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Letter

*Communication Theory*

# Average output SNR of equal-gain diversity receivers over correlative Weibull fading channels

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SUMMARY

In this letter, capitalising on a recently presented expression for the joint probability density function of the Weibull fading statistical model, a novel closed-form expression is derived for the average output signal-to-noise ratio (SNR) of an $L$-branch receiver employing equal-gain combining (EGC). The obtained expression includes the case of correlative fading with arbitrary correlation among the input branches, as well as non-identical fading statistics. Numerical results depict the effects of the fading severity, the fading correlation and the number of diversity branches, on EGC performance. An interesting outcome is that the average output SNR is not a clear and meaningful measure for the EGC receivers performance, when these operate in correlative fading channels. Copyright © 2005 AEIT.

## 1. INTRODUCTION

Antenna diversity is widely applied in wireless communication systems to reduce the effects of fading and to provide increased signal strength at the receiver. Various techniques are known to combine the signals received from multiple diversity branches. The most popular of them are selection combining (SC), equal-gain combining (EGC), maximal-ratio combining (MRC) and a combination of SC and MRC called generalised-selection combining (GSC) [1]. An SC receiver chooses the branch with the highest instantaneous signal-to-noise ratio (SNR), while MRC provides optimum performance, at the expense of implementation complexity, since it requires knowledge of all channel parameters. EGC is an intermediate solution as far as the performance and the implementation complexity are concerned. In EGC receivers, the signals in all branches are weighted with the same factor, irrespective of the signal amplitude. Moreover, co-phasing of all input signals is needed to avoid output signal cancellation.

The performance of EGC receivers has been extensively studied in the past for several well-known fading-channel models, such as Rayleigh, Rice, Nakagami-$m$ and Hoyt, assuming independent or correlative fading [1]. Surprisingly, another well-known fading-channel model, namely the Weibull model, has not yet received as much attention as the previously mentioned models, despite the fact that it provides a very good fit to experimental fading channel measurements for both indoor [2, 3] as well as for outdoor [4, 5] environments. Previously published works related to the performance analysis of digital receivers over Weibull fading channels include the following. In Reference [6],

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the Weibull fading-channel model has been considered for the evaluation of the first two moments of the output SNR of GSC receivers. In a more recent work [7], the performances of switched and stay diversity receivers in Weibull fading has been studied. In Reference [8], dual SC receivers in correlated Weibull fading have been considered, while in Reference [9], important performance measures such as the outage probability and the average output SNR have been studied, for $L$-branch SC receivers over independent and identically distributed Weibull fading channels. However, to the best of the authors' knowledge, the average output SNR, which is one of the most important and best-understood performance criteria in wireless communication theory, has not been addressed for EGC receivers operating in correlated Weibull fading channels.

In this letter, the average output SNR of $L$-branch EGC receivers operating over correlated, but not necessarily identically distributed, Weibull fading channels is studied. Numerical results demonstrate the effects of the system and channel parameters on EGC average output SNR. The remainder of this letter is as follows. In Section 2, after presenting the system and channel model, the EGC average output SNR is derived in closed-form. In Section 3, several numerically evaluated results are presented, while some useful concluding remarks are provided in Section 4.

## 2. CHANNEL MODEL AND EGC ANALYSIS

This section presents the correlated Weibull fading-channel model, and the EGC average output SNR is obtained in closed-form. A detailed discussion, concerning the effect of the correlation coefficient on the derived performance metric is also provided.

### 2.1. The Weibull fading model

Considering $L$ Weibull distributed fading channels, the cumulative distribution function (CDF) of the $\ell$th envelope, $R_\ell$, ($\ell = 1, 2, \ldots, L$) is given by [10]

$$F_{R_\ell}(r) = 1 - \exp\left[-\left(\frac{r}{\omega_\ell}\right)^{\beta}\right] \tag{1}$$

where $\omega_\ell^2 = \varepsilon\langle R_\ell^2\rangle/\Gamma(d_2)$, $\Gamma(\cdot)$ is the Gamma function [11, Equation (6.1.1)], $d_k = 1 + k/\beta$ with $k$ being a positive real value and $\varepsilon\langle R_\ell^2\rangle$ is the average fading power, with $\varepsilon\langle\cdot\rangle$ denoting statistical averaging. Moreover, $\beta$ is the Weibull fading parameter ($\beta > 1$). As $\beta$ increases, the severity of fading decreases, while for $\beta = 2$, Equation (1) reduces to the well-known Rayleigh CDF.

