Outage probability of relayed free space optical communication systems

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Free space optical (FSO) communication is highly sensitive to atmospheric turbulence. The use of relays where the distance between the transmitter and the receiver is scaling down via multi-hop routing can improve the performance of an FSO link. In this reported work, the outage probability of a multi-hop FSO communication system with amplify-and-forward relays, assuming strong turbulence fading channels, is analytically derived. The turbulence-induced fading is modelled as a multiplicative random process which follows the K or the negative exponential distribution.

System and channel model: A multi-hop free space optical (FSO) communication system with intensity modulation/direct direction links-hops using on-off keying is considered. We consider N-1 channel state information (CSI)-based relays where each one has knowledge of the CSI of its preceding hop. For such a system the equivalent end-to-end signal-to-noise ratio (SNR), i.e. the SNR at the receiver, can be written as:

$$\mu = \frac{\prod_{i=1}^{N} s_i^2 g_{i-1}^2}{\sum_{i=1}^{N} n_i \left(\prod_{j=i+1}^{N} g_{j-1}^2 s_j^2\right)} \tag{1}$$

where $s_i = \eta I_i$ denotes the instantaneous gain of the ith hop, g_i is the gain of the ith relay, n_i is the additive white Gaussian noise signal at the input of the ith relay with power N_0, η the effective photocurrent conversion ratio of the receiver and I_i the turbulence induced light intensity at the ith hop. A choice for the gain which ignores the noise of the preceding hop was proposed in [1] as $g_i^2 = 1/s_i^2$. Hence (1) takes the form

$$\mu = \left(\sum_{i=1}^{N} \frac{1}{\mu_i}\right)^{-1} \tag{2}$$

where $\mu_i = \eta^2 I_i^2 / N_0$ is the instantaneous electrical SNR of the ith hop.

Outage probability: The equivalent end-to-end outage probability is generally given by

$$\begin{aligned} P_{out} &= \Pr(\mu \le \mu_{th}) = \Pr\left(\frac{1}{\mu} \ge \frac{1}{\mu_{th}}\right) \\ &= 1 - L^{-1}\left(\frac{M_{1/\mu}(s)}{s}\right)\bigg|_{1/\mu_{th}} \end{aligned} \tag{3}$$

where μ is the instantaneous SNR, μ_{th} a specified threshold, $L^{-1}(\bullet)$ is the inverse Laplace transform and $M_{1/\mu}(\bullet)$ is the moment generating function (MGF) of $1/u$. Using (2) and due to the independency of the turbulence-induced fading channels we have:

$$M_{1/\mu}(s) = \prod_{i=1}^{N} M_{1/\mu_i}(s) \tag{4}$$

A. K channel model: The probability density function (pdf) of the K distribution is given as in [2]

$$f_{I_i}(I) = \frac{2\alpha^{(\alpha+1)/2}}{\Gamma(\alpha)\bar{I}_i}\left(\frac{I}{\bar{I}_i}\right)^{(\alpha-1)/2} K_{a-1}\left(2\sqrt{\alpha\left(\frac{I}{\bar{I}_i}\right)}\right), \qquad I \ge 0 \tag{5}$$

where α is a channel parameter related to the effective number of discrete scatterers, $\Gamma(\cdot)$ is the well-known Gamma function, $\bar{I}_i$ denotes the average irradiance of the ith channel and $K_\nu(\cdot)$ is the νth-order modified Bessel function of the second kind, defined in [3, equation (8.432)]. After a simple power transformation the pdf of the electrical SNR of each hop, μ_i, can be derived as

$$f_{\mu_i}(\mu) = \frac{\alpha^{(\alpha+1)/2}}{\Gamma(\alpha)\sqrt{\bar{\mu}_i \mu}}\left(\sqrt{\frac{\mu}{\bar{\mu}_i}}\right)^{(\alpha-1)/2} K_{\alpha-1}\left(2\sqrt{\alpha\sqrt{\frac{\mu}{\bar{\mu}_i}}}\right) \tag{6}$$

where $\bar{\mu}_i = (\eta E[I])^2/N_0$ is the electrical average SNR defined in [4]. Using (6) and the definition of the MGF of $1/\mu_i$ we have

$$\begin{aligned} M_{1/\mu_i(s)} = {} & \frac{a^{(a+1)/2}}{2\Gamma(\alpha)\sqrt{\bar{\mu}_i^{\alpha}}}\int_0^{\infty} \mu^{(\alpha-3)/4} G_{1,0}^{0,1}\left[\frac{\mu}{s}\Big|_{-}^{1}\right] G_{0,2}^{2,0} \\ & \times \left[a\sqrt{\frac{\mu}{\bar{\mu}_i}}\,\middle|\, \begin{matrix} - \\ \frac{a-1}{2}, \frac{1-a}{2} \end{matrix}\right] d\mu \end{aligned} \tag{7}$$

where the $K_\nu(\cdot)$ [5, equation (14)] and exponential function [5, equation (11)], [3, equation (9.31.2)] are written in terms of the Meijer's G-function, defined in [3, equation (9.301)]. The integral in (7) can be evaluated with the help of [5, equation (21)], yielding to

$$\begin{aligned} M_{1/\mu_i(s)} = {} & \frac{a^{(a+1)/2} s^{(\alpha+1)/4}}{4\pi\Gamma(\alpha)^4\sqrt{\bar{\mu}_i^{\alpha+1}}} G_{0,5}^{5,0} \\ & \times \left[\frac{sa^2}{16\bar{\mu}_i}\,\middle|\, \begin{matrix} - \\ \frac{a-1}{2}, \frac{a+1}{4}, \frac{1-a}{4}, \frac{3-a}{4}, -\frac{a+1}{4} \end{matrix}\right] \end{aligned} \tag{8}$$

