

which using (23), yields

$$\mathcal{E}\left\langle\prod_{i=1}^L Z_i^{n_i}\right\rangle=(1-\rho)^{1+\sum_{j=1}^L n_j/\beta_j}\left(\prod_{j=1}^L\Omega^{n_j/\beta_j}\right) \times\sum_{k_1,k_2,\ldots,k_{L-1}=0}^{\infty}\left[\prod_{q=1}^{L-1}\frac{\rho^{k_q}}{(k_q!)^2}\right]\Gamma(k_1+d_{n_1,1}) \times\Gamma(k_{L-1}+d_{n_L,L})\prod_{i=2}^{L-1}\frac{\Gamma(k_i+k_{i-1}+d_{n_i,i})}{(1+\rho)^{k_i+k_{i-1}+d_{n_i,i}}}. \tag{25}$$

3) Joint mgf: The joint mgf of $\overrightarrow{Z}$ can be derived as

$$\mathcal{M}_{\overrightarrow{Z}}(\overrightarrow{s})=\mathcal{E}\langle\exp(-\overrightarrow{s}\cdot\overrightarrow{Z})\rangle \tag{26}$$

where $\overrightarrow{s}=[s_1\ s_2\ \cdots\ s_L]$ and the term $\overrightarrow{s}\cdot\overrightarrow{Z}$ denotes the inner product of $\overrightarrow{s}$ and $\overrightarrow{Z}$, i.e., $\overrightarrow{s}\cdot\overrightarrow{Z}=\sum_{i=1}^L s_iZ_i$. By substituting (23) in (26), some integrals of the form of (7) appear. Hence, by using (8),

$$f_{\overrightarrow{\boldsymbol{Z}}}(\overrightarrow{\boldsymbol{r}}) = \frac{1}{(4\pi\Omega)^L} \frac{\prod_{i=1}^{L} \beta_i r_i^{\beta_i - 1}}{(1-\rho)^{L-1}[1+(L-1)\rho]} \exp\left\{ -\frac{[1+(L-2)\rho]\sum_{i=1}^{L} r_i^{\beta_i}}{2(1-\rho)[1+(L-1)\rho]\Omega} \right\} \times \underbrace{\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\cdots\int_{-\pi}^{\pi}}_{L\text{-fold}} \exp\left\{ \frac{\rho \sum_{\substack{i,j=1 \\ i<j}}^{L} r_i^{\beta_i/2} r_j^{\beta_j/2} \cos(\varphi_i - \varphi_j)}{(1-\rho)[1+(L-1)\rho]\Omega} \right\} d\varphi_1\, d\varphi_2 \cdots d\varphi_L \quad (29)$$

the joint mgf of the multivariate Weibull distribution with exponential correlation can be obtained as in (27)

$$\begin{aligned} &\mathcal{M}_{\overrightarrow{\boldsymbol{Z}}}(\overrightarrow{\boldsymbol{s}}) \\ &= \frac{\prod_{i=1}^{L}\beta_i}{\Omega^L(1-\rho)^{L-1}} \sum_{k_1,k_2,\ldots,k_{L-1}=0}^{\infty} \left\{ \prod_{i=1}^{L-1} \frac{1}{(k_i!)^2} \left[\frac{\sqrt{\rho}}{\Omega(1-\rho)}\right]^{2k_i} \right\} \\ &\quad \times s_1^{-(k_1+1)\beta_1} \Upsilon\left[\frac{s_1^{-\beta_1}}{\Omega(1-\rho)}, (k_1+1)\beta_1 \right] \\ &\quad \times s_L^{-(k_{L-1}+1)\beta_L} \Upsilon\left[\frac{s_L^{-\beta_L}}{\Omega(1-\rho)}, (k_{L-1}+1)\beta_L \right] \\ &\quad \times \prod_{i=2}^{L-1} s_i^{-\beta_i(k_i+k_{i-1}+1)} \Upsilon\left[\frac{(1+\rho)s_i^{-\beta_i}}{\Omega(1-\rho)}, (k_i+k_{i-1}+1)\beta_i \right]. \end{aligned} \quad (27)$$

4) Joint cdf: The multivariate cdf of the Weibull distributed RVs $\{Z_\ell\}$, with exponential correlation, can be derived by using (23) and the definition of the lower incomplete Gamma function $\gamma(\cdot,\cdot)$ [38, eq. (8.350/1)] as

$$\begin{aligned} F_{\overrightarrow{\boldsymbol{Z}}}(\overrightarrow{\boldsymbol{r}}) = (1-\rho) &\sum_{k_1,k_2,\ldots,k_{L-1}=0}^{\infty} \gamma\left[k_1+1, \frac{r_1^{\beta_1}}{\Omega(1-\rho)}\right] \\ &\times \gamma\left[k_{L-1}+1, \frac{r_L^{\beta_L}}{\Omega(1-\rho)}\right] \left[\prod_{i=1}^{L-1} \frac{\rho^{k_i}}{(k_i!)^2}\right] \\ &\times \frac{\prod_{j=2}^{L-1} \gamma\left[k_{j-1}+k_j+1, r_j^{\beta_j}\frac{(1+\rho)}{\Omega(1-\rho)}\right]}{(1+\rho)^{L-2+k_1+k_{L-1}+2\sum_{l=2}^{L-2} k_l}}. \end{aligned} \quad (28)$$

It is useful to note that when the first argument of $\gamma(k,x)$ is an integer and x an arbitrary positive number, this function can be simplified to standard functions as [38, eq. (8.352/1)]

$$\gamma(k+1,x) = k!\left[1-\exp(-x)\sum_{l=0}^{k} x^l/l!\right].$$

Although such simplifications may be applied in (28) as well as in several expressions following next, they have not been performed for simplicity of the presentation.

D. The Multivariate Weibull Distribution With Constant Correlation

The constant correlation model, first proposed by Aalo in [2, Sec. II.A], [43], refers to the situation of L i.d. channels, where the spatial correlation is a function of the distance among the antennas. Hence, this model may be applied to digital receivers having equidistant antennas such as an arrays of three antennas placed on a equilateral triangle or from closely placed antennas on other than linear arrays.

The multivariate pdf of the i.d. Rayleigh distributed RVs with constant correlation $\{R_\ell\}_{\ell=1}^{L}$ is given by [9, eqs. (47) and (16)]. Let $\rho_{i,j}$ be the correlation coefficient between R_i^2 and R_j^2 with $i,j=1,2,\ldots,L$. Then, $\rho_{i,j}=\rho_{j,i}=\rho$, when $i \neq j$, while $\rho_{i,j}=1$, when $i=j$. By applying the transformation of (2) in the multivariate Rayleigh pdf with constant correlation and by following a similar method such that for the derivation of (22), the joint pdf of the Weibull distributed RVs $\{Z_\ell\}$ with constant correlation can be obtained as in (29) at the top of the page, with $\Omega = \mathcal{E}\langle Z_\ell^{\beta_\ell}\rangle\ \forall \ell$ and marginal pdfs given by (3) for $\ell = 1, 2, \ldots, L$. The (Weibull) correlation coefficient between Z_i^2 and Z_j^2 is given by $\varrho_{i,j}=\varrho_{j,i}=\varrho$, when $i \neq j$, while $\varrho_{i,j}=1$, when $i=j$, with $i,j=1,2,\ldots,L$ and ϱ given by (15).