The complementary CDF (or survival function) of the bivariate Weibull distribution for the correlated signal envelopes $R_i$ and $R_j$ $(i,j = 1, 2, \ldots, L)$, which has been presented for the first time in the open technical literature in the field of telecommunications in Reference [8], is given by

$$\tilde{F}_{R_i,R_j}(x,y) = \exp\left\{-\left[\left(\frac{x}{\omega_i}\right)^{\beta/\delta_{i,j}} + \left(\frac{y}{\omega_j}\right)^{\beta/\delta_{i,j}}\right]^{\delta_{i,j}}\right\} \tag{2}$$

with marginal CDFs as in Equation (1). The parameter $\delta_{i,j}$ $(0 < \delta_{i,j} \leqslant 1)$ is connected with the correlation coefficient between the $i$th and $j$th branches $\rho_{i,j}$ $(0 \leqslant \rho_{i,j} \leqslant 1)$ as Reference [12]

$$\rho_{i,j} = \frac{\Gamma^2\left(d_{\delta_{i,j}}\right)\Gamma(d_2) - \Gamma^2(d_1)\Gamma\left(d_{2\delta_{i,j}}\right)}{\Gamma\left(d_{2\delta_{i,j}}\right)\left[\Gamma(d_2) - \Gamma^2(d_1)\right]} \tag{3}$$

with $\rho_{i,j} = 1$, when $i = j$ and $\rho_{i,j} = \rho_{j,i}$, when $i \neq j$. An interesting property of Equation (3) is that the uncorrelated case $(\rho_{i,j} = 0)$ is obtained when $\delta_{i,j} = 1$, since in that case Equation (2) can be written as a product of two marginal complementary CDFs. Moreover, the average product of $R_i$ and $R_j$ is given by [12]

$$\varepsilon\langle R_i R_j\rangle = \omega_i\,\omega_j\,\frac{\Gamma^2\left(d_{\delta_{i,j}}\right)\Gamma(d_2)}{\Gamma\left(d_{2\delta_{i,j}}\right)} \tag{4}$$

### 2.2. Average output SNR

We consider an $L$-branch EGC receiver operating in the fading environment described in Subsection 2.1. The instantaneous SNR measured at the output of the diversity receiver is given by [1]

$$\gamma_{\text{egc}} = \frac{E_{\text{s}}}{LN_0}\left(\sum_{i=1}^{L} R_i\right)^2 \tag{5}$$

The instantaneous SNR at the $\ell$th input branch is

$$\gamma_\ell = R_\ell^2\,\frac{E_{\text{s}}}{N_0} \tag{6}$$

where $E_{\text{s}}$ is the transmitted symbol energy and $N_0$ is the single-sided power spectral density of the additive white Gaussian noise (AWGN), while the corresponding average input SNR is

$$\bar{\gamma}_\ell = \varepsilon\langle R_\ell^2\rangle\,\frac{E_{\text{s}}}{N_0} = \omega_\ell^2\,\Gamma(d_2)\,\frac{E_{\text{s}}}{N_0} \tag{7}$$

 

Using Equation (5), the EGC average output SNR can be written as

$$\bar{\gamma}_{\text{egc}} = \frac{E_s}{LN_0}\varepsilon\left\langle\left(\sum_{i=1}^{L} R_i\right)^2\right\rangle \tag{8}$$

which using Equation (6) can be further expressed as

$$\bar{\gamma}_{\text{egc}} = \frac{1}{L}\varepsilon\left\langle\left(\sum_{i=1}^{L}\sqrt{\gamma_i}\right)^2\right\rangle \tag{9}$$

