Bounds for Multihop Relayed Communications in Nakagami-m Fading

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***Abstract*—We present closed-form lower bounds for the performance of multihop transmissions with nonregenerative relays over not necessarily identically distributed Nakagami-m fading channels. The end-to-end signal-to-noise ratio is formulated and upper bounded by using an inequality between harmonic and geometric means of positive random variables (RVs). Novel closed-form expressions are derived for the moment generating function, the probability density function, and the cumulative distribution function of the product of rational powers of statistically independent Gamma RVs. These statistical results are then applied to studying the outage probability and the average bit-error probability for phase- and frequency-modulated signaling. Numerical examples compare analytical and simulation results, verifying the tightness of the proposed bounds.**

***Index Terms*—Average bit-error probability (ABEP), Gamma random variables (RVs), multihop relayed communications, Nakagami-m fading, outage probability.**

I. Introduction

MULTIHOP systems have a number of advantages over traditional communication networks in the areas of deployment, connectivity, and capacity, while minimizing the need for fixed infrastructure. Relaying techniques enable network connectivity where traditional architectures are impractical due to location constraints, and can be applied to cellular, wireless local area networks (WLANs), and hybrid networks. In multihop systems, the source terminal communicates with the destination terminal through a number of relay terminals. Therefore, multihop systems have the advantage of broadening the coverage without using large transmitting power [1]–[5]. Recently, the concept of cooperative diversity, where the mobile users cooperate/collaborate with each other in order to exploit the benefits of spatial diversity without the need for using physical antenna arrays, has gained great interest. In general, cooperative networks are multihop communication networks, where the destination terminal combines the signals received from both the source terminal and relays [6]–[9].

The performance analysis of multihop wireless communication systems operating in fading channels has been an important field of research in the past few years. Hasna and Alouini have presented a useful and semianalytical framework for the evaluation of the end-to-end outage probability of multihop wireless systems with nonregenerative channel state information (CSI)-assisted relays over Nakagami-m fading channels [3]. Moreover, the same authors have studied the outage and the error performance of dual-hop systems with regenerative and nonregenerative (CSI-assisted or fixed-gain) relays over Rayleigh [1], [4] and Nakagami-m [2] fading channels. Recently, Boyer *et al.* [5] have proposed and characterized four channel models for multihop wireless communications, and have also introduced the concept of multihop diversity. Finally, Karagiannidis has studied the performance bounds for multihop relayed transmissions with blind (fixed-gain) relays over Nakagami-n (Rice), Nakagami-q (Hoyt), and Nakagami-m fading channels [10] using the moments-based approach [11]. However, to the best of the authors' knowledge, the performance of multihop relayed systems has never been addressed in terms of tabulated functions in Nakagami-m fading.

In this letter, using the well-known inequality between harmonic and geometric means of positive random variables (RVs), we present performance bounds for the end-to-end signal-to-noise ratio (SNR) of multihop wireless communication systems with CSI-assisted or fixed-gain relays operating in nonidentical Nakagami-m fading channels. Motivated by the fact that the proposed bounds, in their general form, are products of rational powers of statistically independent squared Nakagami-m RVs (or equivalently, Gamma RVs), we derive novel closed-form expressions for their moment generating function (MGF), the probability density function (PDF), and the cumulative distribution function (CDF). These statistical results are then applied to the study of important system performance metrics. Closed-form lower bounds are derived for the outage probability, and the average bit-error probability (ABEP) for binary phase-shift keying (BPSK) and binary frequency-shift keying (BFSK) modulation schemes. Numerical and computer simulation examples verify the accuracy of the presented mathematical analysis and show the tightness of the proposed bounds.

The remainder of this letter is organized as follows. In Section II, closed-form expressions for the MGF, PDF, and CDF of the product of rational powers of Gamma RVs are presented. Next, Section III introduces the multihop system and channel model under consideration. In the same section, closed-form upper bound expressions for the statistics of the end-to-end SNR are proposed both for CSI-assisted and fixed-gain relayed systems. These results are applied in Section IV to evaluate the end-to-end performance metrics of multihop wireless communication systems. Finally, some concluding remarks are given in Section V.

