Dual-hop wireless communications with combined gain relays

T.A. Tsiftsis, G.K. Karagiannidis and S.A. Kotsopoulos

Abstract: A dual-hop relayed wireless communication system is presented where the gain of the relay, called combined gain relay (CGR), is produced after combining the channel state information from both hops, depending on the mean hop's signal-to-noise ratio (SNR). The proposed scheme can be efficiently applied in dual-hop transmissions with unbalanced mean SNRs due to the long-term fading effects produced by the movement of the user in the area served by the wireless network. The overall system performance is studied in Rayleigh fading channels. Closed-form expressions are derived for important system performance metrics, such as average end-to-end SNR, average error probability and outage probability. The CGR's average power consumption is investigated which in certain cases is lower compared with existed relays. Numerical results and simulations show an improvement in the end-to-end system performance.

1 Introduction

Multihop relaying technology is a promising solution for throughput and high data-rate coverage requirements in future cellular and *ad-hoc* wireless communication systems without the need to use large power at the transmitter, and to combat fading and shadowing through spatial/multiuser [1–4]. Nowadays, there is great interest in the research community on the potential of multihop and especially on dual-hop transmissions [4–8]. Dual-hop transmission can be classified into two main categories, regenerative or non-regenerative, depending on the relay type used. Nonregenerative relays just amplify and retransmit the information signal, as opposed to regenerative relaying nodes which decode the signal and then retransmit the detected version to the next node [5, 6]. Additionally, relays of nonregenerative systems are classified in two main subcategories, as channel state information (CSI)-assisted relays, where they use the CSI from the previous hop to produce their gain leading to a power control of the retaransmitted signal, and fixed-gain relays with lower complexity compared with CSI-assisted ones, and which introduce a fixed gain and thus a variable signal power at the output.

Looking through the recent open technical literature, the performance of dual-hop wireless communication systems is studied in [1, 5–8]. Hasna and Alouini have presented a useful and semianalytical framework for the evaluation of the end-to-end outage probability of multihop wirelsess systems with nonregenerative CSI-assisted relays over Nakagami-*m* fading channels [1]. Moreover, the same authors have studied the outage and the error performance of dual-hop systems with regenerative and nonregenerative CSI-assisted relays over Rayleigh [5] and Nakagami-*m* [6] fading channels. The analysis in [1, 5, 6] is based on an upper bound for the end-to-end signal-to-noise ratio (SNR) which leads to lower bounds for the system's outage and average error probability. This bound corresponds to an ideal relay capable of inverting the channel in the previous hop (regardless of the fading state of that hop) without limiting the output power. Furthermore, in [7] the end-to-end performance of dual-hop systems equipped with nonregenerative fixed-gain relays is investigated and a specific relay is proposed, called semiblind, that benefits from the knowledge of the first hop's average fading power. Tsiftsis *et al.* presented a new upper bound for the end-to-end SNR and efficiently evaluated the average error probability in dual-hop collaborative diversity systems, especially at low SNRs [8]. Anghel and Kaveh in [3] have studied the error performance of a co-operative network of dual-hop transmissions with parallel CSI-assisted relays in Rayleigh fading, where multiuser spatial diversity is used to combat the signal's impairments. Karagiannidis has studied the performance bounds for multihop relayed transmissions with blind (fixed-gain) relays over Nakagami-*n* (Rice), Nakagami-*q* (Hoyt) and Nakagami-*m* fading channels [4].

In this paper we present a dual-hop transmission system, where the gain of the relay, called combined gain relay (CGR), is produced using CSI from both hops, depending on the mean hop's SNR. The proposed scheme can be efficiently used in dual-hop wireless transmissions with unbalanced mean SNRs between the hops due to the long-term fading effects produced by the movement of the user in the area served by the wireless network. The overall system performance is studied in Rayleigh fading channels as follows: Closed-form expressions for the moments, the average bit error probability (ABEP) and the outage probability of the end-to-end SNR are derived. We study the average power consumed by the CGR and it is shown that in certain cases it is less compared with existing relays. Moreover, numerical examples and Monte Carlo simulations show that CGR results in a significant improvement in the end-to-end system performance, compared with existing gain relay schemes.