III. Performance Analysis of Dual-Branch Diversity Receivers

We consider a dual-branch ($L=2$) diversity receiver operating over correlated Weibull fading channels described by the joint pdf expression given by (11). The baseband received signal in the ℓth ($\ell=1$ and 2) antenna is $\zeta_\ell = wh_\ell + n_\ell$, where w is the complex transmitted symbol, $E_w = \mathcal{E}\langle|w|^2\rangle$ is the transmitted average symbols energy, h_ℓ is the complex channel fading envelope with its magnitude $Z_\ell = |h_\ell|$ being modeled as a Weibull distributed RV, and n_ℓ is the additive white Gaussian noise (AWGN) with single-sided power spectral density N_0. The usual assumptions are made that the phase of h_ℓ can be accurately tracked and that the AWGN is uncorrelated among the input diversity branches.

The instantaneous SNR per symbol of the ℓth diversity channel can be expressed as

$$\gamma_\ell = Z_\ell^2 \frac{E_s}{N_0} \quad (30)$$

with its corresponding average SNR being

$$\bar{\gamma}_\ell = \mathcal{E}\left\langle Z_\ell^2\right\rangle \frac{E_s}{N_0} = \Gamma(d_{2,\ell})\Omega_\ell^{2/\beta_\ell}\frac{E_s}{N_0}. \quad (31)$$

Based on an interesting property of the Weibull distribution, that the nth power of a Weibull distributed RV with parameters (β_ℓ,Ω_ℓ) is another Weibull distributed RV with parameters $(\beta_\ell/n,\Omega_\ell)$, it can be easily concluded that γ_ℓ is also a Weibull distributed RV with parameters $(\beta_\ell/2,(a_\ell\bar{\gamma}_\ell)^{\beta_\ell/2})$ and $a_\ell = 1/\Gamma(d_{2,\ell})$. Hence, using the formulas of $\{Z_\ell\}$ presented in Section II, corresponding expressions for $\{\gamma_\ell\}$ can be easily derived by replacing β_ℓ with $\beta_\ell/2$ and Ω_ℓ with $(a_\ell\bar{\gamma}_\ell)^{\beta_\ell/2}$, helpful in the study of the performance of diversity receivers operating over Weibull correlated fading channels.

A. Dual-Branch SC Receivers

The instantaneous SNR per symbol at the output of a dual-branch SC receiver will be the one with the highest instantaneous value between the two branches [44], i.e.,

$$\gamma_{\mathrm{sc}} = \max\{\gamma_1,\gamma_2\}. \quad (32)$$

1) Outage Probability: By using (17), the cdf of γ_{sc} can be obtained in closed form as in (33)

$$\begin{aligned} &F_{\gamma_{\mathrm{sc}}}(\gamma) \\ &= 1-\exp\left[-\left(\frac{\gamma}{a_1\bar{\gamma}_1}\right)^{\beta_1/2}\right] \\ &\quad \times Q_1\left[\sqrt{\frac{2}{1-\rho}}\left(\frac{\gamma}{a_2\bar{\gamma}_2}\right)^{\beta_2/4}, \sqrt{\frac{2\rho}{1-\rho}}\left(\frac{\gamma}{a_1\bar{\gamma}_1}\right)^{\beta_1/4}\right] \\ &\quad -\exp\left[-\left(\frac{\gamma}{a_2\bar{\gamma}_2}\right)^{\beta_2/2}\right]\left\{1-Q_1\left[\sqrt{\frac{2\rho}{1-\rho}}\left(\frac{\gamma}{a_2\bar{\gamma}_2}\right)^{\beta_2/4},\right.\right. \\ &\qquad \left.\left.\sqrt{\frac{2}{1-\rho}}\left(\frac{\gamma}{a_1\bar{\gamma}_1}\right)^{\beta_1/4}\right]\right\}. \end{aligned} \quad (33)$$

$$\begin{aligned}\mu_n = \sum_{k=0}^{\infty}\frac{\rho^k}{k!}\Bigg\{&(a_1\bar{\gamma}_1)^n(1-\rho)^{d_{2n,1}}\left[\Gamma(k+d_{2n,1}) - \sum_{l=0}^{k}\frac{1}{l!}\left(\frac{a_1\bar{\gamma}_1}{a_2\bar{\gamma}_2}\right)^{l\beta_2/2}(1-\rho)^{(\beta_2/\beta_1-1)l}\right.\\ &\left.\times\,\Upsilon\left[(1-\rho)^{(\beta_2/\beta_1-1)}\left(\frac{a_1\bar{\gamma}_1}{a_2\bar{\gamma}_2}\right)^{\beta_2/2}, d_{2n,1}+k+l\frac{\beta_2}{\beta_1}\right]\right]\\ &+(a_2\bar{\gamma}_2)^n(1-\rho)^{d_{2n,2}}\left[\Gamma(k+d_{2n,2}) - \sum_{l=0}^{k}\frac{1}{l!}\left(\frac{a_2\bar{\gamma}_2}{a_1\bar{\gamma}_1}\right)^{l\beta_1/2}(1-\rho)^{(\beta_1/\beta_2-1)l}\right.\\ &\left.\times\,\Upsilon\left[(1-\rho)^{(\beta_1/\beta_2-1)}\left(\frac{a_2\bar{\gamma}_2}{a_1\bar{\gamma}_1}\right)^{\beta_1/2},\ d_{2n,2}+k+l\frac{\beta_1}{\beta_2}\right]\right]\Bigg\}\end{aligned} \tag{39}$$

$$\begin{aligned}\mu_n = (1-\rho)^{d_{2n}}\sum_{k=0}^{\infty}\frac{\rho^k}{k!}\Gamma(k+d_{2n})\Bigg\{&(a\bar{\gamma}_1)^n\left[1-\frac{[1+(\bar{\gamma}_1/\bar{\gamma}_2)^{\beta/2}]^{-k-d_{2n}}}{\Gamma(k+d_{2n})}\sum_{m=0}^{k}\frac{1}{m!}\frac{\Gamma(m+k+d_{2n})}{[1+(\bar{\gamma}_2/\bar{\gamma}_1)^{\beta/2}]^m}\right]\\ &+(a\bar{\gamma}_2)^n\left[1-\frac{[1+(\bar{\gamma}_2/\bar{\gamma}_1)^{\beta/2}]^{-k-d_{2n}}}{\Gamma(k+d_{2n})}f\sum_{m=0}^{k}\frac{1}{m!}\frac{\Gamma(m+k+d_{2n})}{[1+(\bar{\gamma}_1/\bar{\gamma}_2)^{\beta/2}]^m}\right]\Bigg\}\end{aligned} \tag{40}$$

Since the Marqum's Q-function is not included as a built-in function in most of the well-known mathematical software packages, alternatively, it can be written in the form of an infinite series representation. Hence, by using (19), the cdf of $\gamma_{\rm sc}$ can be derived as

$$F_{\gamma_{\rm sc}}(\gamma) = \int_0^{\gamma}\int_0^{\gamma} f_{\gamma_1,\gamma_2}(\gamma_1,\gamma_2)\,d\gamma_1\,d\gamma_2 \tag{34}$$

which using [38, eq. (8.351/1)], yields

$$F_{\gamma_{\rm sc}}(\gamma) = (1-\rho)\sum_{k=0}^{\infty}\frac{\rho^k}{(k!)^2}\prod_{i=1}^{2}\gamma\left[k+1, \frac{1}{1-\rho}\left(\frac{\gamma}{a_i\bar{\gamma}_i}\right)^{\beta_i/2}\right]. \tag{35}$$