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II. Statistical Background

Theorem 1: (MGF of the product of rational powers of Gamma RVs) Let $\{X_i\}_{i=1}^N$ be N independent, but not necessarily identically distributed (i.n.i.d.), Gamma RVs, with the PDF given by

$$f_{X_i}(x) = \frac{x^{\alpha_i - 1}}{\beta_i^{\alpha_i}\Gamma(\alpha_i)} \exp\left(-\frac{x}{\beta_i}\right) \tag{1}$$

where $\Gamma(\cdot)$ is the Gamma function [12, eq. (8.310/1)], and α_i, β_i are positive real numbers. Then, the MGF of the new RV Y_1, defined as the product of powers of N RVs X_i, i.e.,

$$Y_1 \triangleq \prod_{i=1}^N X_i^{\ell_i/k} \tag{2}$$

with $\ell_1, \ell_2, \ldots, \ell_N$ and k being positive integers, can be expressed in closed form as

$$\begin{aligned}\mathcal{M}_{Y_1}(s) &= \frac{\sqrt{k}\prod_{i=1}^N \ell_i^{\alpha_i - 1/2}}{\left(\sqrt{2\pi}\right)^{r-N+k-1}\prod_{i=1}^N \Gamma(\alpha_i)} \\ &\times G_{r,k}^{k,r}\left[\frac{(-1)^k \left(\frac{s}{k}\right)^k}{\prod_{i=1}^N (\beta_i \ell_i)^{-\ell_i}} \middle| \begin{matrix}\Delta(\ell_1, 1-\alpha_1), \ldots, \Delta(\ell_N, 1-\alpha_N) \\ \Delta(k, 0)\end{matrix}\right]\end{aligned} \tag{3}$$

where $r = \sum_{i=1}^N \ell_i$, $\Delta(k,u) \triangleq u/k, (u+1)/k, \ldots, (u+k-1)/k$, with u real, and $G[\cdot]$ is the Meijer G-function [12, eq. (9.301)].

Note that Meijer's G-function is a standard built-in function in most of the well-known mathematical software packages, such as MAPLE, MATHEMATICA, and MATLAB. In addition, using [13, eq. (18)], Meijer's G-function can be written in terms of the more familiar generalized hypergeometric functions [12, eq. (9.14.1)].

Proof: See Appendix I. ■

Corollary 1: (PDF of the product of rational powers of Gamma RVs) The PDF of Y_1 is given by

$$\begin{aligned}f_{Y_1}(y) &= \frac{k\, y^{-1}\prod_{i=1}^N \ell_i^{\alpha_i - 1/2}}{\left(\sqrt{2\pi}\right)^{r-N}\prod_{i=1}^N \Gamma(\alpha_i)} \\ &\times G_{0,r}^{r,0}\left[y^k \prod_{i=1}^N \left(\frac{1}{\beta_i \ell_i}\right)^{\ell_i} \middle| \begin{matrix} - \\ \Phi_1, \Phi_2, \ldots, \Phi_N\end{matrix}\right]\end{aligned} \tag{4}$$

where $\Phi_i \triangleq \Delta(\ell_i, \alpha_i)$.

Proof: The PDF of Y_1 can be derived as $f_{Y_1}(y) = \mathcal{L}^{-1}\{M_{Y_1}(-s); y\}$, where $\mathcal{L}^{-1}(\cdot;\cdot)$ denotes the inverse Laplace transform. Using the formula for the inverse Laplace transform of the Meijer G-function [14, eq. (3.38.1)], we obtain (4). ■

It must be mentioned here that $f_{Y_1}(\cdot)$ represents a valid PDF, since it is a nonnegative function, and using [13, eq. (24)] and [15, eq. (6.1.20)], it can be easily verified that $\int_0^\infty f_{Y_1}(y)dy = 1$. Moreover, in Fig. 1, Monte Carlo simulations are performed to show the accuracy of (4). From this figure, an exact match is evident between simulations and analytical results. For the case of $\ell_i = k = 1$, the PDF $f_{Y_2}(y)$ of the product of N i.n.i.d. Gamma RVs, $Y_2 \triangleq \prod_{i=1}^N X_i$, can be written as