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2 Combined gain relay scheme

2.1 System and channel model

A dual-hop wireless communication system with a non-regenerative relaly operating over independent but not necessarily identically distributed Rayleigh fading channels is shown in Fig. 1. The source terminal S communicates with the destination terminal D through the terminal R which acts as relay. Assuming that terminal S transmits a signal with an average power normalized to unity, the received signal at terminal R can be written as

$$r_R = \alpha_1 s + n_1 \tag{1}$$

where α_1 is the fading amplitude of the channel between terminals S and R (first hop), modelled as a Rayleigh random variable and following the probability density function (PDF)

$$f_{\alpha_i}(\alpha_i) = \frac{2\alpha_i}{\Omega_i}\exp\left(-\frac{\alpha_i^2}{\Omega_i}\right), \quad i = 1, 2 \tag{2}$$

where $\Omega_i = \overline{\alpha_i^2}$ is the averaging fading power on the ith hop and n_1 is the additive white Gaussian noise (AWGN) with single-sided power spectral density (PSD) N_0. The signal r_R is then multiplied by the gain g of terminal R and retransmitted to terminal D. The received signal at terminal D can be written as

$$r_D = g\alpha_2(\alpha_1 s + n_1) + n_2 \tag{3}$$

where α_2 is the fading amplitude of the channel between R and D (second hop), following the PDF in (2) and n_2 is the AWGN with single-sided PSD N_0. We have omitted the time index in (1) and (3) for brevity. Using (3) the overall instantaneous SNR at the receiving end can be written as

$$\gamma_{end} = \frac{[\alpha_2 g \alpha_1]^2}{\left[(\alpha_2 g)^2 + 1\right]N_0} = \frac{\frac{\alpha_1^2}{N_0}\frac{\alpha_2^2}{N_0}}{\frac{\alpha_2^2}{N_0} + \frac{1}{g^2 N_0}} \tag{4}$$

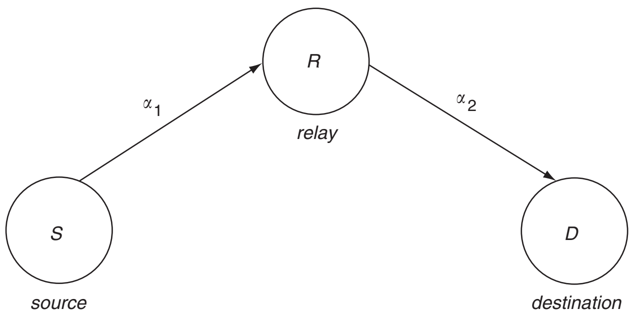

Fig. 1 *Dual-hop wireless communication system*

2.2 Mode of operation

When terminal R has available CSI from the first hop, one kind of gain relay proposed and studied in previously published works [9, 10] is given by

$$g_1^2 = \frac{1}{\alpha_1^2 + N_0} \tag{5}$$

The choice of this gain aims to limit the output power of the relay if the fading amplitude of the first channel α_1 is low.

Next we propose an alternative mode of operation for the relay-node R, called combined gain relay (CGR), as follows:

Step 1: Periodically, and in synchronisation with terminal D, terminal R estimates the average SNR of the first hop $\bar{\gamma}_1$ and terminal D estimates the average SNR of the second hop $\bar{\gamma}_2$ and sends it to terminal R.

Step 2: Terminal R generates the new gain relay, according to the following rule:

$$g^2 = \begin{cases} g_1^2 : 1/(\alpha_1^2 + N_0), & \text{if } \xi < \xi_{th} \quad (6) \\ g_2^2 : 1/(\alpha_2^2 + N_0), & \text{if } \xi > \xi_{th} \quad (7) \end{cases}$$

where $\xi = \bar{\gamma}_1/\bar{\gamma}_2$ denotes the degree of the average SNR unbalance. The CGR mode of operation is schematically depicted in Fig. 2. The parameter ξ_{th} is the threshold which signals the transition between the two available gains and depends on the performance criterion under consideration (i.e. either the average end-to-end SNR or the error probability or the outage probability). The way to choose values for ξ_{th} is discussed in Section 5. Note that the choice of g_2 does not limit the instantaneous output power of the relay.