The outage probability $P_{\rm out}$ is defined as the probability that the SC output SNR falls below a given outage threshold $\gamma_{\rm th}$. This probability can be obtained by replacing γ with $\gamma_{\rm th}$ in (33) or (35), i.e.,

$$P_{\rm out}(\gamma_{\rm th}) = F_{\gamma_{\rm sc}}(\gamma_{\rm th}). \tag{36}$$

2) Moments of the Output SNR: The nth-order moment of the SC output SNR can be derived as [39]

$$\mu_n = \mathcal{E}\left\langle\gamma_{\rm sc}^n\right\rangle = \int_0^{\infty}\gamma^n f_{\gamma_{\rm sc}}(\gamma)\,d\gamma. \tag{37}$$

By taking the first derivative of (35), the pdf of $\gamma_{\rm sc}$ can be obtained as

$$\begin{aligned}f_{\gamma_{\rm sc}}(\gamma) = \frac{1}{2}\sum_{k=0}^{\infty}&\frac{\rho^k}{(k!)^2(1-\rho)^k}\\ \times\Bigg\{&\frac{\beta_1\gamma^{-1+(k+1)\beta_1/2}}{(a_1\,\bar{\gamma}_1)^{(k+1)\beta_1/2}}\exp\left[-\frac{1}{1-\rho}\left(\frac{\gamma}{a_1\bar{\gamma}_1}\right)^{\beta_1/2}\right]\\ &\times\gamma\left[k+1, \frac{1}{1-\rho}\left(\frac{\gamma}{a_2\bar{\gamma}_2}\right)^{\beta_2/2}\right]\\ &+\frac{\beta_2\gamma^{-1+(k+1)\beta_2/2}}{(a_2\bar{\gamma}_2)^{(k+1)\beta_2/2}}\exp\left[-\frac{1}{1-\rho}\left(\frac{\gamma}{a_2\bar{\gamma}_2}\right)^{\beta_2/2}\right]\\ &\times\gamma\left[k+1, \frac{1}{1-\rho}\left(\frac{\gamma}{a_1\bar{\gamma}_1}\right)^{\beta_1/2}\right]\Bigg\}.\end{aligned} \tag{38}$$

By substituting (38) in (37) and using [45, eq. (21)], the nth-order moment of $\gamma_{\rm sc}$ can be derived as in (39) at the top of the page, where in that equation $\lambda/\kappa = \beta_2/\beta_1$ holds instead of (9) with β_2/β_1 being assumed to belong to rationals. By using [38, eq. (3.326/2)], for $\beta_1 = \beta_2 = \beta$, μ_n can be significantly reduced to (40) also at the top of the page, where $d_\tau = 1 + \tau/\beta$ and $a = 1/\Gamma(d_2)$. Note, that for independent and i.d. input branches, (40) reduces to an earlier known result [27, eq. (8)].

The SC average output SNR, $\bar{\gamma}_{\rm sc}$, is a useful performance measure which serves as an excellent indicator of the overall system's fidelity. By setting $n = 1$ in (39), i.e., $\bar{\gamma}_{\rm sc} = \mu_1$, $\bar{\gamma}_{\rm sc}$ can be obtained as in (41) at the top of the following page, while for $\beta_1 = \beta_2 = \beta$ reduces to (42) also at the top of the following page.The AoF is defined as

$$A_F \triangleq \frac{\operatorname{var}(\gamma_{\rm sc})}{\bar{\gamma}_{\rm sc}^2} \tag{43}$$

and is considered as a unified measure of the severity of fading [1]. Typically, this performance criterion is independent of the average fading power. Using (40), the AoF of the SC output can be easily expressed in a simple closed-form expression as

$$A_F = \frac{\mu_2}{\mu_1^2} - 1. \tag{44}$$

It is important to underline that the higher order moments $(n \geq 3)$ are especially useful in signal processing algorithms for signal detection, classification, and estimation of wideband communications in the presence of fading [46].

3) ASEP and Outage Probability: The mgf of the SC output SNR can be expressed as

$$\mathcal{M}_{\gamma_{\rm sc}}(s) = \mathcal{E}\langle\exp(-s\gamma_{\rm sc})\rangle \tag{45}$$

where for $\beta_1 = \beta_2 = \beta$ and by substituting (38), resulting in (46) at the top of the following page.

Using the above mgf expression of the dual-branch SC output SNR, the ASEP of noncoherent binary frequency-shift keying (NBFSK) and binary differential phase-shift keying (BDPSK) modulation signaling can be directly calculated (e.g., for BDPSK $\bar{P}_{\rm be} = 0.5\mathcal{M}_{\gamma_{\rm sc}}(1)$), since for other types of modulation formats, including binary phase-shift keying (BPSK), M-ary phase-shift keying (M-PSK), quadrature amplitude modulation (M-QAM), amplitude modulation (M-AM), and differential phase-shift keying (M-DPSK), single integrals with finite limits and integrands composed of elementary (exponential and trigonometric) functions have to be readily evaluated via numerical integration [1].

B. Dual-Branch EGC Receivers

For a dual-branch EGC receiver, the instantaneous output signal envelope is [1], [47], [48]

$$Z = \frac{1}{\sqrt{2}}(Z_1 + Z_2) = \sqrt{\frac{N_0}{2E_s}}(\sqrt{\gamma_1} + \sqrt{\gamma_2}). \tag{47}$$

$$\bar{\gamma}_{\rm sc} = \sum_{k=0}^{\infty} \frac{\rho^k}{k!} \left\{ a_1 \bar{\gamma}_1 (1-\rho)^{d_{2,1}} \left[\Gamma(k+d_{2,1}) - \sum_{l=0}^{k} \frac{1}{l!} \left(\frac{a_1\bar{\gamma}_1}{a_2\bar{\gamma}_2} \right)^{l\beta_2/2} (1-\rho)^{(\beta_2/\beta_1-1)l} \right.\right.$$
$$\left.\times\, \Upsilon_1 \left[(1-\rho)^{(\beta_2/\beta_1-1)} \left(\frac{a_1\bar{\gamma}_1}{a_2\bar{\gamma}_2} \right)^{\beta_2/2}, d_{2,1}+k+l\frac{\beta_2}{\beta_1} \right]\right]$$
$$+ a_2\bar{\gamma}_2 (1-\rho)^{d_{2,2}} \left[\Gamma(k+d_{2,2}) - \sum_{l=0}^{k} \frac{1}{l!} \left(\frac{a_2\bar{\gamma}_2}{a_1\bar{\gamma}_1} \right)^{l\beta_1/2} (1-\rho)^{(\beta_1/\beta_2-1)l} \right.$$
$$\left.\left.\times\, \Upsilon_2 \left[(1-\rho)^{(\beta_1/\beta_2-1)} \left(\frac{a_2\bar{\gamma}_2}{a_1\bar{\gamma}_1} \right)^{\beta_1/2}, d_{2,2}+k+l\frac{\beta_1}{\beta_2} \right]\right]\right\} \quad (41)$$

$$\bar{\gamma}_{\rm sc} = a(1-\rho)^{d_2} \sum_{k=0}^{\infty} \frac{\rho^k}{k!} \Gamma(k+d_2) \left\{ \bar{\gamma}_1 \left[1 - \frac{[1+(\bar{\gamma}_1/\bar{\gamma}_2)^{\beta/2}]^{-k-d_2}}{\Gamma(k+d_2)} \sum_{m=0}^{k} \frac{1}{m!} \frac{\Gamma(m+k+d_2)}{[1+(\bar{\gamma}_2/\bar{\gamma}_1)^{\beta/2}]^m} \right] \right.$$
$$\left. + \bar{\gamma}_2 \left[1 - \frac{[1+(\bar{\gamma}_2/\bar{\gamma}_1)^{\beta/2}]^{-k-d_2}}{\Gamma(k+d_2)} \sum_{m=0}^{k} \frac{1}{m!} \frac{\Gamma(m+k+d_2)}{[1+(\bar{\gamma}_1/\bar{\gamma}_2)^{\beta/2}]^m} \right] \right\} \quad (42)$$