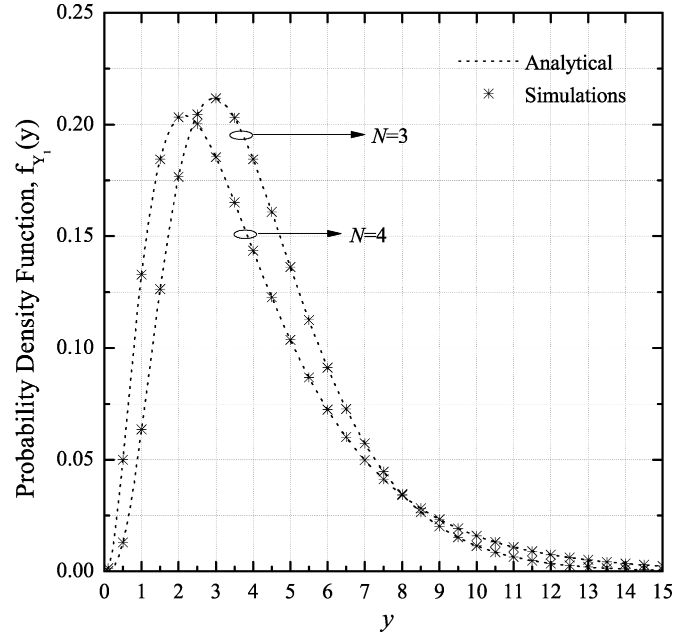

Fig. 1. Comparison between analytical results and Monte Carlo simulations for the PDF formulated by (4) ($k = 3$, $\ell_i = i$, $m_i = i$, $\beta_1 = 10$, $\beta_2 = \beta_3 = 1.05$, $\beta_4 = 0.25$, and 200 000 iterations).

$$f_{Y_2}(y) = \frac{y^{-1}}{\prod_{i=1}^N \Gamma(\alpha_i)} G_{0,N}^{N,0}\left[\frac{y}{\prod_{i=1}^N \beta_i} \middle| \begin{matrix} - \\ \alpha_1, \alpha_2, \ldots, \alpha_N\end{matrix}\right]. \tag{5}$$

Corollary 2: (CDF of the product of rational powers of Gamma RVs) The CDF of Y_1 is given by

$$\begin{aligned}F_{Y_1}(y) &= \frac{\prod_{i=1}^N \ell_i^{\alpha_i - 1/2}}{\left(\sqrt{2\pi}\right)^{r-N}\prod_{i=1}^N \Gamma(\alpha_i)} \\ &\times G_{1,r+1}^{r,1}\left[y^k \prod_{i=1}^N \left(\frac{1}{\beta_i \ell_i}\right)^{\ell_i} \middle| \begin{matrix} 1 \\ \Phi_1, \Phi_2, \ldots, \Phi_N, 0\end{matrix}\right].\end{aligned} \tag{6}$$

Proof: Following the definition, $F_{Y_1}(y) = \int_0^y f_{Y_1}(z)dz$ and using [13, eq. (26)] yields (6). ■

For the case of $\ell_i = k = 1$, the CDF of Y_2 is given by

$$F_{Y_2}(y) = \frac{G_{1,N+1}^{N,1}\left[\frac{y}{\prod_{i=1}^N \beta_i} \middle| \begin{matrix} 1 \\ \alpha_1, \alpha_2, \ldots, \alpha_N, 0\end{matrix}\right]}{\prod_{i=1}^N \Gamma(\alpha_i)}. \tag{7}$$

III. An Upper Bound for the End-to-End SNR

In this section, we derive upper bounds for the distributions of the end-to-end SNR for the CSI-assisted and fixed-gain relay implementations of a multihop communication system.

A. System and Channel Model

We consider an N-hop wireless communication system which operates over i.n.i.d. Nakagami-m fading channels. The source terminal S communicates with the destination terminal

D through $N-1$ nodes terminals $R_1, R_2, \ldots, R_{N-1}$. These terminals relay the signal only from one hop to the next, acting as nonregenerative relays. It is also assumed that all node relays can simultaneously receive and transmit (in the same frequency band), and no delay is incurred in the whole chain of transmissions. Assume that terminal S is transmitting a signal with an average power normalized to unity. Then the end-to-end SNR, i.e., the SNR at D, can be written as [3]

$$\gamma_{\text{end}} = \frac{\prod_{i=1}^{N} \upsilon_i^2 \, g_{i-1}^2}{\sum_{i=1}^{N} N_{0,i} \left(\prod_{j=i+1}^{N} g_{j-1}^2 \upsilon_j^2 \right)} \tag{8}$$

where υ_i is the fading amplitude of the ith hop, $N_{0,i}$ is the one-sided power spectral density at the input of the ith relay, and g_i is the gain of the ith relay with $g_0 = 1$.

