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Over-the-air Computing in OFDM Systems

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***Abstract*—In next-generation networks, computing and related applications are expected to be enabled by communications. To accommodate a large number of devices and achieve better resource utilization, over-the-air (OTA) computing has been proposed as an attractive scheme to enable efficient computing. As such, OTA computing could be a candidate for computing applications that are susceptible to frequency-selective fading channels. In current communication systems, orthogonal frequency division multiplexing (OFDM) is the most common approach for dealing with such fading conditions, thus it is important to investigate whether a similar approach can be used for OTA computing. With this in mind, we model an orthogonal frequency domain (OFD) multi-user system where the users utilize OTA computing in each subcarrier. To account for practical scenarios, we study the presence of inter-carrier interference (ICI) among the subcarriers, and an optimization problem is formulated and solved to extract the optimal power allocation policy for the proposed OFD OTA computing system.**

***Index Terms*—over-the-air computing, orthogonal frequency division multiplexing, resource allocation, 6G**

I. Introduction

The next-generation communication systems are expected to enable many new applications, especially in the context of IoT networks [1]. The latter typically consist of a large number of devices, and in many cases data aggregation is of interest rather than individual knowledge of each device's data. In this regard, over-the-air (OTA) computing has been proposed as an interesting alternative that leverages the superposition principle of multiple access channels (MACs) to achieve wireless data aggregation [2]. Based on superposition and the nomographic form of multivariate functions, OTA computing was facilitated in [3] as an effective way to approximate any multivariate function, while in the seminal work [4], it was proved that analog transmission is optimal for OTA computing. With this in mind, the authors in [5] extracted the optimal power allocation scheme when perfect channel state information (CSI) is available at the fusion center (FC) to optimize the accuracy of OTA computing, measured by the mean squared error (MSE) between the ideal and the received signal, while in [6], the effects of imperfect CSI were investigated and a channel re-estimation approach was proposed to improve the MSE.

The distributed nature of OTA computing has also attracted considerable attention as a means to enable distributed learning techniques, such as federated learning (FL) [7]. Indeed, OTA computing has been shown to be an effective method for such techniques, where a global update of parameters is performed by simultaneously transmitting multiple distributed calculated parameters. As demonstrated in [8], OTA computing can ensure the convergence of FL models even when the number of transmitting devices varies from time to time. Apart from that, the resource efficiency of OTA computing has also made it a point of research in cooperation with other advancing technologies. For this scope, in [9], [10], multiple-input multiple-output (MIMO) systems have been explored as a method that can allow OTA computing to perform simultaneous multi-function computing as well as different tasks such as integrated sensing and computing.

However, many applications related to computing, such as autonomous driving, are expected to operate in frequency-selective fading conditions that cause intersymbol interference (ISI). In order to mitigate the effect of the latter, an optimal power allocation scheme was extracted and a deep neural network (DNN) was proposed for the generation of a new optimal waveform in [11]. To further counter ISI, orthogonal frequency division multiplexing (OFDM) is mostly implemented by modern systems. In this direction, in [12], an OFDM model for OTA computing under imperfect CSI conditions was studied and optimal power policies were extracted. However, to the authors' knowledge, no work has investigated the effect of inherent problems such as inter-carrier interference (ICI) in such models. With this in mind, we present an orthogonal frequency domain (OFD) OTA computing system that incorporates frequency errors and extract a form for the received signal that integrates ICI factors due to the multiple subcarriers of the system. Based on the expression for the latter, an optimization problem is formulated and solved by alternating optimization and using Slater's condition to determine the optimal power allocation policy of the studied system. Finally, simulation results are provided to illustrate the performance gain of the proposed policy over other state-of-the-art techniques, thus demonstrating the significance of the present work.

II. System Model

We consider an OTA computing system consisting of a receiver, which acts as an FC, and multiple transmitting devices. Let K be the number of transmitting devices in the system, where the data of each of them is independent between them. We define the set of all devices $\mathcal{K} = \{1, \cdots, K\}$, where the order of the devices is taken with respect to the order of their respective channel gains. For the computing part, we assume that we want to calculate a function $f : \mathbb{R}^K \to \mathbb{R}$ of all transmitted data, denoted as $f(x_1, x_2, \cdots, x_K)$. If f is a nomographic function of the data, there are suitable pre-processing functions $\varphi_k : \mathbb{R} \to \mathbb{R}$, $\forall k \in \{1, \cdots, K\}$ and

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a post-processing function $\psi : \mathbb{R} \rightarrow \mathbb{R}$ such that the target function f can be expressed as

$$f(x_1, x_2, \cdots, x_k) = \psi\left(\sum_{k=1}^{K} \varphi_k(x_k)\right), \quad (1)$$

where x_k denotes the sample of the k-th device.

