Error performance for equal-gain combiners over Rayleigh fading channels

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An alternative approach for the evaluation of error probability (ERRP) in L-branch BPSK equal-gain combiners in the presence of Rayleigh fading is presented. The ERRP is evaluated directly, via a simple closed form expression, using the properties of the characteristic function of a random variable. Results and comparison with other existing techniques are also presented.

Introduction: Diversity is one of the most commonly used techniques in mobile radio for combatting signal fading. Moreover, equal-gain combining (EGC) has been accepted as an attractive diversity tool due to its improved error performance and the simplicity of its implementation [1]. Two effective techniques have been proposed for the evaluation of error probability (ERRP) in equal-gain combiners in Rayleigh fading channels. The first such useful approach was made by Beaulieu [2] where the ERRP was obtained through the determination of the probability density function (PDF) of the sum of L Nakagami signals arriving in the L branches of the combiner and Rayleigh fading was treated as a special case. The proposed closed forms contain two kinds of confluent hypergeometric functions. Zhang [3, 4] presented a simpler approach, and the ERRP for coherent and non-coherent modulation schemes was derived directly from the characteristic function (CHF) of the decision variable using a special lemma (Gil-Palaez) and avoiding the lengthy process of deriving the PDF of the sum of L Rayleigh variates. This is an important result since the proposed closed formulation is much simpler than formulations in previously published techniques, but it also needs the calculation of a confluent hypergeometric function. In this Letter an alternative approach for the evaluation of ERRP in BPSK systems with EGC diversity and additive uncorrelated white noise is proposed. The ERRP is evaluated directly using the definition and the properties of the characteristic function of a random variable and a final closed formula is derived. This formula is very simple, can be evaluated using the Hermite numerical integration method and does not require any complicated calculations (such as the confluent hypergeometric functions in previous techniques).

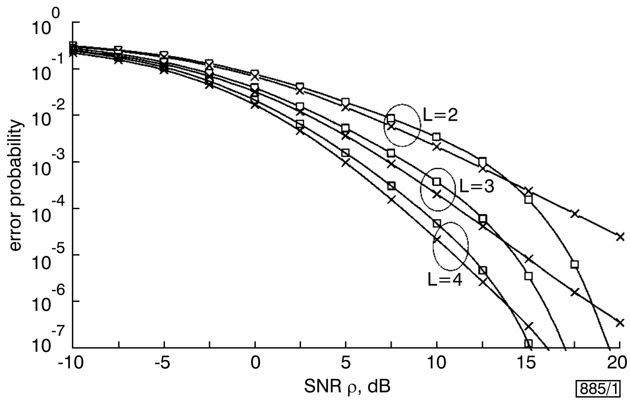

Fig. 1 *ERRP against SNR ρ at reference branch using Zhang's technique, and approach proposed in this Letter*

□ Zhang's technique
+ new approach

System model and error performance analysis: In an ECG system with coherent detection the signals received in each branch are co-phased, summed and coherently demodulated. The decision variable γ for a coherent BPSK can be formulated as [5]

$$\gamma = \pm \sum_{k=1}^{L} x_k + \sum_{k=1}^{L} w_k \qquad (1)$$

where x_k is the output signal amplitude at the kth branch and w_k represents the complex Gaussian noise at the kth branch with zero mean and variance N_k. It is assumed that x_k remains constant within a symbol duration but changes from symbol to symbol following a Rayleigh distribution with PDF given by

$$p_{x_k}(t) = \frac{2\cdot t}{\Omega_k}\cdot \exp\left(-\frac{t^2}{\Omega_k}\right) \qquad (2)$$

with Ωk being the average signal energy. Let $\Phi_\gamma(s)$, $\Phi_{x_k}(s)$ and $\Phi_{w_k}(s)$ be the CHF of parameters γ, x_k and w_k, respectively. Then from eqn. 1 and due to the independence between the desired signals and noise in each branch, $\Phi_\gamma(s)$ can be written as

$$\Phi_\gamma(s) = \prod_{k=1}^{L} \Phi_{x_k}(s)\cdot \Phi_{w_k}(s) \qquad (3)$$

or from the definition of the CHF

$$\Phi_\gamma(s) = \prod_{i=1}^{L} \Phi_{w_k}(s)\cdot \prod_{k=1}^{L}\int_0^\infty \exp(jst_k)\cdot p_{x_k}(t_k)dt_k \qquad (4)$$

Following the same analysis as in [3, 4] the average bit energy to average noise density ratio (SNR) in the kth branch is defined as

$$\rho_k = \frac{\Omega_k}{\eta_L/L} \qquad (5)$$

with η_L being the total (in all branches) power of the Gaussian noise. The CHF of the Gaussian variate w_k is well known and can be expressed as

$$\Phi_{w_k}(s) = \exp\left(-\frac{N_k}{4}\cdot s^2\right) \qquad (6)$$

which is the CHF of a Gaussian PDF with zero mean and variance $N_\kappa/2$. Hence, from eqn. 4 and after the transformation $t_k = \sqrt{\Omega_k}\cdot r_k$, $\Phi_\gamma(s)$ can be written as

$$\Phi_\gamma(s) = \Phi_{NORM(0,\eta_L/2)}(s)\int_0^\infty\cdots\int_0^\infty \exp\left(-js\sum_{k=1}^{L} - \sqrt{\Omega_k}r_k\right) \cdot 2^L \prod_{k=1}^{L} r_k \exp\left(-\sum_{k=1}^{L} r_k^2\right) dr_1\cdots dr_L \qquad (7)$$

where $\Phi_{NORM(0,\eta_L}/2)$ is the CHF of a normal distribution with zero mean and variance $\eta_L/2$. The ERRP for coherent BPSK detection is defined as [4, 5]

$$P_e = \mathrm{Prob}(\gamma < 0) \qquad (8)$$

or

$$P_e = \int_{-\infty}^{0} p_\gamma(\tau)d\tau = \frac{1}{2\pi}\cdot\int_{-\infty}^{0}\int_{-\infty}^{\infty}\Phi_\gamma(s)\cdot\exp(-js\tau)ds d\tau \qquad (9)$$

where τ is another auxiliary variable and $p_\gamma(\tau)$ is the PDF of parameter γ.