Due to the fact that υ_i is Nakagami-m distributed, the corresponding instantaneous SNR γ_i, defined as $\gamma_i = \upsilon_i^2 / N_{0,i}$, is Gamma distributed, with the PDF given by [16]

$$f_{\gamma_i}(\gamma) = \frac{m_i^{m_i}}{\overline{\gamma}_i^{m_i} \, \Gamma(m_i)} \gamma^{m_i - 1} \exp\left(-\frac{m_i \gamma}{\overline{\gamma}_i} \right) \tag{9}$$

where $m_i \geq 1/2$ is a parameter describing the fading severity of the ith hop, and $\overline{\gamma}_i$ is the average SNR, i.e., $\overline{\gamma}_i = E\left\langle \upsilon_i^2 \right\rangle / N_{0,i}$, with $E\langle \cdot \rangle$ denoting expectation. It is obvious that setting $\alpha_i = m_i$ and $\beta_i = \overline{\gamma}_i / m_i$ in (1) yields (9).

B. CSI-Assisted Relays

One choice for the gain is proposed in [9, eq. (9)] to be

$$g_i^2 = \frac{1}{\upsilon_i^2 + N_{0,i}}. \tag{10}$$

This gain aims to invert the fading state of the preceding channel, while limiting the instantaneous output power of the relay if the fading amplitude of the preceding hop, υ_i, is low. By substituting (10) in (8), the derived equivalent SNR at terminal D can given by [3, eq. (2)], which is not easily tractable due to the complexity in finding the statistics. However, another choice of relay gain is set to

$$g_i^2 = \frac{1}{\upsilon_i^2} \tag{11}$$

where the relay just amplifies the incoming signal with the inverse of the channel of the previous hop, regardless of the noise of that hop. As mentioned in [3], such a relay serves as a benchmark for all practical multihop systems employing nonregenerative relays. Additionally, a comparison of the outage probability between multihop systems with the two relay gains of (10) and (11) showed a similar performance in the high-SNR region.

By applying (11) to (8), the end-to-end SNR becomes

$$\gamma_{\text{end}} = \left(\sum_{i=1}^{N} \frac{1}{\gamma_i} \right)^{-1}. \tag{12}$$

In order to study important performance metrics of the end-to-end SNR, (12) should be expressed in a more mathematically tractable form. To achieve it, we propose an upper bound for (12) using the well-known inequality between geometric and harmonic means for $x_1, x_2, \ldots, x_N$, given by

$$\mathcal{H}_N \leq \mathcal{G}_N \tag{13}$$

where $\mathcal{H}_N \triangleq N(\sum_{i=1}^{N} 1/x_i)^{-1}$ and $\mathcal{G}_N \triangleq \prod_{i=1}^{N} x_i^{1/N}$ are the harmonic and geometric means, respectively. In (13), the equality holds only when $x_1 = x_2 = \cdots = x_N$. Using (12) and (13), an upper bound for the end-to-end SNR, γ_b, for multihop systems with CSI-assisted relays can be obtained as

$$\gamma_{\text{end}} \leq \gamma_b = \frac{1}{N} \prod_{i=1}^{N} \gamma_i^{1/N}. \tag{14}$$

By applying (4) and (6) in (14), the PDF and CDF of γ_b can be written in closed form as

$$f_{\gamma_b}(\gamma) = \frac{N\gamma^{-1} G_{0,N}^{N,0}\left[(\gamma N)^N \prod_{i=1}^{N} \frac{m_i}{\overline{\gamma}_i} \,\middle|\, \begin{matrix} - \\ m_1, m_2, \ldots, m_N \end{matrix} \right]}{\prod_{i=1}^{N} \Gamma(m_i)} \tag{15}$$

and

$$F_{\gamma_b}(\gamma) = \frac{G_{1,N+1}^{N,1}\left[(\gamma N)^N \prod_{i=1}^{N} \frac{m_i}{\overline{\gamma}_i} \,\middle|\, \begin{matrix} 1 \\ m_1, m_2, \ldots, m_N, 0 \end{matrix} \right]}{\prod_{i=1}^{N} \Gamma(m_i)} \tag{16}$$

respectively.