To mitigate the effect of frequency-selective channels, modern communication systems utilize multi-carrier transmission schemes. The most efficient of these schemes is OFDM due to the better spectrum utilization it achieves as a result of the overlapping subcarriers it uses. Because of its efficiency, an OFD OTA system can be an interesting practical implementation. As in current digital OFDM systems, in the OFD OTA computing system, each device in $\mathcal{K}$ can utilize a set of subcarriers $\mathcal{L} = \{0, \cdots, L-1\}$ to transmit data for aggregation per subcarrier. To implement OFD OTA, we assume that each device in $\mathcal{K}$ is subject to selective fading and that the associated delay spread caused by the latter is such that the wireless channel results in interference between μ symbols. Therefore, the channel of each device can be considered as a finite impulse response (FIR) filter $h_k[n],\ 0 \leq n \leq \mu$ of length $\mu + 1$. Due to the number of subcarriers, each device has a set of L symbols to transmit, which are denoted as $\boldsymbol{X}_k = [X_k[0], \cdots, X_k[L-1]]$, however for the OFD symbol of the k-th device to counter ISI, an additional amount of μ symbols is required due to the cyclic prefix (CP). Therefore, the transmitted symbol is $\tilde{\boldsymbol{X}}_k = [X_k[L-\mu], \cdots, X_k[L-1], X_k[0], \cdots, X_k[L-1]]$. For the first μ symbols of $\tilde{\boldsymbol{x}}_k$ no power is used. Similarly, the available transmit power of the k-th user at each subcarrier is denoted as $\boldsymbol{b}_k = [b_k[0], \cdots, b_k[L-1]]$ and the transmit power for each user and subcarrier is denoted by $\boldsymbol{B} = [\boldsymbol{b_1} \cdots \boldsymbol{b_K}]$. Let $\boldsymbol{P}_k = [P_k[0], \cdots, P_k[L-1]] = \langle \boldsymbol{X}_k, \boldsymbol{b}_k \rangle$ be the vector transmitted by the k-th device for the data symbols, i.e., not including the CP zero symbols, where $\langle \cdot, \cdot \rangle$ symbolizes the inner product operation between two vectors. At the FC, a different receiver gain can be used for each subcarrier, which is denoted as $\boldsymbol{a} = [a[0], \cdots, a[L-1]]$.

For the rest of this work, we assume that the samples are distributed such that $\mathbb{E}[X_k[n]] = 0$ and $\mathbb{E}[X_k^2[n]] = 1$ holds for each sample. We also consider additive white Gaussian noise (AWGN), $w[n]$, to be the n-th noise sample at the receiver side with $\mathbb{E}[w[n]] = 0$ and $\mathbb{E}[w^2[n]] = \sigma^2$, where σ^2 is the noise power. Without loss of generality, we further assume that the receiver and all transmitters are equipped with a single antenna. We also assume that perfect CSI is available at the devices and the FC.

III. OFD OTA Computing

Because of its ability to mitigate the effect of ISI, an OFD approach can be attractive for OTA systems. However, a system using many subcarriers may be affected by phenomena such as Doppler shift and frequency synchronization errors, which cause frequency shifts of the transmitted signals, resulting in ICI. In this way, each subcarrier of the OFD system can be conceptualized as having a frequency offset Δf from its ideal transmit frequency, which is normalized with respect to the bandwidth of two consecutive subcarriers and modeled as a Gaussian random variable $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$.

For the transmitted symbols $P_k[l]$ of each device and subcarrier, the OFD OTA system produces its inverse fast Fourier transform (IFFT) equivalent, and the n-th symbol transmitted by the k-th device is given by

$$p_k[n] = \frac{1}{\sqrt{L}} \sum_{l=0}^{L-1} P_k[l] e^{j2\pi n \frac{l}{L}},\ \forall n \in \mathcal{L}, \quad (2)$$

where the normalized version of the IFFT is used. Hence, the received signal at the n-th sample can be written as

$$r[n] = \sum_{k=1}^{K} p_k[n] * h_k[n] e^{j2\pi n \frac{\epsilon}{L}} + w[n],\ \forall n \in \mathcal{L}, \quad (3)$$