$$\mathcal{M}_{\gamma_{\rm sc}}(s) = \frac{1}{2} \sum_{k=0}^{\infty} \frac{\rho^k}{k!(1-\rho)^k} \left\{ \frac{\beta_1}{(sa_1\bar{\gamma}_1)^{(k+1)\beta_1/2}} \left[\Upsilon\left[\frac{(sa_1\bar{\gamma}_1)^{-\beta_1/2}}{1-\rho}, (k+1)\frac{\beta_1}{2} \right] \right.\right.$$
$$\left. - \sum_{m=0}^{k} \frac{(1-\rho)^{-m}}{m!} \frac{1}{(sa_2\bar{\gamma}_2)^{m\beta_2/2}}\, \Upsilon\left[\frac{1}{1-\rho} \sum_{i=1}^{2} \frac{1}{(sa_i\bar{\gamma}_i)^{\beta_i/2}}, (m+k+1)\frac{\beta_i}{2} \right]\right]$$
$$+ \frac{\beta_2}{(sa_2\bar{\gamma}_2)^{(k+1)\beta_2/2}} \left[\Upsilon\left[\frac{(sa_2\bar{\gamma}_2)^{-\beta_2/2}}{1-\rho}, (k+1)\frac{\beta_2}{2} \right] \right.$$
$$\left.\left. - \sum_{m=0}^{k} \frac{(1-\rho)^{-m}}{m!} \frac{1}{(sa_1\bar{\gamma}_1)^{m\beta_1/2}} \times \Upsilon\left[\frac{1}{1-\rho} \sum_{i=1}^{2} \frac{1}{(sa_i\bar{\gamma}_i)^{\beta_i/2}}, (m+k+1)\frac{\beta_i}{2} \right]\right]\right\} \quad (46)$$

1) Moments of the Output SNR: By using the binomial identity [38, (1.111)], the nth-order moment of $\gamma_{\rm egc} = Z^2 E_s/N_0$ can be derived as

$$\mu_n = \mathcal{E}\left\langle \gamma_{\rm egc}^n \right\rangle = \frac{1}{2^n} \sum_{k=0}^{2n} \binom{2n}{k} \mathcal{E}\left\langle \gamma_1^{k/2} \gamma_2^{n-k/2} \right\rangle \quad (48)$$

which, by substituting (13) and after some straightforward simplifications, yields

$$\mu_n = \frac{1}{2^n} \sum_{k=0}^{2n} \binom{2n}{k} (1-\rho)^{1+k/\beta_1+(2n-k)/\beta_2} (a_2\bar{\gamma}_2)^{n-k/2}$$
$$\times (a_1\bar{\gamma}_1)^{k/2} \Gamma\left(1+\frac{k}{\beta_1}\right) \Gamma\left(1+\frac{2n-k}{\beta_2}\right)$$
$$\times\, {}_2F_1\left(1+\frac{k}{\beta_1}, 1+\frac{2n-k}{\beta_2}; 1; \rho\right). \quad (49)$$

The average output SNR $\bar{\gamma}_{\rm egc}$ can be obtained by setting $n=1$ in (49), i.e., $\bar{\gamma}_{\rm egc} = \mu_1$, resulting in

$$\bar{\gamma}_{\rm egc} = \frac{1}{2}(\bar{\gamma}_2+\bar{\gamma}_1) + \sqrt{\bar{\gamma}_1\bar{\gamma}_2}(1-\rho)^{1+1/\beta_1+1/\beta_2}$$
$$\times \frac{\Gamma(1+1/\beta_1)\Gamma(1+1/\beta_2)}{\sqrt{\Gamma(1+2/\beta_1)\Gamma(1+2/\beta_2)}}\, {}_2F_1\left(1+\frac{1}{\beta_1}, 1+\frac{1}{\beta_2}; 1; \rho\right) \quad (50)$$

while utilizing both the first- and second-order moments, the AoF can be also obtained in closed form.

2) ASEP: Using (21), the mgf of Z_1+Z_2 can be derived as

$$\mathcal{M}_{Z_1+Z_2}(s) = \mathcal{M}_{Z_1,Z_2}(s,s). \quad (51)$$

The characteristic function of Z can be derived using

$$\Phi_Z(s) = \mathcal{M}_{Z_1+Z_2}(\jmath s\sqrt{E_s/(2N_0)}) \quad (52)$$

in conjunction with (51) as

$$\Phi_Z(s) = \beta_1\beta_2 \sum_{q=0}^{\infty} \frac{\rho^q}{(q!)^2(1-\rho)^{2q+1}}$$
$$\times \prod_{i=1}^{2} \frac{1}{[(a_i\bar{\gamma}_i/2)^{\beta_i/2}(\jmath s)^{\beta_i}]^{q+1}}$$
$$\times\, \Upsilon\left[\frac{(a_i\bar{\gamma}_i/2)^{-\beta_i/2}}{(\jmath s)^{\beta_i}(1-\rho)}, (q+1)\beta_i\right]. \quad (53)$$

Based on the Parseval's theorem approach [49], the ASEP for several coherent and noncoherent modulation schemes can be evaluated as

$$\bar{P}_{\rm se} = \frac{1}{\pi} \int_0^{\infty} \Re\{\Phi_Z(s) P^*(s)\}\, ds \quad (54)$$

where $P^*(\cdot)$ is the complex conjugate of the Fourier transform of the conditional symbol error probability and the notation $\Re\{\cdot\}$ denotes the real-part operator. For example, for BDPSK

$$P(s) = \frac{\sqrt{\pi}}{4} \exp\left(-\frac{s^2}{2}\right) + \jmath\frac{1}{2}\mathcal{D}\left(\frac{s}{\sqrt{2}}\right) \quad (55)$$

with $\mathcal{D}(\cdot)$ denoting the Dawson's integral defined as $\mathcal{D}(x) = \exp(-x^2)\int_0^x \exp(t^2) dt$.

C. Dual-Branch MRC Receivers

The instantaneous SNR per symbol measured at the output of a dual-branch MRC receiver is

$$\gamma_{\rm mrc} = \gamma_1 + \gamma_2. \quad (56)$$

1) Moments of the Output SNR: By using the binomial identity, the corresponding nth-order moment, $\mu_n = \mathcal{E}\langle \gamma_{\rm mrc}^n \rangle$, can be obtained as

$$\mu_n = \mathcal{E}\langle (\gamma_1+\gamma_2)^n \rangle = \sum_{k=0}^{n} \binom{n}{k} \mathcal{E}\left\langle \gamma_1^k \gamma_2^{n-k} \right\rangle \tag{57}$$

which by using (13), (30), and (31), yields

$$\begin{aligned}\mu_n = &\sum_{k=0}^{n} \binom{n}{k} (1-\rho)^{1+2k/\beta_1+2(n-k)/\beta_2} (a_2\bar{\gamma}_2)^{n-k} \\ &\times (a_1\bar{\gamma}_1)^k \Gamma\left(1+\frac{2k}{\beta_1}\right) \Gamma\left[1+\frac{2(n-k)}{\beta_2}\right] \\ &\times {}_2F_1\left[1+\frac{2k}{\beta_1}, 1+\frac{2(n-k)}{\beta_2}; 1; \rho\right].\end{aligned} \tag{58}$$

By substituting both the first- (average SNR), $\mu_1 = \bar{\gamma}_1 + \bar{\gamma}_2$, and the second-order μ_2 moments of the output SNR in (44), the AoF at the output of MRC can be also obtained in closed form.