C. Fixed-Gain Relays

The fixed-gain relays provide reduced implementation complexity in the CSI part, at the expense of the requirements for high-transmission-power amplifiers, which may be very costly in practice. Nonregenerative relays introduce fixed gains to the received signal given by

$$g_i^2 = \frac{1}{C_i N_{0,i}} \tag{17}$$

where C_i is a positive constant ($C_0 = 1$). Following the same procedure as in [3] and using (17), the end-to-end SNR can be expressed as [10]

$$\begin{aligned} \gamma'_{\text{end}} &= \left(\frac{1}{\gamma_1} + \frac{C_1}{\gamma_1 \gamma_2} + \cdots + \frac{C_1 C_2 \cdots C_{N-1}}{\gamma_1 \gamma_2 \gamma_3 \cdots \gamma_N} \right)^{-1} \\ &= \left(\sum_{n=1}^{N} \prod_{j=1}^{n} \frac{C_{j-1}}{\gamma_j} \right)^{-1} \end{aligned} \tag{18}$$

and by using (13) in (18), an upper bound for the γ'_{end} can be derived as

$$\gamma'_{\text{end}} \leq \gamma'_b = \frac{1}{N} \prod_{i=1}^{N} C_i^{-(N-i)/N} \gamma_i^{(N+1-i)/N}. \tag{19}$$

Moreover, taking into account *Corollaries 1* and *2* and after some algebra, the PDF and CDF of γ'_b can be obtained as

$$f_{\gamma'_b}(\gamma) = N \mathcal{P} \gamma^{-1} G_{0,\varrho}^{\varrho,0}\left[\mathcal{R}\gamma^N \,\middle|\, \begin{matrix} - \\ \Lambda_1, \Lambda_2, \ldots, \Lambda_N \end{matrix} \right] \tag{20}$$

and

$$F_{\gamma'_b}(\gamma) = \mathcal{P}\, G_{1,\varrho+1}^{\varrho,1}\left[\mathcal{R}\gamma^N \,\middle|\, \begin{matrix} 1 \\ \Lambda_1, \Lambda_2, \ldots, \Lambda_N, 0 \end{matrix} \right] \tag{21}$$

respectively, where

$$\begin{aligned} \varrho &= \frac{N(N+1)}{2} \\ \mathcal{P} &= \frac{\prod_{i=1}^{N} (N+1-i)^{m_i - 1/2}}{\left(\sqrt{2\pi}\right)^{N(N-1)/2} \prod_{i=1}^{N} \Gamma(m_i)} \\ \Lambda_i &= \Delta(N+1-i, m_i) \end{aligned}$$

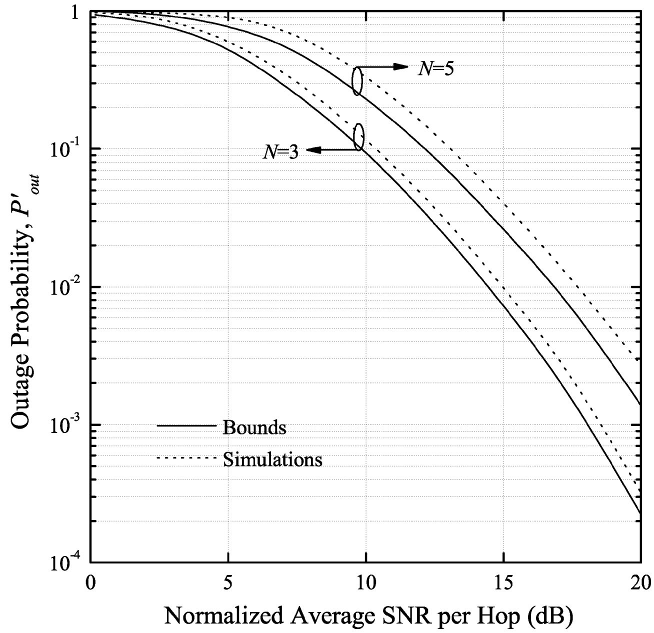

Fig. 2. Outage probability bounds for a multihop system with fixed-gain relays ($\overline{\gamma}_i = \overline{\gamma}$, $C_i = 1.7$, and $m_i = m = 2.7$).