where $*$ denotes the convolution operation caused by the frequency-selective nature of the wireless channels. Applying the fast Fourier transform (FFT) to the received samples, the equivalent received signal of the l-th subcarrier is

$$\begin{aligned} R[l] &= \frac{1}{L} \sum_{k=1}^{K} \sum_{n=0}^{L-1} \sum_{m=0}^{L-1} P_k[m] H_k[m] e^{-j2\pi n \frac{m-\epsilon}{L}} e^{j2\pi n \frac{l}{L}} + W[l] \\ &= \sum_{k=1}^{K} \left(P_k[l] H_k[l] S(0) + \sum_{\substack{m=0, \\ m \neq l}}^{L-1} P_k[m] H_k[m] S(l-m) \right) + W[l], \end{aligned} \quad (4)$$

where the FFT of the noise at the l-th subcarrier is given by $W[l] = \frac{1}{\sqrt{L}} \sum_{n=0}^{L-1} w[n] e^{-j2\pi n \frac{l}{L}}$, while the channel frequency response of the k-th device at the m-th subcarrier is given as $H_k[m] = \frac{1}{\sqrt{L}} \sum_{n=0}^{L-1} h_k[n] e^{-j2\pi n \frac{m}{L}}$ which is calculated for the L-point zero-padded version of the channel response. Furthermore, $S(l-m)$ are coefficients of the ICI terms which are given as

$$\begin{aligned} S(l-m) &= \frac{1}{L} \sum_{n=0}^{L-1} e^{j2\pi(l-m+\epsilon)\frac{n}{L}} \\ &= \frac{\sin\left(\pi(l-m+\epsilon)\right)}{L \sin\left(\pi\left(\frac{l-m+\epsilon}{L}\right)\right)} e^{j\pi(l-m-\epsilon)\left(1-\frac{1}{L}\right)}, \end{aligned} \quad (5)$$

where the sum of the first L terms of geometric series has been used to get the final expression in (5). Note that due to the normalized version of the FFT definition used here, the statistics of the noise remain unchanged after the FFT is applied to the original noise samples. Then, for the l-th subcarrier, the OTA calculation is equal to $\hat{Y}[l] = a[l]R[l]$, while the ideally received signal is $Y[l] = \sum_{k=1}^{K} X_k[l]$.

To estimate the performance of the OFD OTA computing system, the MSE must be considered. Due to the multiple subcarriers, the MSE to be minimized is given as the average of the MSE of all subcarriers. Using (4) and (5), the combined MSE of all subcarriers is given as

$$\begin{aligned}\mathrm{MSE}^{\mathrm{ICI}} =& \sum_{l=0}^{L-1} a^2[l] \sum_{k=1}^{K} \sum_{m=0}^{L-1} b_k^2[m] H_k^2[m] \mathbb{E}[|S(l-m)|^2] \\ &- 2\sum_{l=0}^{L-1} a[l] \sum_{k=1}^{K} b_k[l] H_k[l] \mathbb{E}[|S(0)|] \mathbb{E}[\cos(\arg\{S(0)\})] \quad (6) \\ &+ LK + \frac{\sigma^2}{L}\sum_{l=0}^{L-1} a^2[l],\end{aligned}$$

where $\arg\{\cdot\}$ symbolizes the argument of the specified complex number. Then, based on (6), we can formulate the following problem under the total power constraint for each user

$$\begin{aligned}\min_{\boldsymbol{a},\boldsymbol{B}} \quad & \mathrm{MSE}^{\mathrm{ICI}}, \\ \textbf{s.t.} \quad & \mathrm{C}_1 : \|\boldsymbol{b_k}\|_2^2 \leq P,\ \forall k \in \mathcal{K}.\end{aligned} \qquad \textbf{(P1)}$$

As can be seen from the expression for MSE in (6), the optimization problem (**P1**) is non-convex with respect to $\boldsymbol{a}, \boldsymbol{B}$. However, we can observe that (6) is convex for one set of variables when the other is fixed. The total power constraint is also convex with respect to $\boldsymbol{B}$. Thus, alternating optimization can be used to break problem (**P1**) into two separate convex optimization problems that are easier to solve. To find the optimal solutions to these problems, we use the statistical mean of the ICI coefficients between subcarriers $S(l-m),\ \forall l, m \in \mathcal{L}$ in (6). Thus, the solution resulting from the alternating optimization in (**P1**) provides a very good approximation of the MSE performance on average, but is not specifically optimal for each frequency error.