2) ASEP and Outage Probability: By using (21), the mgf of the dual-branch MRC output SNR per symbol can be obtained as

$$\mathcal{M}_{\gamma_{\rm mrc}}(s) = \mathcal{M}_{\gamma_1,\gamma_2}(s,s). \tag{59}$$

Based on the above equation and the mgf approach to the performance analysis, the ASEP of dual-branch MRC receivers can be studied.

If $\gamma_{\rm th}$ is a certain specified threshold, then the outage probability is defined as the probability that $\gamma_{\rm mrc}$ falls below $\gamma_{\rm th}$ and is given by [1]

$$\begin{aligned}P_{\rm out}(\gamma_{\rm th}) &= F_{\gamma_{\rm mrc}}(\gamma_{\rm th}) \\ &= \mathcal{L}^{-1}\left[\frac{\mathcal{M}_{\gamma_{\rm mrc}}(s)}{s}; \gamma_{\rm mrc}\right]\Bigg|_{\gamma_{\rm mrc}=\gamma_{\rm th}}\end{aligned} \tag{60}$$

where $F_{\gamma_{\rm mrc}}(\cdot)$ is the cdf of $\gamma_{\rm mrc}$ and $\mathcal{L}^{-1}[\cdot;\cdot]$ denotes the inverse Laplace transform. Due to the complicated form of $\mathcal{M}_{\gamma_{\rm mrc}}(s)/s$ in (60), the so-called Euler summation-based algorithm for the inversion of cdfs may be applied [1, Appendix 9B.1], [50].

IV. Performance Analysis of Multibranch Diversity Receivers

Let us consider an L-branch diversity receiver operating over exponentially correlated Weibull fading channels ($\ell = 1, 2, \ldots, L$). Based on the theoretical framework of Section II-C, several performance criteria such as average output SNR, AoF, outage probability, and ASEP are derived.

A. Multibranch SC Receivers

The instantaneous SNR per symbol at the output of the SC receiver will be the one with the highest instantaneous value among the L branches, i.e.,

$$\gamma_{\rm sc} = \max\{\gamma_1, \gamma_2, \ldots, \gamma_L\}. \tag{61}$$

By using (28), the cdf of $\gamma_{\rm sc}$ can be obtained as

$$F_{\gamma_{\rm sc}}(\gamma) = F_{\overrightarrow{\gamma}}(\underbrace{\gamma, \gamma, \ldots, \gamma}_{L}) \tag{62}$$

with $\overrightarrow{\gamma} = [\gamma_1, \gamma_2, \ldots, \gamma_L]$.

The outage probability, $P_{\rm out}$, can be obtained by replacing γ with $\gamma_{\rm th}$ in (62) as

$$P_{\rm out}(\gamma_{\rm th}) = F_{\gamma_{\rm sc}}(\gamma_{\rm th}). \tag{63}$$

B. Multibranch EGC Receivers

By considering a multibranch EGC receiver, the received signal envelope is

$$Z = \frac{1}{\sqrt{L}} \sum_{i=1}^{L} Z_i = \sqrt{\frac{N_0}{LE_s}} \sum_{i=1}^{L} \sqrt{\gamma_i}. \tag{64}$$

By using the multinomial identity [51, eq. (24.1.2)], the nth-order moment of $\gamma_{\rm egc} = Z^2 E_s/N_0$ can be derived as

$$\begin{aligned}\mu_n &= \mathcal{E}\left\langle \gamma_{\rm egc}^n \right\rangle = \frac{1}{L^n} \mathcal{E}\left\langle \left(\sum_{i=1}^{L} \sqrt{\gamma_i}\right)^{2n} \right\rangle \\ &= \frac{(2n)!}{L^n} \sum_{\substack{k_1,k_2,\ldots,k_L=0 \\ k_1+k_2+\cdots+k_L=2n}}^{2n} \frac{\mathcal{E}\left\langle \gamma_1^{k_1/2} \gamma_2^{k_2/2} \ldots \gamma_L^{k_L/2} \right\rangle}{k_1! k_2! \ldots k_L!}\end{aligned} \tag{65}$$

which, using (25) and after some straightforward simplifications, yields

$$\begin{aligned}\mu_n = &\bar{\gamma}^n \frac{(2n)!}{L^n} \sum_{\substack{k_1,k_2,\ldots,k_L=0 \\ k_1+k_2+\cdots+k_L=2n}}^{2n} \frac{(1-\rho)^{1+\sum_{j=1}^{L} k_j/\beta_j}}{\prod_{j=1}^{L} k_j! a_j^{-k_j/2}} \\ &\times \sum_{t_1,t_2,\ldots,t_{L-1}=0}^{\infty} \left[\prod_{q=1}^{L-1} \frac{\rho^{t_q}}{(t_q!)^2}\right] \Gamma(t_1 + d_{k_1,1}) \\ &\times \Gamma(t_{L-1} + d_{k_L,L}) \prod_{i=2}^{L-1} \frac{\Gamma(t_i + t_{i-1} + d_{k_i,i})}{(1+\rho)^{t_i+t_{i-1}+d_{k_i,i}}}\end{aligned} \tag{66}$$

where $\bar{\gamma} = \Omega E_s/N_0$ is the average input SNR per symbol identical to all branches $\bar{\gamma}_\ell = \bar{\gamma} \forall \ell$. The average output SNR, $\bar{\gamma}_{\rm egc}$, can be obtained by setting $n = 1$ in (66), i.e., $\bar{\gamma}_{\rm egc} = \mu_1$, resulting in

$$\begin{aligned}\bar{\gamma}_{\rm egc} = &\bar{\gamma} \frac{2}{L} \sum_{\substack{k_1,k_2,\ldots,k_L=0 \\ k_1+k_2+\cdots+k_L=2}}^{2} \frac{(1-\rho)^{1+\sum_{j=1}^{L} k_j/\beta_j}}{\prod_{j=1}^{L} k_j! a_j^{-k_j/2}} \\ &\times \sum_{t_1,t_2,\ldots,t_{L-1}=0}^{\infty} \left[\prod_{q=1}^{L-1} \frac{\rho^{t_q}}{(t_q!)^2}\right] \Gamma(t_1 + d_{k_1,1}) \\ &\times \Gamma(t_{L-1} + d_{k_L,L}) \prod_{i=2}^{L-1} \frac{\Gamma(t_i + t_{i-1} + d_{k_i,i})}{(1+\rho)^{t_i+t_{i-1}+d_{k_i,i}}}\end{aligned} \tag{67}$$

while by utilizing both the first- and second-order moments, AoF can also be obtained.