Expanding the term $\left(\sum_{i=1}^{L}\sqrt{\gamma_i}\right)^2$, using the multinomial identity [11, Chapter 24.1.2], Equation (9) can be written as

$$\bar{\gamma}_{\text{egc}} = \frac{2}{L}\sum_{\substack{h_1=0\\h_1+h_2+\cdots+h_L=2}}^{2}\sum_{h_2=0}^{2}\cdots\sum_{h_L=0}^{2}\varepsilon\left\langle\prod_{i=1}^{L}\frac{\gamma_i^{h_i/2}}{h_i!}\right\rangle \tag{10}$$

which can be easily expressed as

$$\bar{\gamma}_{\text{egc}} = \frac{1}{L}\left(\sum_{i=1}^{L}\bar{\gamma}_i + 2\sum_{\substack{h_1=0\\h_1+h_2+\cdots+h_L=2}}^{1}\sum_{h_2=0}^{1}\cdots\sum_{h_L=0}^{1}\varepsilon\left\langle\prod_{i=1}^{L}\gamma_i^{h_i/2}\right\rangle\right) \tag{11}$$

Despite the complicated form of the above formula, it can be easily recognised that only terms of the form $\varepsilon\langle\sqrt{\gamma_i\gamma_j}\rangle$ need to be evaluated. These terms can be easily obtained using Equation (4), with Equations (6) and (7), as

$$\varepsilon\left\langle\sqrt{\gamma_i\,\gamma_j}\right\rangle = \sqrt{\bar{\gamma}_i\,\bar{\gamma}_j}\,\frac{\Gamma^2\left(d_{\delta_{i,j}}\right)}{\Gamma\left(d_{2\delta_{i,j}}\right)} \tag{12}$$

Hence, by substituting Equation (12) in Equation (11) and after some straightforward mathematical manipulations, the average output SNR of an $L$-branch EGC receiver operating over correlated, but not necessarily identically distributed, Weibull fading channels can be obtained in a simple closed-form expression as

$$\bar{\gamma}_{\text{egc}} = \frac{1}{L}\left[\sum_{i=1}^{L}\bar{\gamma}_i + 2\sum_{i=2}^{L}\sum_{j=1}^{i-1}\frac{\Gamma^2\left(d_{\delta_{i,j}}\right)}{\Gamma\left(d_{2\delta_{i,j}}\right)}\sqrt{\bar{\gamma}_i\,\bar{\gamma}_j}\right] \tag{13}$$

Note, that Equation (13) may be used with arbitrary correlation between the diversity input branches, such as constant, exponential etc.

### 2.3. *Extreme cases discussion*

In order to reveal the effect of correlation on the average SNR, assuming identically distributed input branches $\bar{\gamma}_\ell = \bar{\gamma}$, we further study two extreme cases. The first one is when the EGC input branches are same ($\rho_{i,j} = 1$), where Equation (13) simplifies to

$$\frac{\bar{\gamma}_{\text{egc}}}{\bar{\gamma}} = L \tag{14}$$

meaning that $\bar{\gamma}_{\text{egc}}$ is independent on $\rho$. It is interesting to mention that the average output SNR of $L$-branch MRC receivers $\bar{\gamma}_{\text{mrc}}$ is also given by Equation (14). The contradictory result of Equation (14) can be explained as follows. Due to the specific combining technique applied in EGC, cross-product terms of the input-fading envelopes contribute to the average output SNR. These terms are directly related to its correlation coefficient. Hence, the higher its correlation coefficient is, the higher the average output SNR yields.

The other case arises when the EGC input branches are uncorrelated ($\rho_{i,j} = 0$), where Equation (13) simplifies to

$$\frac{\bar{\gamma}_{\text{egc}}}{\bar{\gamma}} = 1 + \frac{\Gamma^2(d_1)}{\Gamma(d_2)}(L-1) < L \tag{15}$$

In this case, the EGC average output SNR is always less than that in Equation (14). Hence, contrary to the prior knowledge that the branch-fading correlation degrades all the performance measure characteristics, in case of EGC receivers the average output SNR increases with an increase of the correlation coefficient between the branches, without necessarily meaning that it directly translates into an improved EGC receiver performance.