Fixing $\boldsymbol{a}$ and solving in terms of $b_k[m],\ \forall k \in \mathcal{K}, m \in \mathcal{L}$, the optimal power coefficients can be obtained by considering the Lagrangian function of the expression in (**P1**), which is given as

$$\begin{aligned}\mathcal{F} =& \sum_{l=0}^{L-1} a^2[l] \sum_{k=1}^{K} \sum_{m=0}^{L-1} b_k^2[m] H_k^2[m] \mathbb{E}[|S(l-m)|^2] \\ &- 2\sum_{l=0}^{L-1} a[l] \sum_{k=1}^{K} b_k[l] H_k[l] \mathbb{E}[|S(0)|] \mathbb{E}[\cos(\arg\{S(0)\})] \\ &+ LK + \frac{\sigma^2}{L}\sum_{l=0}^{L-1} a^2[l] + \sum_{k=1}^{K} \lambda_k \left(\sum_{m=0}^{L-1} b_k^2[m] - P\right).\end{aligned} \quad (7)$$

Then, differentiating (7) with respect to $b_k[m]$ and considering the power constraint with respect to each subcarrier, we have

$$b_k[m] = \frac{a[m] H_k[m] \mathbb{E}[|S(0)|] \mathbb{E}[\cos(\arg\{S(0)\})]}{\sum_{l=0}^{L-1} a^2[l] H_k^2[l] \mathbb{E}[|S(l-m)|^2] + \lambda_k}. \quad (8)$$

Taking into consideration constraint C_1 and the Karush-Kuhn-Tucker (KKT) conditions for each user results in an equation of the form $\lambda_k(\sum_{m=0}^{L-1} b_k^2[m] - P) = 0$, which can be solved arithmetically. If the solution of this equation is such that $\lambda_k < 0$, then by Slater's condition the optimal value is $\lambda_k = 0$ and $b_k[m]$ takes values according to (8). Fixing $\boldsymbol{B}$ and solving in terms of $a[l],\ \forall l \in \mathcal{L}$, it is easy to obtain by the Lagrangian function (7) that the optimal MSE is achieved for

$$a[l] = \frac{\sum_{k=1}^{K} b_k[l] H_k[l] \mathbb{E}[|S(0)|] \mathbb{E}[\cos(\arg\{S(0)\})]}{\sum_{k=1}^{K} \sum_{m=0}^{L-1} b_k^2[m] H_k^2[m] \mathbb{E}[|S(l-m)|^2] + \sigma^2/L}. \quad (9)$$

Iterating between $\boldsymbol{B}$ and $\boldsymbol{a}$ yields a final solution for both transmitting equalization and receiver gain factors.

IV. Simulation Results

In this section, we present the simulation results of the OFD OTA system model. Regarding the simulation parameters of the system, unless otherwise stated, we assume that the delay spread of the channel of each device is such that $\mu = 4$. The channel gains of all K devices and L subcarriers $H_k[n],\ \forall k \in \mathcal{K},\ \forall n \in \mathcal{L}$ are modeled as circularly symmetric complex Gaussian random variables with variance equal to 1, i.e., $H_k[n] \sim \mathcal{CN}(0,1)$. The maximum transmit power for each device is set to $P = 10$, so that the average transmit SNR is 10 dB. Unless otherwise specified, the number of devices and the number of subcarriers are set to $K = 20$ and $L = 64$, respectively, while the normalized frequency offset variance is considered to be $\sigma_\epsilon^2 = 0.01$. In general, we assume that the ISI symbols due to the delay spread of the channel are less than the CP added symbols, which are chosen so that the CP overhead μ/L is equal to 6.25%, which is typical for modern communication systems [13].

In order to provide an objective performance evaluation of the proposed power allocation scheme, two commonly used benchmarks are used for comparison. The first benchmark is the "optimal per subcarrier" scheme, which extracts the optimal power allocation for each subcarrier [5], not considering the joint power constraint in all subcarriers, but assuming a maximum transmit power of $\sqrt{P/L}$ for each subcarrier. The "full power" scheme is also considered, where the full available power is equally distributed among all subcarriers, i.e. $\sqrt{P/L}$.