C. Multibranch MRC Receivers

The instantaneous SNR per symbol measured at the output of a multibranch MRC receivers is

$$\gamma_{\rm mrc} = \sum_{i=1}^{L} \gamma_i. \tag{68}$$

TABLE I
NUMBER OF TERMS FOR CONVERGENCE OF THE ABEP OF SC IN RANGE OF $\pm 2\%$ (BDPSK, $\beta = 3, \bar{\gamma}_\ell = \bar{\gamma}$, AND $L = 2$)

$\bar{\gamma}$ (dB)	$\rho = 0.1$	$\rho = 0.5$	$\rho = 0.9$
-5	6	8	15
0	5	7	12
5	5	6	9
10	5	5	6
15	5	5	5

TABLE II
NUMBER OF TERMS FOR CONVERGENCE OF THE OUTAGE PROBABILITY OF SC IN RANGE OF $\pm 2\%$ ($\beta = 2.5, \bar{\gamma}_\ell = \bar{\gamma}$, AND $\rho = 0.5$)

$\gamma_{th}/\bar{\gamma}$ (dB)	$L = 2$	$L = 3$	$L = 4$
10	5	3	3
5	4	2	3
0	2	2	3
-5	1	1	1
-10	1	1	1
-15	1	1	1
-20	1	1	1

1) Moments of the Output SNR: Based on the multinomial identity, the corresponding nth-order moment of $\gamma_{\rm mrc}, \mu_n = \mathcal{E}\langle\gamma_{\rm mrc}^n\rangle$ can be obtained as

$$\mu_n = n! \sum_{\substack{k_1,k_2,\ldots,k_L=0\\ k_1+k_2+\cdots+k_L=n}}^{n} \frac{\mathcal{E}\left\langle \gamma_1^{k_1}\gamma_2^{k_2}\ldots\gamma_L^{k_L}\right\rangle}{k_1!k_2!\ldots k_L!} \tag{69}$$

which by using (25), (30), and (31), yields

$$\begin{aligned}\mu_n = n!\bar{\gamma}^n & \sum_{\substack{k_1,k_2,\ldots,k_L=0\\ k_1+k_2+\cdots+k_L=n}}^{n} \frac{(1-\rho)^{1+2\sum_{j=1}^{L} k_j/\beta_j}}{\prod_{j=1}^{L} k_j! a_j^{-k_j}} \\ & \times \sum_{t_1,t_2,\ldots,t_{L-1}=0}^{\infty} \left[\prod_{q=1}^{L-1} \frac{\rho^{t_q}}{(t_q!)^2}\right] \Gamma\left(t_1 + d_{2k_1,1}\right) \\ & \times \Gamma\left(t_{L-1} + d_{2k_L,L}\right) \prod_{i=2}^{L-1} \frac{\Gamma\left(t_i + t_{i-1} + d_{2k_i,i}\right)}{(1+\rho)^{t_i+t_{i-1}+d_{2k_i,i}}}. \end{aligned} \tag{70}$$

By utilizing both the first- and second-order moments of $\gamma_{\rm mrc}$, the AoF at the output of MRC can also be obtained.

2) ASEP and Outage Probability: Using (27), the mgf of the multibranch MRC output SNR per symbol with exponential correlation can be obtained as

$$\mathcal{M}_{\gamma_{\rm mrc}}(s) = \mathcal{M}_{\overrightarrow{\gamma}}(\underbrace{s, s, \ldots, s}_{L}). \tag{71}$$

The above expression can be useful in the study of various performance characteristics of multibranch MRC receivers operating in a Weibull fading environment, such as the ASEP and the outage probability using (60).

V. NUMERICAL RESULTS

In this section, by using the previous mathematical analysis, numerical results are presented for the performance of diversity receivers over correlated Weibull fading channels, where without loss of generality we consider that $\beta_\ell = \beta \forall \ell$. The numerical evaluation of several expressions in Sections III and IV require the summation of an infinite number of terms. Tables I and II summarize the number of terms needed for SC receivers, in order to achieve an accuracy better than $\pm 2\%$ after the truncation of the infinite series. As Table I indicates, an increase in ρ leads to an increase of the required number of terms that are needed to be summed in order to obtain the target accuracy. Furthermore, the number of the required terms depends strongly on the average input SNR ($\bar{\gamma}_1 = \bar{\gamma}_2 = \bar{\gamma}$). An increase on $\bar{\gamma}$ decreases the number of terms that are required to be summed. Similar conclusions can be also extracted from Table II, where it is interesting to be mentioned, that for $\gamma_{\rm th}/\bar{\gamma} \leq -5$ dB, only one term is enough for the numerical evaluation of $P_{\rm out}$ of multibranch SC receivers.

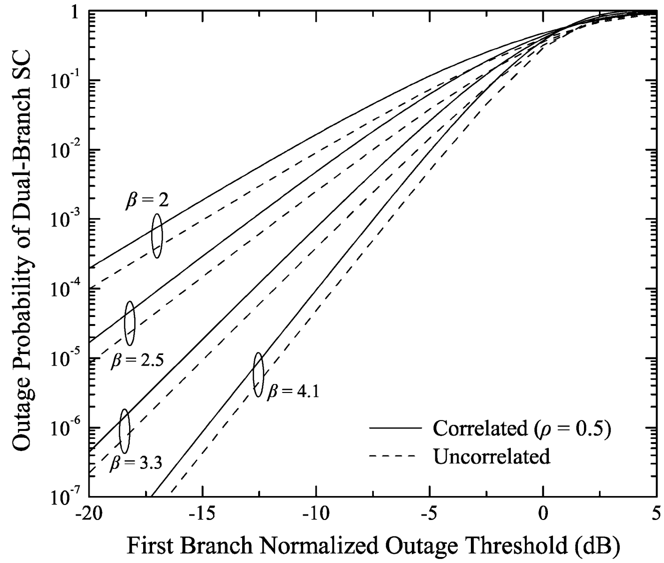

Fig. 2. Outage probability of a dual-branch SC as a function of the first branch normalized outage threshold for $\bar{\gamma}_2 = 1.25\bar{\gamma}_1$.

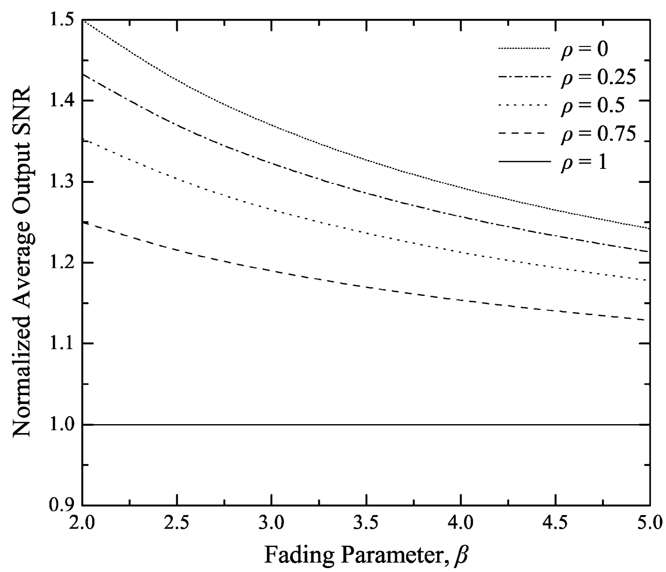

Fig. 3. Normalized average dual-branch SC output SNR as a function of the fading parameter.