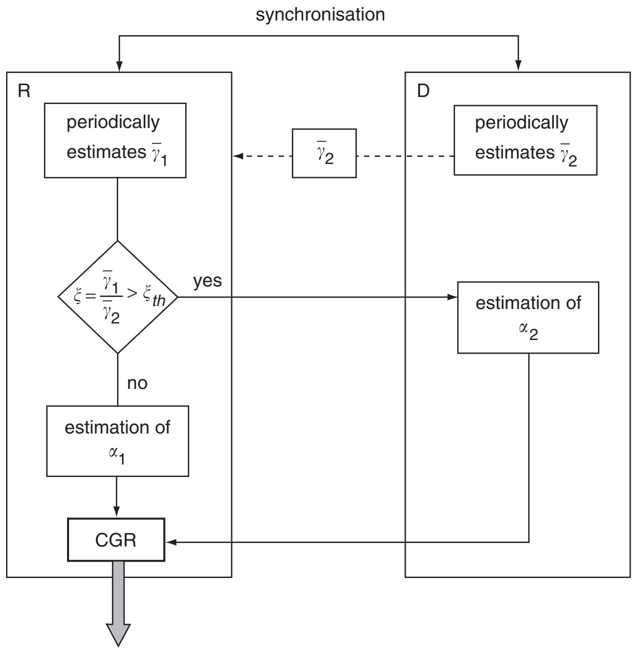

Fig. 2 *CGR mode of operation*

The ability of relay R to generate the appropriate gain, depending on the fading conditions in both hops, leads to an end-to-end SNR, which using (4) with (6) and (7) is formulated as

$$\gamma_{end} = \begin{cases} \gamma_1\gamma_2/(\gamma_1 + \gamma_2 + 1), & \text{if } \xi < \xi_{th} \quad (8) \\ \gamma_1\gamma_2/(2\gamma_2 + 1), & \text{if } \xi > \xi_{th} \quad (9) \end{cases}$$

where γ_i is the instantaneous SNR of the ith hop, following the exponential PDF defined as

$$f_{\gamma_i}(\gamma_i) = \frac{1}{\bar{\gamma}_i}\exp\left(-\frac{\gamma_i}{\bar{\gamma}_i}\right), \quad i = 1, 2 \tag{10}$$

As mentioned in Section 1 the performance analysis of dual-hop systems with relay given by (6) has been extensively studied in the literature [1, 3, 5, 6, 8, 10]. Thus, in the following, only the case of $g = g_2$ is studied further.

3 Performance analysis

3.1 Moments of end-to-end SNR

The first- and the second-order moments of the end-to-end SNR are statistical parameters which can be efficiently used to evaluate important performance system measures, such as average output SNR and variance. The higher-order moments (higher than the second) are also useful in signal processing algorithms for signal detection, classification and estimation and they play a fundamental role in understanding the performance of wideband communication systems in the presence of fading [11].

By definition the nth moment of $\gamma_{\rm end}$ for $g = g_2$ is given by

$$E\langle \gamma_{end}^n \rangle = \int_0^\infty \int_0^\infty \left(\frac{\gamma_1 \gamma_2}{2\gamma_2 + 1} \right)^n f_{\gamma_1}(\gamma_1) f_{\gamma 2}(\gamma_2) d\gamma_1 d\gamma_2 \quad (11)$$

where $E\langle\cdot\rangle$ denotes expectation and using (10) into (11), $E\langle \gamma_{end}^n \rangle$ can be written as

$$E\langle \gamma_{end}^n \rangle = \frac{1}{\bar{\gamma}_1 \bar{\gamma}_2} \int_0^\infty \gamma_1^n \exp\left(-\frac{\gamma_1}{\bar{\gamma}_1} \right) d\gamma_1 \times \int_0^\infty \left(\frac{\gamma_2}{2\gamma_2 + 1} \right)^n \exp\left(-\frac{\gamma_2}{\bar{\gamma}_2} \right) d\gamma_2 \quad (12)$$

where the solution of (12) is given by (see Appendix, Section 8)

$$E\langle \gamma_{end}^n \rangle = \frac{(n!)^2}{2n} \bar{\gamma}_1^n U\left(n, 0, \frac{1}{2\bar{\gamma}_2} \right) \quad (13)$$

where U $(\cdot,\cdot,\cdot)$ is the Kummer's function defined in [12, eqn. 13.1.3]

3.1.1 Average end-to-end SNR:

Using (13) for $n = 1$, the average end-to-end SNR can be obtained as

$$\bar{\gamma}_{end} = \frac{\bar{\gamma}_1}{2} \exp\left(\frac{1}{2\bar{\gamma}_2} \right) E_2\left(\frac{1}{2\bar{\gamma}_2} \right) \quad (14)$$

where $E_k(\cdot)$ is the exponential integral [13, eqn. 5.1.4] with k being a positive integer.