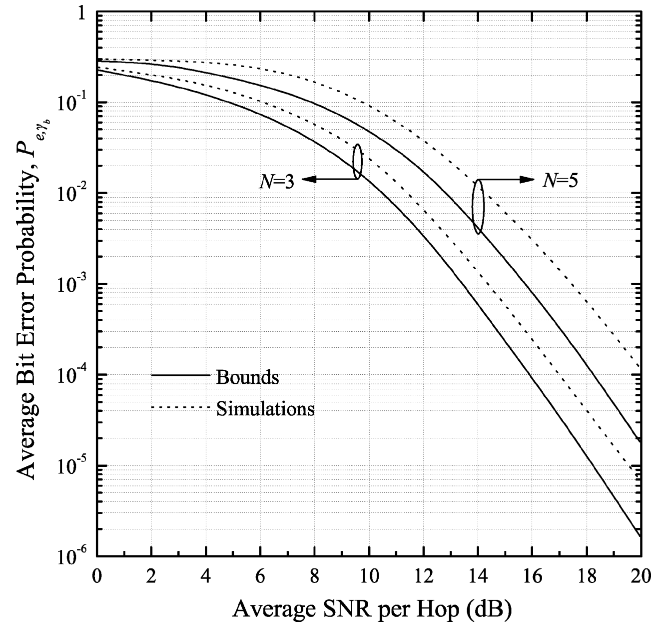

Fig. 3. BPSK error bounds for a multihop system with CSI-assisted relays in i.i.d. Nakagami-m fading channels ($\overline{\gamma}_i = \overline{\gamma}$ and $m_i = m = 2.7$).

$$\mathcal{R} = N^N \prod_{i=1}^{N} C_i^{N-i} \prod_{i=1}^{N} \left[\frac{m_i}{\overline{\gamma}_i \, (N+1-i)} \right]^{N+1-i}. \quad (22)$$

IV. Performance Metrics

A. Outage Probability

The probability of outage is defined as the probability that the instantaneous SNR falls below a specified threshold γ_{th}. This threshold is a protection value of the SNR, above which the quality of service is satisfactory. In the case of the multihop system under consideration, the use of upper bounds γ_b or γ_b' leads to lower bounds for the outage probability in the destination terminal D, expressed as $P_{\text{out}} \geq F_{\gamma_b}(\gamma_{\text{th}})$ for CSI-assisted relays, and $P'_{\text{out}} \geq F_{\gamma_b'}(\gamma_{\text{th}})$ for fixed-gain relays.

As an indicative example for the proposed bounds, assuming fixed-gain relays and equal average SNRs per hop (for all hops, $\overline{\gamma}_i = \overline{\gamma}$), lower bounds for the outage probability are plotted in Fig. 2 as a function of the inverse normalized to outage threshold $\overline{\gamma}/\gamma_{\text{th}}$. The obtained results clearly show that the outage performance degrades with an increase of the number of hops. Additionally, the lower the value of N, the tighter the proposed bounds, even for high SNR values.

B. Average Bit-Error Probability

For coherent binary signal constellations, the ABEP $\overline{P}_e$ can be formulated as [16]

$$\overline{P}_e = \frac{1}{2} E \left\langle \operatorname{erfc}\left(\sqrt{\xi \gamma}\right) \right\rangle \quad (23)$$

where $\operatorname{erfc}(\cdot)$ is the complementary error function [12, eq. (8.250.4)], and $\xi = (1-\varepsilon)/2$, ε being the correlation coefficient between the two signaling waveforms. Thus, for $\varepsilon = -1$, $\xi = 1$ for coherent BPSK, and for $\varepsilon = 0$, $\xi = 1/2$ for coherent orthogonal BFSK.