Having numerically evaluated (36), in Fig. 2, the outage probability $P_{\rm out}$ is plotted as a function of the first branch normalized outage threshold $\gamma_{\rm th}/\bar{\gamma}_1$, for a dual-branch SC, with unequal input SNRs per symbol ($\bar{\gamma}_2 = 1.25\bar{\gamma}_1$), $\rho = 0.5$ and for different values of β. For

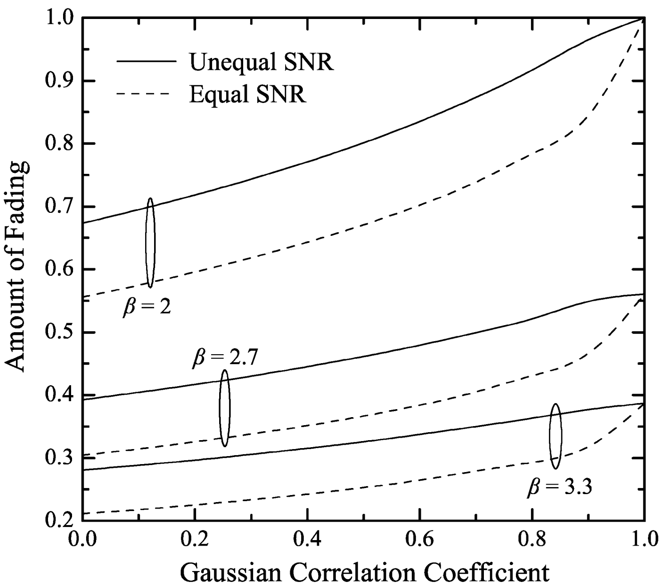

Fig. 4. AoF of dual-branch SC as a function of the Gaussian correlation coefficient, ρ, for $\bar{\gamma}_2 = 0.5\bar{\gamma}_1$.

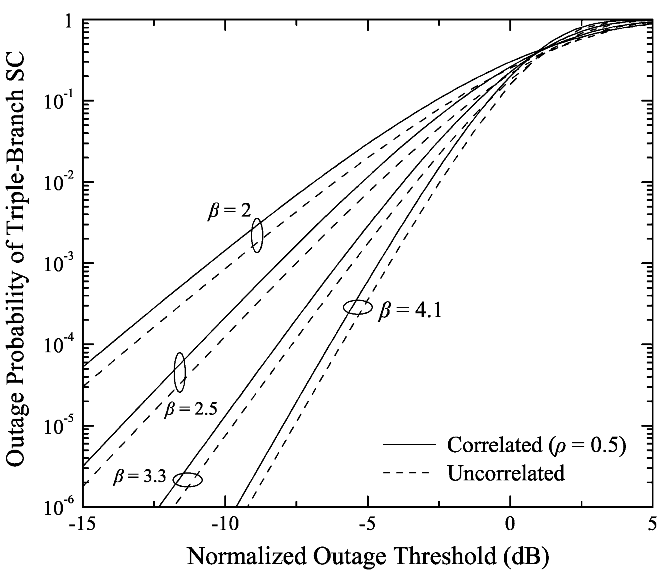

Fig. 5. Outage probability of triple-branch SC as a function of the normalized outage threshold.

comparison purposes, the curve for $\rho = 0$ is also included as a special case for best performance. The obtained results clearly show, that $P_{\rm out}$ degrades with an increase of ρ, $\gamma_{\rm th}/\bar{\gamma}_1$, and/or fading severity.

By using (42), in Fig. 3, $\bar{\gamma}_{\rm sc}/\bar{\gamma}_1$ is plotted as a function of β, for equal input average SNRs $(\bar{\gamma}_1 = \bar{\gamma}_2)$ and for several values of ρ. As expected, the diversity gain increases as β and/or ρ decreases. It is interesting to note, that $\bar{\gamma}_{\rm sc}/\bar{\gamma}_1$ degrades more rapidly for lower values of β and ρ. For the limiting case of $\rho = 0$, the SNR gain of the combiner takes its maximum value, while for $\rho \to 1$ the corresponding gain approaches to unity.

Based on (40) and (44), Fig. 4 demonstrates the numerically evaluated results for the AoF, A_F, of a dual-branch SC receiver as a function of ρ for equal and unequal input average SNRs $(\bar{\gamma}_2 = 0.5\bar{\gamma}_1)$. This figure indicates, that an increase of ρ and/or a decrease of β, leads to a corresponding increase of A_F resulting in performance degradation.

By using (62), in Fig. 5, $P_{\rm out}$ is plotted as a function of the normalized outage threshold $\gamma_{\rm th}/\bar{\gamma}$, for a triple-branch SC, with i.d. average

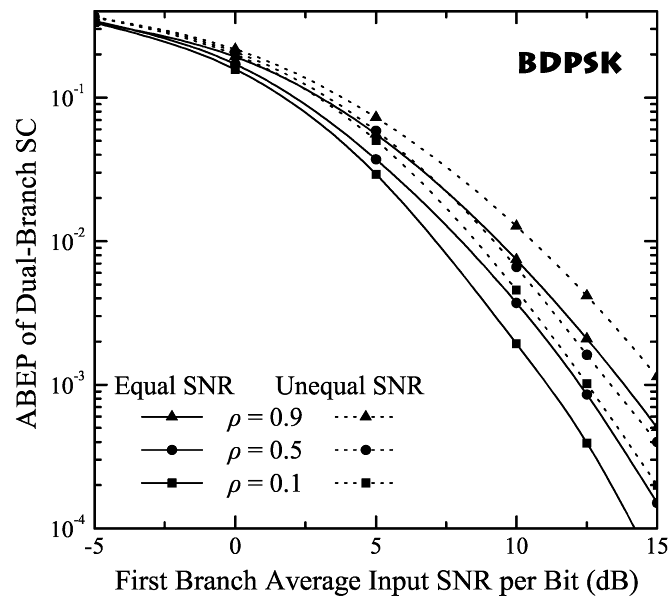

Fig. 6. ABEP of dual-branch SC as a function of the average input SNR for equal $(\bar{\gamma}_1 = \bar{\gamma}_2)$ and unequal $(\bar{\gamma}_1 = 0.5\bar{\gamma}_2)$ branch average SNRs and $\beta = 3$.

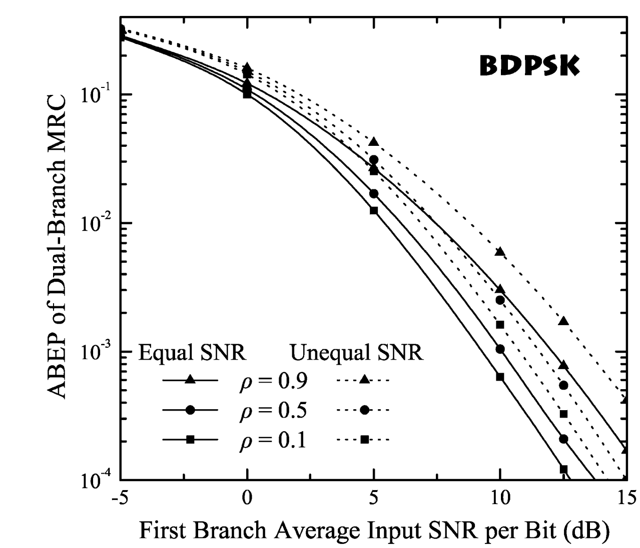

Fig. 7. ABEP of dual-branch MRC as a function of the average input SNR for equal $(\bar{\gamma}_1 = \bar{\gamma}_2)$ and unequal $(\bar{\gamma}_1 = 0.5\bar{\gamma}_2)$ branch average SNRs and $\beta = 3$.

input SNRs per symbol, exponential correlation, and same parameters as in Fig. 2, in which the same findings are also extracted.