B. Negative (NE) channel model: Assuming an NE turbulence model, the pdf of the channel irradiance, can be written as

$$f_{I_i}(I) = \frac{I}{\bar{I}_i}\exp\left(-\frac{I}{\bar{I}_i}\right), \qquad I \ge 0 \tag{9}$$

Then, the pdf of μ_i, after power transformation of I_i, can be simply derived as

$$f_{\mu_i}(\mu) = \frac{1}{2\sqrt{\bar{\mu}_i \mu}}\exp\left(-\sqrt{\frac{\mu}{\mu_i}}\right) \tag{10}$$

Following the same method as previously we have

$$M_{1/\mu_i(s)} = \sqrt{\frac{s}{4\pi\bar{\mu}_i}}\, G_{0,3}^{3,0}\left[\frac{s}{4\bar{\mu}_i}\,\middle|\, \begin{matrix} - \\ \frac{-1}{2}, 0, \frac{1}{2} \end{matrix}\right] \tag{11}$$

By substituting (8) or (11) in (4), $M_{1/\mu}(s)$ is derived in closed-form. The outage probability can then be evaluated using any numerical method for the inverse Laplace transform (e.g. as the one used in [1]).

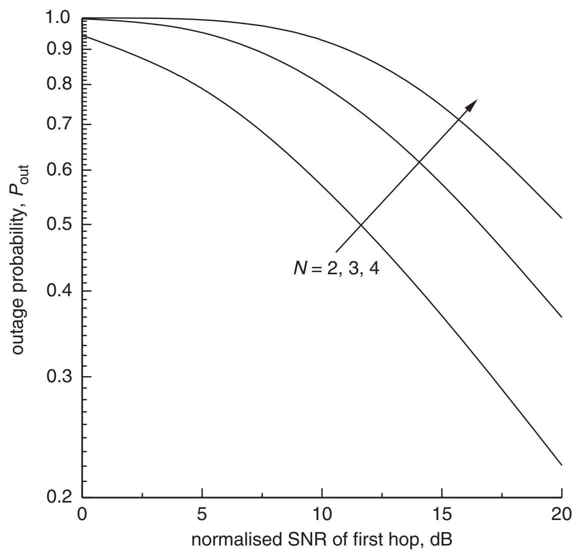

Fig. 1 *End-to-end outage probability of N-hop FSO communication system over non-identical NE channels against $\bar{\mu}_1/\mu_{th}$*

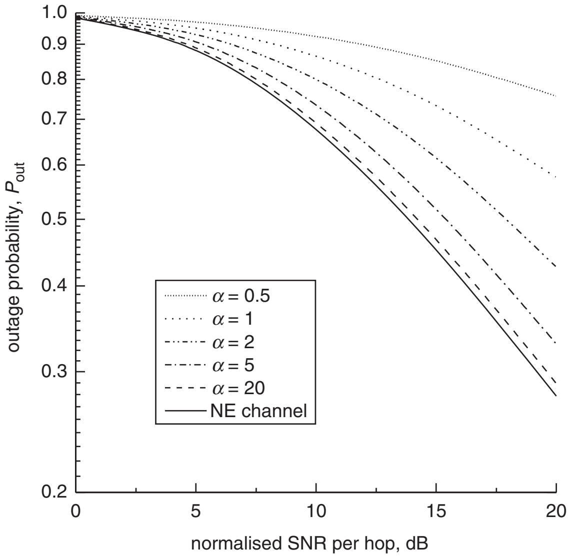

Fig. 2 *End-to-end outage probability of triple-hop FSO communication system over identical K distributed channels against* $\bar{\mu}/\mu_{th}$

Numerical results: Fig. 1 shows the outage probability against the normalised to outage threshold average SNR of the first hop, $\bar{\mu}_1/\mu_{th}$ assuming non-identical NE channels (i.e. $\bar{\mu}_i = \bar{\mu}/i$). We notice that the outage performance of the system degrades as the number of hops increases, this being similar behaviour as in RF multi-hop systems. Fig. 2 shows outage probability against normalised average SNR per hop over identical K distributed channels (i.e. $\bar{\mu}_i = \bar{\mu}$) for several values of parameter α. The system performance is not significantly improved, even for high values of α. This happens because K distribution is less sensitive at large values of α. Moreover, it is also observed that the outage probability when $\alpha = 20$ is quite close to that for the NE channel. This was expected since K distribution tends to NE one as $a \rightarrow \infty$.

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