3.2 Average symbol error probability (ASEP)

The error performance for several digital modulation schemes can be efficiently studied using the well-known moment generating function (MGF)-based approach [14]. The MGF, defined here as

$$\mathcal{M}_{\gamma_{end}}(s) \triangleq E\langle \exp(-s\gamma_{end}) \rangle \quad (15)$$

can be evaluated in closed-form solving the following double integral:

$$\mathcal{M}_{\gamma_{end}}(s) = \int_0^\infty \int_0^\infty \exp\left(-s \frac{\gamma_1 \gamma_2}{2\gamma_2 + 1} \right) f_{\gamma_1}(\gamma_1) f_{\gamma_2}(\gamma_2) d\gamma_1 d\gamma_2 \quad (16)$$

Substituting $f_{\gamma_i}(\gamma_i)$ into (16) yields

$$\mathcal{M}_{\gamma_{end}}(s) = \frac{1}{\bar{\gamma}_1 \bar{\gamma}_2} \int_0^\infty \int_0^\infty \exp\left(-s \frac{\gamma_1 \gamma_2}{2\gamma_2 + 1} \right) \times \exp\left(-\frac{\gamma_1}{\bar{\gamma}_1} \right) \exp\left(-\frac{\gamma_2}{\bar{\gamma}_2} \right) d\gamma_1 d\gamma_2 \quad (17)$$

The first integral in (17) (i.e. the one on γ_1) is of the form

$$\mathcal{I}_1 = \frac{1}{\bar{\gamma}_1 \bar{\gamma}_2} \int_0^\infty \exp\left(-s \frac{\gamma_1 \gamma_2}{2\gamma_2 + 1} \right) \exp\left(-\frac{\gamma_1}{\bar{\gamma}_2} \right) d\gamma_1 \quad (18)$$

which can be solved using [13, eqn. 3.310] as

$$\mathcal{I}_1 = \frac{1}{\bar{\gamma}_2} \frac{2\gamma_2 + 1}{s\gamma_2 \bar{\gamma}_1 + 2\gamma_2 + 1} \quad (19)$$

The second integral in (17) (i.e. the one or γ_2) can now be written using (19) as

$$\mathcal{I}_2 = \frac{1}{\bar{\gamma}_2} \int_0^\infty \frac{2\gamma_2 + 1}{s\gamma_2 \bar{\gamma}_1 + 2\gamma_2 + 1} \exp\left(-\frac{\gamma_2}{\bar{\gamma}_2} \right) d\gamma_2 \quad (20)$$

Using [13, eqns. 3.352.4 and 3.353.5] and [12, eqn. 5.1.45], (20) yields

$$\mathcal{M}_{\gamma_{end}}(s) = \frac{2}{2 + s\bar{\gamma}_1} + \frac{s\bar{\gamma}_1}{\bar{\gamma}_2 (2 + s\bar{\gamma}_1)^2} \times \Gamma\left(0, \frac{1}{2\bar{\gamma}_2 + s\bar{\gamma}_1 \bar{\gamma}_2} \right) \exp\left(\frac{1}{2\bar{\gamma}_2 + s\bar{\gamma}_1 \bar{\gamma}_2} \right) \quad (21)$$

where $\Gamma(x, y)$ is the incomplete gamma function, defined as $\Gamma(x, y) = \int_y^\infty t^{x-1} e^{-t} dt$.

With the aid of $\mathcal{M}_{\gamma_{end}}(s)$, and using the MGF-based approach for the performance evaluation of digital modulations over fading channels presented in [14], the error rates can be calculated directly for noncoherent binary signalling, such as BFSK and DPSK, while for other cases including M-QAM and M-PSK single integrals with finite limits and integrands composed of elementary functions have to be readily evaluated via numerical integration.