Using (46) and (59) or (71), the error performances of dual-branch SC and dual- or multibranch MRC receivers, respectively, can be obtained for several modulation schemes. As a typical example, in Fig. 6, the ABEP performance of SC receivers with BDPSK signaling is plotted as a function of $\bar{\gamma}_1$, for equal $\bar{\gamma}_1 = \bar{\gamma}_2$ and unequal $\bar{\gamma}_1 = 0.5\bar{\gamma}_2$ input average SNRs, $\beta = 3$, and for several values of ρ. Moreover, in Fig. 7, the ABEP of dual-branch MRC receivers for the same modulation format and same input SNRs unbalance, as in Fig. 6, is plotted as a function of $\bar{\gamma}_1$ for several values of ρ. From both figures, the obtained performance evaluation results show, that the ABEP improves with a decrease of ρ. For a fixed ρ, the best performance is observed for equally balanced average input SNRs. As expected, by comparing Figs. 6 and 7, the ABEP of a dual-branch MRC receiver

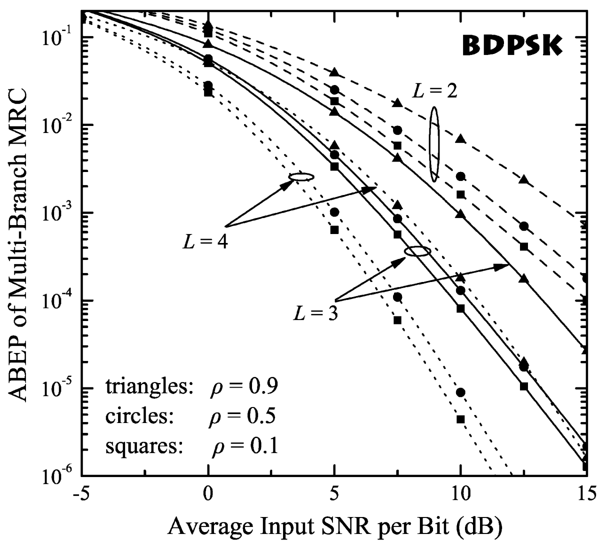

Fig. 8. ABEP of multibranch MRC as a function of the average input SNR for equal average branch SNRs, exponential correlation, and $\beta = 2.5$.

is better than SC. Additionally, by using (71), the BDPSK ABEP of multibranch MRC receivers with exponential correlation is also plotted, in Fig. 8, for $L = 2, 3$, and 4 i.d. input diversity branches and $\beta = 2.5$. As expected, as L increases, the ABEP of MRC receiver significantly improves.

VI. Conclusion

A theoretical framework for modeling fading channels with a Gaussian class of multivariate Weibull distributions as well as some indicative applications for wireless communications systems were presented. Novel analytical pdf, cdf, and mgf expressions for the bivariate Weibull distribution as well as the Weibull correlation coefficient were derived. Moreover, for the multivariate Weibull distributions with exponential and constant correlation matrixes, useful analytical formulas for the joint pdf, cdf, mgf, and product moments were obtained. The derived theoretical results were applied to analyze the performance of multibranch SC, EGC, and MRC receivers, operating in correlated Weibull fading environment. Several performance criteria, such as moments of the output SNR, average output SNR, AoF, and outage probability, were analytically derived, while the ASEP for several coherent and noncoherent modulation formats was also studied. Various numerically evaluated results were presented, showing the effects of fading severity as well as the correlation coefficient on the system's performance. It was shown, that only a few terms are needed, in order to accurately numerically evaluate formulas, which are in the form of infinite series representation.

Appendix I
Transformation Method of Meijer's G-Function to Generalized Hypergeometric Functions

Meijer's G-functions of the form

$$I = G^{\kappa,\lambda}_{\lambda,\kappa}\left[z \,\middle|\, \begin{matrix} a_1, a_2, \dots, a_\lambda \\ b_1, b_2, \dots, b_\kappa \end{matrix}\right] \tag{I-1}$$

with $\lambda > \kappa$ can be expressed in terms of more familiar generalized hypergeometric functions ${}_pF_q(\cdot;\cdot;\cdot)$ [38, eq. (9.14/1)] (with p and q being positive integers) following the next two steps.

Step 1: By using [38, eq. (9.31/2)], (I-1) can be written as

$$I = G^{\lambda,\kappa}_{\kappa,\lambda}\left[\frac{1}{z} \,\middle|\, \begin{matrix} 1-b_1, 1-b_2, \dots, 1-b_\kappa \\ 1-a_1, 1-a_2, \dots, 1-a_\lambda \end{matrix}\right]. \tag{I-2}$$

Step 2: By using [38, eq. (9.304)], (I-2) can be rewritten as

$$I = \sum_{w=1}^{\lambda} \prod_{\substack{j=1 \\ j\neq w}}^{\lambda} [\Gamma(a_w - a_j)] \prod_{j=1}^{\kappa} [\Gamma(1 + b_j - a_w)] \times z^{a_w - 1} {}_\kappa F_{\lambda-1}\left[\Lambda_w; \Delta_w; \frac{(-1)^\lambda}{z}\right] \tag{I-3}$$

where

$$\Lambda_w = 1 - a_w + b_1, 1 - a_w + b_2, \dots, 1 - a_w + b_\kappa$$

and

$$\Delta_w = 1 - a_w + a_1, \; 1 - a_w + a_2, \dots, 1 - a_w + a_{w-1}, \; 1 - a_w + a_{w+1}, \dots, 1 - a_w + a_\lambda.$$

Appendix II
The Bivariate Rayleigh Distribution

The pdf, cdf, and $(n+m)$th-order product moment of Rayleigh RVs R_1 and R_2 can be mathematically expressed as [1], [4]

$$f_{R_1,R_2}(r_1, r_2) = \frac{4 r_1 r_2}{\Omega_1 \Omega_2 (1-\rho)} \exp\left[-\frac{1}{1-\rho}\left(\frac{r_1^2}{\Omega_1} + \frac{r_2^2}{\Omega_2}\right)\right] \times I_0\left[\frac{2\sqrt{\rho} r_1 r_2}{(1-\rho)\sqrt{\Omega_1 \Omega_2}}\right] \tag{II-1}$$

$$F_{R_1,R_2}(r_1, r_2) = 1 - \exp\left(-\frac{r_1^2}{\Omega_1}\right) Q_1\left(\sqrt{\frac{2}{1-\rho}}\frac{r_2}{\sqrt{\Omega_2}}, \sqrt{\frac{2\rho}{1-\rho}}\frac{r_1}{\sqrt{\Omega_1}}\right) - \exp\left(-\frac{r_2^2}{\Omega_2}\right)\left[1 - Q_1\left(\sqrt{\frac{2\rho}{1-\rho}}\frac{r_2}{\sqrt{\Omega_2}}, \sqrt{\frac{2}{1-\rho}}\frac{r_1}{\sqrt{\Omega_1}}\right)\right] \tag{II-2}$$

and

$$\mathcal{E}\left\langle R_1^n R_2^m \right\rangle = (1-\rho)^{1+(n+m)/2} \Omega_1^{n/2} \Omega_2^{m/2} \Gamma\left(1 + \frac{n}{2}\right) \times \Gamma\left(1 + \frac{m}{2}\right) {}_2F_1\left(1 + \frac{n}{2}, 1 + \frac{m}{2}; 1; \rho\right) \tag{II-3}$$

respectively. In the above equations, $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind [38, eq. (8.406/1)], $Q_1(\cdot,\cdot)$ is the first-order Marcum's Q-function [1, eq. (4.33)], ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the Gauss hypergeometric function [38, eq. (9.100)], $\Omega_\ell = \mathcal{E}\langle R_\ell^2 \rangle$, and $0 \leq \rho < 1$ is the (Gaussian) power correlation coefficient between R_1^2 and R_2^2 defined as

$$\rho \triangleq \frac{\operatorname{cov}\left(R_1^2, R_2^2\right)}{\sqrt{\operatorname{var}\left(R_1^2\right)}\sqrt{\operatorname{var}\left(R_2^2\right)}}. \tag{II-4}$$

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