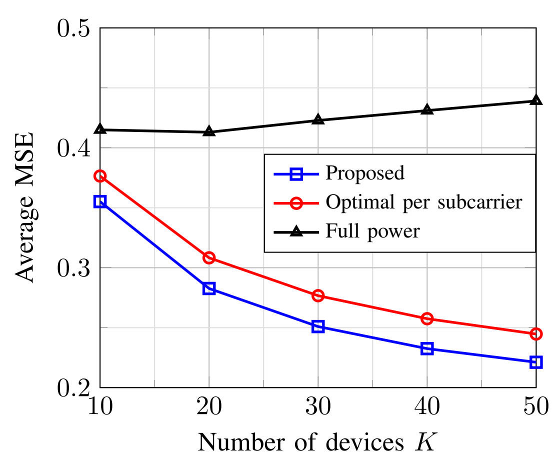

Fig. 1. MSE performance for varying number of devices K

In Fig. 1, we observe the behavior of the OFD OTA system when it is affected by ICI and the number of transmitting devices varies. As shown, the average MSE of the proposed scheme decreases as the number of devices increases, which

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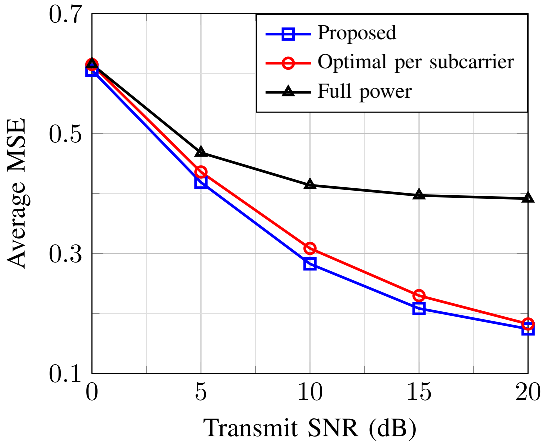

Fig. 2. MSE performance for varying transmit SNR

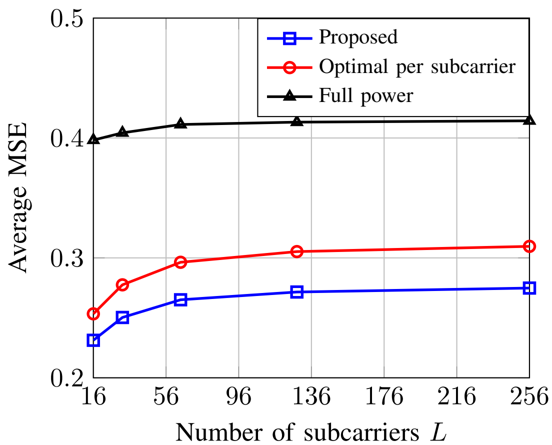

Fig. 3. MSE performance for varying number of subcarriers L and number of CP added symbols for constant overhead 6.25%

means that the accuracy performance of the system is also improved in contrast to the full power scheme. It is important to emphasize that the decreasing behavior of the MSE is crucial for many applications, as it is equivalent to convergence for scenarios where FL is utilized for DNN training. Furthermore, it is important to note that the proposed scheme outperforms the previously optimal policy for each subcarrier, with the performance gap increasing with the number of devices, reaching up to a 10% improvement gain for $K = 50$ devices.

In Fig. 2, similar findings can be extracted when the number of devices remains constant and the transmit SNR varies. As expected, increasing the transmit SNR leads to a greater improvement in the average MSE compared to the varying number of devices. This result is extremely useful as it can allow a designed system to calculate a required transmit power threshold to achieve the desired MSE performance and thus the desired accuracy levels. Again, the proposed scheme always outperforms the other two policies, with its greater improvement observed for medium transmit SNR values, which is expected since for small SNR values the maximum transmit power is usually preferred for both policies, while for large SNR values the performance gain remains stable around 6% over the second best policy.

Fig. 3 shows the MSE performance for increasing number of subcarriers when the CP overhead remains constant. As observed, all policies display an MSE increasing behavior, however it is important to note that the MSE increase grows smaller as the number of subcarriers increases. This is a consequence of ICI caused by multiple subcarriers, which results in more terms being present in the MSE calculation expressed by (6). Although this behavior causes the MSE to increase, which is not beneficial, we observe that the proposed power policy outperforms the other policies by at least 10%, while at the MSE loss remains small compared to the fact that ISI is completely eliminated.

V. Conclusions

In this work, we model an OFD OTA computing system that aims to tackle frequency selective fading channels. Considering the most practical case of frequency offset, we incorporate ICI in the proposed model and formulate an optimization problem to minimize the average MSE under sum-power constraints for each device. The solution of the proposed formulated problem yields increased performance over state-of-the-art techniques, thus highlighting the significance of the derived optimal power allocation policy.

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