For example, the ABEP of DPSK is given by [14, eqn. 8.201] $\bar{P}_b(E) = \frac{1}{2}\mathcal{M}_{\gamma_{end}}(1)$, which for the CGR can be written in closed-form, using (21), as

$$\bar{P}_b(E) = \frac{1}{2 + \bar{\gamma}_1} + \frac{\bar{\gamma}_1}{2\bar{\gamma}_2 (2 + \bar{\gamma}_1)^2} \times \Gamma\left(0, \frac{1}{2\bar{\gamma}_2 + \bar{\gamma}_1 \bar{\gamma}_2} \right) \exp\left(\frac{1}{2\bar{\gamma}_2 + \bar{\gamma}_1 \bar{\gamma}_2} \right) \quad (22)$$

For the case of BPSK modulation scheme which is used as an example in Section 5, the ABEP can be evaluated via numerical integration as [14, eqn. 9.12]

$$\bar{P}_b(E) = \frac{1}{\pi} \int_0^{\pi/2} \mathcal{M}_{\gamma_{end}}\left(-\frac{1}{\sin^2 \phi} \right) d\phi \quad (23)$$

using any of the well-known mathematical software packages, such as Mathematica or Maple.

3.3 Outage probability

If γ_{th} is a certain specified threshold ratio, then for nonregenerative multihop transmissions the outage probability is defined as the probability that the instantaneous SNR at the final destination falls below γ_{th} and is expressed as

$$P_{out} = \Pr[\gamma_{end} \le \gamma_{th}] = \Pr\left[\frac{\gamma_1 \gamma_2}{2\gamma_2 + 1} \le \gamma_{th} \right] \quad (24)$$

which can be evaluated as

$$P_{out} = \int_0^\infty \Pr\left[\frac{\gamma_1 \gamma_2}{2\gamma_2 + 1} \le \gamma_{th} | \gamma_2 \right] f_{\gamma_2}(\gamma_2) d\gamma_2 \quad (25)$$

Following the same method as in [15] the integral yields

$$P_{out} = 1 - 2\sqrt{\frac{\gamma_{th}}{\bar{\gamma}_1 \bar{\gamma}_2}} \exp\left(-\frac{2\gamma_{th}}{\bar{\gamma}_1} \right) K_1\left(2\sqrt{\frac{\gamma_{th}}{\bar{\gamma}_1 \bar{\gamma}_2}} \right) \quad (26)$$

where $K_1(\cdot)$ is the first-order modified Bessel function of the second kind defined in [12, eqn. 9.6.22].

4 Average power consumption

We quantify the average gain of the relay (i.e. average power consumption) when CGR is considered. If τ is the percentage of the time when g_2 is used ($1-\tau$ corresponds to g_1) then the average power consumed by the relay $\bar{P}_{CGR}$ can be expressed by

$$\bar{P}_{CGR} = \tau\bar{P}_2 + (1-\tau)\bar{P}_1 \quad (27)$$

where $\bar{P}_1$ and $\bar{P}_2$ are the average power consumed by the relay when g_1 or g_2 are used, respectively. For Rayleigh fading channels $\bar{P}_1$ and $\bar{P}_2$ can be written as [7, eqn. 15]

$$\bar{P}_i = E\langle g_i^2\rangle = \frac{e^{1/\bar{\gamma}_i}\Gamma(0, 1/\bar{\gamma}_i)}{\bar{\gamma}_i N_0} \quad (28)$$

5 Numerical results

We provide several representative numerical and simulation examples illustrating the performance of the dual-hop system with CGR over Rayleigh fading channels. These results are compared with a system that uses only the gain relay given by (6).

The threshold ξ_{th} for the transition from g_1 to g_2 depends on the selected performance criterion and it can be determined after equating the formulas relating to this criterion of the gains g_1 and g_2 and solving numerically with respect to ξ. When the criterion considered is the outage probability, then ξ_{th} can be determined by equating (26) to [15, eqn. 14] and solving numerically for ξ_{th}. In Table 1, ξ_{th} is evaluated for several values of $\bar{\gamma}_1$. It can be easily verified that almost the same values for ξ_{th} are observed when other performance criteria as the average end-to-end SNR or ABEP are used.