Now, using eqns. 5, 7 and 9 and taking into account the fact that by definition

$$\int_{-\infty}^{\infty}\Phi_{NORM(0,\eta_L/2)}(s)\cdot\exp\left[-js\left(\tau + \sum_{k=1}^{L} - \sqrt{\Omega_k}\cdot r_k\right)\right]ds = 2\pi\cdot p_{NORM(0,\eta_L/2)}\left(\tau + \sum_{k=1}^{L} - \sqrt{\Omega_k}\cdot r_k\right) \qquad (10)$$

the ERRP can be written after a straightforward procedure as

$$P_e = 2^L\cdot\int_0^\infty\cdots\int_0^\infty P_{NORM(0,1/2)}\left(-\sum_{k=1}^{L}\sqrt{\frac{\rho_k}{L}}\cdot r_k\right) \cdot\prod_{k=1}^{L} r_k\cdot\exp\left(-\sum_{k=1}^{L} r_k^2\right) dr_1\cdots dr_L \qquad (11)$$

which is independent of η_L. $P_{NORM(0,1/2)}(x)$ is the well known normal cumulative distribution function (CDF) with zero mean and variance 1/2. Eqn. 11 involves L integrals for L diversity branches and its second part can be calculated numerically with high desired accuracy using the Hermite numerical integration method [6]. The proposed closed formulation for the ERRP is simpler than previous formulations since it needs the calculation of the well-known normal CDF avoiding the evaluation of complex functions (i.e. confluent hypergeometric). The disadvantage of lengthy computation time arises only when the number of branches is greater than four, but it is counterbalanced by the available accuracy. Moreover, as is shown below and has also been mentioned in [4], higher order diversity values do not affect semantically the ERRP. Hence, three or four antennas give sufficient ERRP since more antennas decrease the applicability due to the high implementational cost. It must be noted here that for the case of perfect coherent detection and no co-channel interference, the ERRP performance of BPSK is identical to that of 4PSK [5]. In the case of coherent BFSK the proposed formulation is also valid when ρ_k is replaced by $\rho_k/2$ in eqn. 11. Moreover, the new proposed approach can also be used perfectly for non-coherent DPSK detection. Analysis and the corresponding results of this case are not presented, due to space limitations in this Letter.

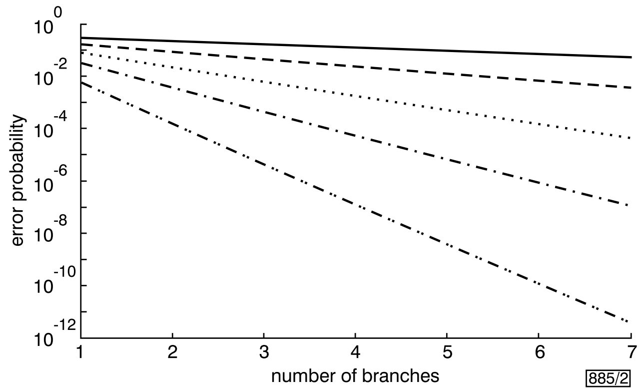

Fig. 2 *ERRP against order of diversity for several values of SNR* ρ, *using approach proposed in this Letter*

——— ρ = –5dB
– – – – ρ = 0dB
········ ρ = 5dB
–·–·–·–·ρ = 10dB
–··–··–·ρ = 15dB

Comparison and results: In Fig. 1 the ERRP for coherent BPSK is depicted as a function of the SNR at the first branch assuming that the signals arriving at each branch have different levels. In this case the SNR at each branch was measured having as a reference point the signal's level in the first branch. The techniques used were that of Zhang [4] and the technique proposed in this Letter. The number of diversity branches was two, three and four. The signal levels for several orders of diversity were assumed to be: [L = 2, ρ = ρdB + (0, 0)], [L = 3, ρ = ρdB + (0, –2, +2)], [L = 4, ρ = ρdB + (0, 0, –1, +1)]. The same Hermite integration order (20) was used in both techniques. The difference in the results between the two techniques ranges from 0.00001 for higher values of SNR to 0.01 for low values of SNR. This difference arises from the converging accuracy of the confluent hypergeometric function, which needs to be calculated in Zhang's method. Moreover, the number of terms which need to be summed there vary depending on the order of diversity and the signal level. This difficulty does not exist in the technique proposed in this Letter, since the calculation of the normal CDF is independent of the number of EGC branches and the signal level. In Fig. 2, the ERRP is depicted in relation to the number of antennas used for several values of SNR. Here, it was assumed that all the branches had the same signal level. It is apparent that the ERRP increases significantly with a diversity order of 2–4 and gradually decreases for higher orders of diversity. These are very useful curves for helping designers to make decisions about the order of diversity taking into account the quality of service (QoS) demands.

Conclusions: A new flexible approach for the evaluation of the error probability in L-branch BPSK equal gain combiners in Rayleigh fading channels has been presented and analysed. The obtained results demonstrate the significance of this approach compared to existing techniques.

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