1) CSI-Assisted Relays: Using (15), (23), the Meijer G-function representation of the $\operatorname{erfc}(\cdot)$ function [13, eq. (12)], and [13, eq. (21)], a lower bound for the ABEP of CSI-assisted relays over Nakagami-m fading channels can be expressed in closed form as

$$\overline{P}_{e,\gamma_b} = \frac{G_{2N,2N}^{N,2N}\left[\left(\frac{N^2}{\xi}\right)^N \prod_{i=1}^{N} \frac{m_i}{\overline{\gamma}_i} \middle| \begin{matrix} \Delta(N,1), \Delta\left(N, \frac{1}{2}\right) \\ m_1, m_2, \ldots, m_N, \Delta(N,0) \end{matrix}\right]}{(\sqrt{2})^{N+1} (\sqrt{\pi})^N \prod_{i=1}^{N} \Gamma(m_i)}. \quad (24)$$

In Fig. 3, lower bounds for the ABEP of a multihop system with CSI-assisted relays are plotted only for $N = 3$ and $N = 5$ to avoid entanglement. Again here, it is evident that the proposed bounds are tight and, as expected, the ABEP deteriorates with an increase in the number of hops. The same results are also observed in Fig. 4, where the error performance is studied for i.n.i.d. Nakagami-m fading channels. However, in this case, it is observed that the proposed bounds lose their tightness in the high SNR region (> 16 dB), compared with the independent and identically distributed (i.i.d.) case. This happens due to the fact that the sharpness of the used harmonic-geometric mean inequality increases when $x_1, x_2, \ldots, x_N$ are close to each other as much as possible. Therefore, the i.i.d. case is the best one in terms of the tightness of the bound.

2) Fixed-Gain Relays: For the case of fixed-gain relays, a lower bound for ABEP can be found using (20) and (23) as

$$\overline{P}_{e,\gamma_b'} = \frac{\mathcal{P}\, G_{2N,\varrho+N}^{\varrho,\,2N}\left[\mathcal{R}N^N \middle| \begin{matrix} \Delta(N,1), \Delta\left(N, \frac{1}{2}\right) \\ \Lambda_1, \ldots, \Lambda_N, \Delta(N,0) \end{matrix}\right]}{(\sqrt{2})^{N+1} (\sqrt{\pi})^N}. \quad (25)$$

V. Conclusion

Performance bounds for multihop transmissions with CSI-assisted or fixed-gain relays operating over i.n.i.d. Nakagami-m fading channels have been presented. The end-to-end SNR is formulated and upper bounded by using the harmonic-geometric mean inequality of positive RVs for both types of relays. Since the proposed bounds are in the form of the product of rational powers of statistically independent Gamma RVs, novel closed-form expressions for the MGF, PDF, and CDF of this product have been

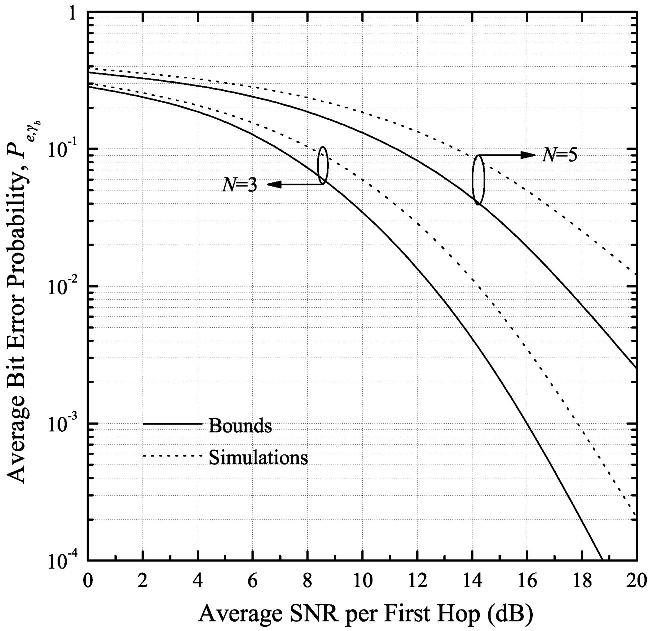

Fig. 4. BPSK error bounds for a multihop system with CSI-assisted relays in i.n.i.d. Nakagami-m fading chanels ($\overline{\gamma}_i = \overline{\gamma}/i$, $m_1 = m_2 = 5$, $m_3 = m_4 = 2.5$, and $m_5 = 1.5$).

derived. Additionally, lower bounds for the outage probability and the average error probability have been presented. Finally, this letter may contribute to a number of open issues for future investigation, such as the variation of the performance loss due to an increase of the number of hops, and the existence of an optimal number of hops when path loss between nodes is considered.