Table 1: Evaluation of ξ_{th} for several values of $\bar{\gamma}_1$

$\bar{\gamma}_1$ (dB)	ξ_{th}
−5	0.6
0	0.54
5	0.58
10	0.68
15	0.84

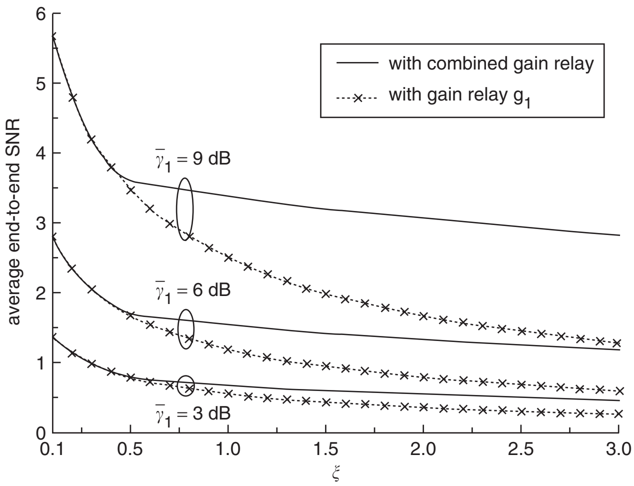

Fig. 3 *Comparison of average end-to-end SNR of dual-hop system with relay gain g_1 and CGR against ξ for several values of $\bar{\gamma}_1$*

In Fig. 3 the average end-to-end SNR for a dual-hop system with CGR is plotted against ξ for several values of mean SNR $\bar{\gamma}_1$. Monte Carlo simulations are also performed for a system using a relay with the gain g_1 and their results are depicted in the same Figure. We observe that depending on the value of ξ, the CGR improves the overall average end-to-end SNR performance. Similar conclusions are also extracted Figs. 4 and 5, where the BPSK average error rate performance and the outage probability are plotted against ξ for several values of $\bar{\gamma}_1$, respectively.

Finally, in Fig. 6 the excess average power consumption expressed by the ratio $\bar{P}_{CGR}/\bar{P}_1$ is evaluated for several values of ξ. It is interesting to observe that when $0.65 \leq \xi \leq 1$ (grey region), although the average power consumption of CGR is less than $\bar{P}_1$ (i.e. $\bar{P}_{CGR}/\bar{P}_1 < 1$), the system's performance is improved. Moreover, as expected, the average power consumption of the CGR is increased when $\xi > 1$. However, by substituting (28) in (27) and using [12, 6.5.31], it can easily verified that $\lim_{\xi\to\infty} \bar{P}_{CGR} = 1/N_0$.

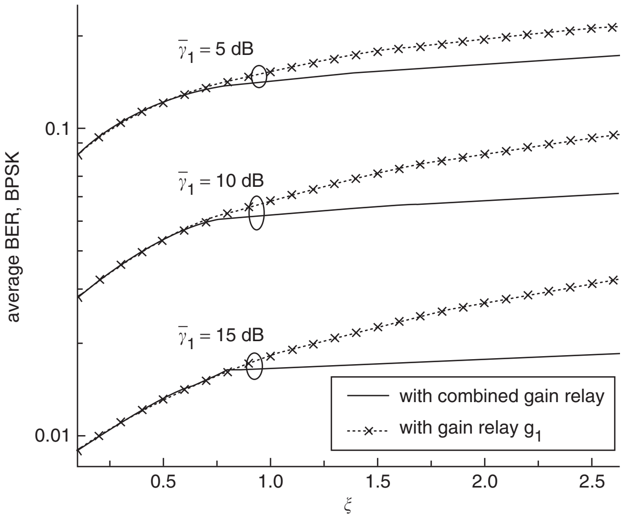

Fig. 4 *Comparison of BPSK average error performance of dual-hop system with relay gain g_i and CGR against ξ for several values of $\bar{\gamma}_1$*