Taking into account the above observations, we may conclude that the average output SNR is not a clear and meaningful performance measure for both EGC and MRC receivers, when these operate in correlative-fading channels. Note, that a similar behaviour, concerning EGC receivers operating in correlative Nakagami-$m$-fading channels, as that described above, has been also observed in Reference [13].

## 3. NUMERICAL RESULTS

Equation (13) is numerically evaluated and the results are illustrated in Figures 1 and 2. The average output SNR of $L$-branch MRC receivers is also plotted in both figures for comparison purposes. Assuming that the receiver operates with an exponentially decaying power delay profile (PDP)

$$\bar{\gamma}_\ell = \bar{\gamma}_1\exp[-\varphi(\ell-1)] \tag{16}$$

in Figure 1, the first branch normalised average output SNR of EGC, $\bar{\gamma}_{\text{egc}}/\bar{\gamma}_1$, with exponential correlated input

 

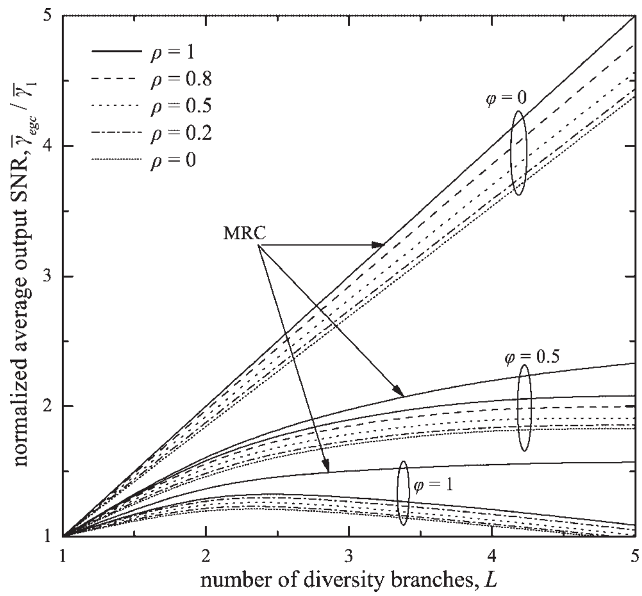


Figure 1. First branch normalised average output SNR of EGC, $\bar{\gamma}_{\mathrm{egc}}/\bar{\gamma}_1$, and MRC, $\bar{\gamma}_{\mathrm{mrc}}/\bar{\gamma}_1$, versus $L$ with exponentially decaying PDP, exponential correlation and $\beta = 2.5$.

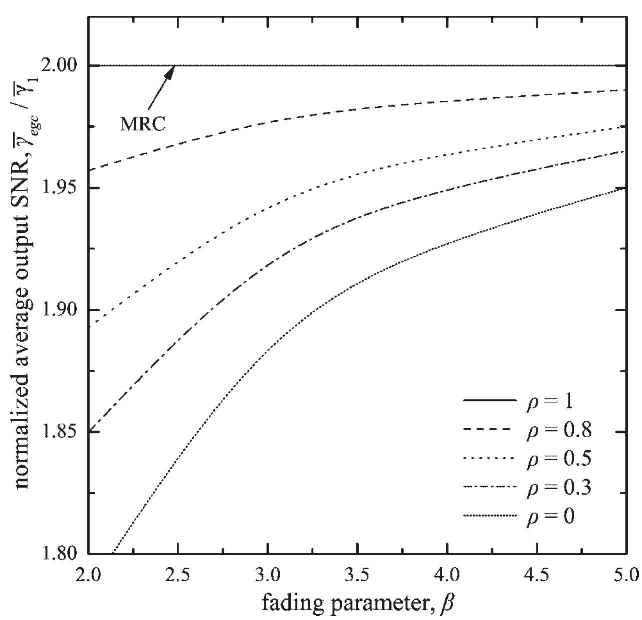


Figure 2. First branch normalised average output SNR of dual-branch EGC, $\bar{\gamma}_{\mathrm{egc}}/\bar{\gamma}_1$, and MRC, $\bar{\gamma}_{\mathrm{mrc}}/\bar{\gamma}_1$, versus $\beta$.