APPENDIX
PROOF OF *THEOREM I*

The MGF of Y_1 in (3) is defined as

$$\mathcal{M}_{Y_1}(-s) = \int_0^\infty \int_0^\infty \cdots \int_0^\infty e^{-s\, x_1^{\ell_1/k}\, x_2^{\ell_2/k} \ldots x_N^{\ell_N/k}} \times f_{X_1}(x_1) \ldots f_{X_N}(x_N)\, dx_1 \ldots dx_N. \quad (26)$$

The first integration in (26), i.e., the one on x_1, is of the form

$$\mathcal{I}_1 = \int_0^\infty x_1^{\alpha_1 - 1} e^{-x_1/\beta_1} e^{-sW_2\, x_1^{\ell_1/k}} dx_1 \quad (27)$$

where $W_i = x_i^{\ell_i/k} x_{i+1}^{\ell_{i+1}/k} \ldots x_N^{\ell_{i+N}/k}$. Using [13], (27) can be written in terms of the Meijer G-function as

$$\mathcal{I}_1 = \int_0^\infty x_1^{\alpha_1 - 1} G_{0,1}^{1,0}\left[\frac{x_1}{\beta_1}\middle| \begin{matrix} - \\ 0 \end{matrix}\right] G_{0,1}^{1,0}\left[sW_2\, x_1^{\ell_1/k}\middle| \begin{matrix} - \\ 0 \end{matrix}\right] dx_1. \quad (28)$$

Using [13, eq. (21)], $\mathcal{I}_1$ can be solved as

$$\mathcal{I}_1 = \frac{\sqrt{k}\, \ell_1^{\alpha_1 - 1/2} G_{\ell_1, k}^{k, \ell_1}\left[\frac{(s\,W_2)^k k^{-k}}{(\beta_1 \ell_1)^{-\ell_1}}\middle| \begin{matrix} \Delta(\ell_1, 1-\alpha_1) \\ \Delta(k, 0) \end{matrix}\right]}{\beta_1^{-\alpha_1} (\sqrt{2\pi})^{\ell_1 - 1 + k - 1}}. \quad (29)$$

The second integration in (26), i.e., the one on x_2, can be now written as

$$\mathcal{I}_2 = \frac{\sqrt{k}\, \ell_1^{m_1 - 1/2}}{\beta_1^{-\alpha_1} (\sqrt{2\pi})^{\ell_1 - 1 + k - 1}} \int_0^\infty x_2^{\alpha_2 - 1} G_{0,1}^{1,0}\left[\frac{x_2}{\beta_2}\middle| \begin{matrix} - \\ 0 \end{matrix}\right] \times G_{\ell_1, k}^{k, \ell_1}\left[\frac{(s\,W_3)^k k^{-k}}{(\beta_1 \ell_1)^{-l_1}} x_2^{\ell_2}\middle| \begin{matrix} \Delta(\ell_1, 1-\alpha_1) \\ \Delta(k, 0) \end{matrix}\right] dx_2. \quad (30)$$

Again, using [13, eq. (21)], the integral $\mathcal{I}_2$ can be solved as

$$\mathcal{I}_2 = \frac{\sqrt{k}\, \ell_1^{\alpha_1 - 1/2} \ell_2^{\alpha_2 - 1/2}}{\beta_1^{-\alpha_1} \beta_2^{-\alpha_2} \left(\sqrt{2\pi}\right)^{\ell_1 + \ell_2 - 2 + k - 1}} \times G_{\ell_1 + \ell_2, k}^{k, \ell_1 + \ell_2}\left[\frac{(s\,W_3)^k k^{-k}}{(\beta_1 \ell_1)^{-\ell_1} (\beta_2 \ell_2)^{-\ell_2}}\middle| \begin{matrix} \Delta(\ell_1, 1-\alpha_1), \Delta(\ell_2, 1-\alpha_2) \\ \Delta(k, 0) \end{matrix}\right]. \quad (31)$$

Following the same procedure, the N-fold integral in (26) can be expressed in closed form as in (3).

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