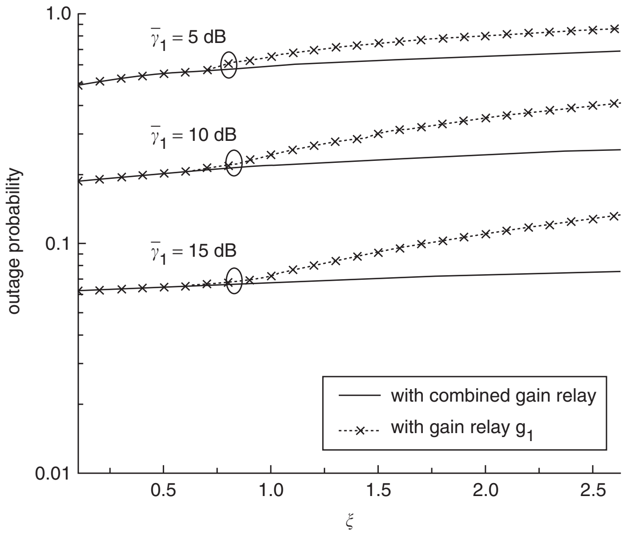

Fig. 5 *Comparison of end-to-end outage probability of dual-hop system with relay gain g_1 and CGR against ξ for several values of $\bar{\gamma}_1$*

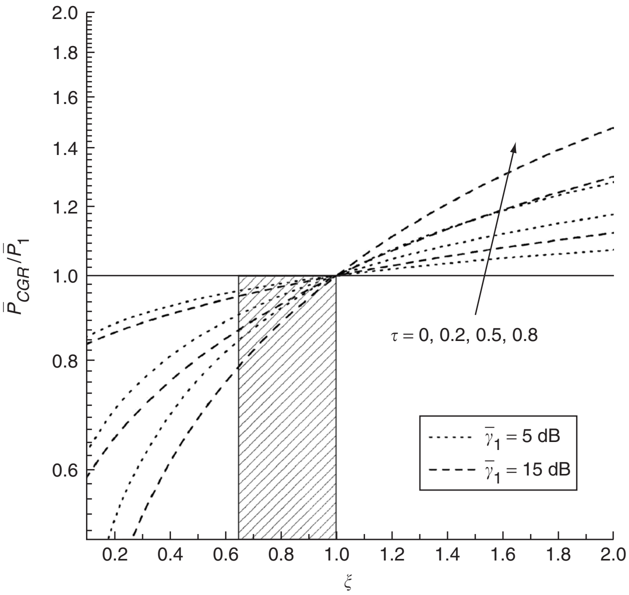

Fig. 6 *Ratio of average power consumption against ξ for several values of τ*

6 Conclusions

A dual-hop wireless communication system with a novel combined gain relay, has been presented. The new type of relay produced a gain using CSI from both hops, depending on the mean hop's SNR. This approach results to an improved end-to-end system performance compared with existing gain relays, but with an increase in average power consumption. However, it was shown that in certain cases, the average power consumed by the CGR is less compared with existing relays. Closed-form expressions for the moments, the average error probability and the outage probability of the end-to-end SNR were also derived.

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8 Appendix

8.1 Evaluation of integral in (12)

The first integral in (12) (i.e. the one on γ_1) can be solved using [13, eqn. 3.326.2] as

$$I_1 = \int_0^\infty \gamma_1^n \exp\left(-\frac{\gamma_1}{\bar{\gamma}_1}\right) d\gamma_1 = n!\bar{\gamma}_1^{n+1} \tag{29}$$

The second integral in (12) (i.e. the one on γ_2) can be written using [16, eqns. 10 and 11] as

$$I_2 \frac{1}{(n-1)!} = \int_0^\infty \gamma_2^n G_{1,1}^{1,1}\left(2\gamma_2 \middle| \begin{matrix} 1-n \\ 0 \end{matrix}\right) G_{0,1}^{1,0}\left(\frac{1}{\bar{\gamma}_2}\gamma_2 \middle| \begin{matrix} - \\ 0 \end{matrix}\right) d\gamma_2 \tag{30}$$

where G $(\cdot)$ is the Meijer's G-function, defined in [13, eqn. 9.301].

Integrals of the form of (30) can be evaluated in closed-form using [16, eqn. 21] as

$$I_2 = \frac{1}{(n-1)!}\bar{\gamma}_2^{n+1} G_{2,1}^{1,2}\left(2\bar{\gamma}_2 \middle| \begin{matrix} 1-n, n \\ 0 \end{matrix}\right) \tag{31}$$

Substituting (31) and (29) in (12) and using [17], the moments of the end-to-end SNR can be expressed in a simple closed-form as (13).