paths is plotted, as a function of $L$, for $\beta = 2.5$, and for several values of $\rho$ and the power decaying factor $\varphi$. It is easily recognised that the combining loss of the receiver gets more accentuated as $\varphi$ increases, while with an increase of $\rho$ better performance is obtained. For $\rho = 1$ and $\varphi = 0$, the performance of EGC receiver is identical with that of MRC, as Equation (14) reads. In Figure 2, the first branch normalised average output SNR of dual-branch EGC is plotted as a function of $\beta$, for identically distributed input branch SNRs and for several values of $\rho_{1,2} = \rho$. It is observed that contrary to the average SNR at the MRC output, which is unaffected by the correlation, the average SNR at the EGC output increases with an increase of the correlation, which is in agreement with the similar result, observed in Figure 1. Moreover, as expected, the receiver performs better with increasing $\beta$. It is also obvious from Figure 2 that while the fading severity decreases (i.e. $\beta$ increases), the average output SNR becomes less sensitive to $\rho$.

## 4. CONCLUSIONS

In this letter, using a simple approach based on the multinomial identity, we studied the average output SNR, for $L$-branch receivers employing EGC diversity operating over correlated, but not necessarily identically distributed, Weibull fading channels. Numerically evaluated results demonstrated that the higher the correlation coefficient the higher the value of the average output SNR, without necessarily translating into an improved EGC receiver performance. Hence, we conclude that the average output SNR is not a clear and meaningful performance measure for EGC (and MRC) receivers, when these operate in correlative fading channels.

REFERENCES

1. Simon MK, Alouini M-S. *Digital Communication over Fading Channels* (2nd edn). John Wiley: New York, 2004.
2. Hashemi H. The indoor radio propagation channel. *IEEE Proceedings* 1993; **81**:943–968.
3. Babich F, Lombardi G. Statistical analysis and characterization of the indoor propagation channel. *IEEE Transactions on Communications* 2000; **48**(3):455–464.
4. Adawi NS, Bertoni HL, Child JR, Daniel WA, Dettra JE, Eckert RP, *et al*. Coverage prediction for mobile radio systems operating in the 800/900 MHz frequency range. *IEEE Transactions on Vehicular Technology* 1988; **37**(1):3–72.
5. Shepherd NH. Radio wave loss deviation and shadow loss at 900 MHz. *IEEE Transactions on Vehicular Technology* 1977; **VT-26**:309–313.
6. Alouini M-S, Simon MK. Performance of generalized selection combining over Weibull fading channels. In *Proceedings of the IEEE Vehicular Technology Conference*, Vol. 3. Rhodes, Greece 2001; pp. 1735–1739.
7. Sagias NC, Zogas DA, Karagiannidis GK, Tombras GS. Performance analysis of switched diversity receivers in Weibull fading. *IEE Electronics Letters* 2003; **39**(20):1472–1474.

 

8. Sagias NC, Karagiannidis GK, Zogas DA, Mathiopoulos PT, Tombras GS. Performance analysis of dual selection diversity in correlated Weibull fading channels. *IEEE Transactions on Communications* 2004; **52**(7):1063–1067.
9. Sagias NC, Mathiopoulos PT, Tombras GS. Selection diversity receivers in Weibull fading: outage probability and average signal-to-noise ratio. *IEE Electronics Letters* 2003; **39**(25):1859–1860.
10. Bury K. *Statistical Distributions in Engineering*. Cambridge University Press: Cambridge, UK, 1999.
11. Abramovitz M, Stegun IA. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables* (9th edn). Dover: New York, 1972.
12. Lu JC, Bhattaacharyya GK. Some new constructions of bivariate Weibull models. *Annals of Institute of Statistical Mathematics* 1990; **42**(3):543–559.
13. Karagiannidis GK, Zogas DA, Kotsopoulos SA. Statistical properties of the EGC output SNR over correlated Nakagami-*m* fading channels. *IEEE Transactions on Wireless Communications* 2004; **3**(5):1764–1769.

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