On the Performance Analysis of Equal-Gain Diversity Receivers over Generalized Gamma Fading Channels

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***Abstract*— A versatile envelope distribution which generalizes many commonly used models for multipath and shadow fading is the so-called generalized Gamma (GG) distribution. By considering the product of N GG random variables (RV)s, novel expressions for its moments-generating, probability density, and cumulative distribution functions are obtained in closed form. These expressions are used to derive a closed-form union upper bound for the distribution of the sum of GG distributed RVs. The proposed bound turns out to be an extremely convenient analytical tool for studying the performance of N-branch equal-gain combining receivers operating over GG fading channels. For such receivers, first the moments of the signal-to-noise (SNR) at the output, including average SNR and amount of fading, are obtained in closed form. Furthermore, novel union upper bounds for the outage and the average bit error probability are derived and evaluated in terms of Meijer's G-functions. The tightness of the proposed bounds is verified by performing comparisons between numerical evaluation and computer simulations results.**

***Index Terms*— Equal-gain combining (EGC), generalized fading channels, generalized Gamma, Lognormal, Nakagami-m, outage probability, sum of random variables, Weibull.**

I. INTRODUCTION

A GENERAL envelope distribution which includes many well-known channel models for both multipath as well as for shadow fading is the so-called generalized Gamma (GG) distribution. This distribution was introduced by Stacy, back in 1962, as a generalization of the (two-parameter) Gamma distribution [1] and it includes the Rayleigh, Nakagami-m, and Weibull as special cases, while it can also describe the Lognormal as a limiting case. Interestingly enough, despite its ability to characterize so many different fading channel models, only very recently the topic of performance analysis of digital receivers over this generalized channel has gained renewed interest. Particularly, in an early work on this topic [2], Coulson *et al.* presented expressions in the form of infinite series for the average bit error probability (ABEP) of single-branch receivers operating over a GG fading environment, with binary phase-shift keying (BPSK) and binary frequency-shift keying (BFSK) modulations. In another related work [3], Yacoub introduced the α-μ distribution and gave a physical justification for the origin of the GG model. More recently, Aalo *et al.* presented a closed-form expression for the ABEP for both coherent and noncoherent/differentially coherent binary digital modulations [4].

The performance of diversity receivers has been extensively studied in past for the most important fading channel models, Rayleigh, Nakagami-m, and Weibull (e.g. see [5]–[8]). However, a performance study of diversity and specifically equal-gain combining (EGC) receivers over GG fading channels has not been presented yet[1]. The main difficulty in studying EGC receivers is that the distribution of the sum of fading envelopes is required. The derivation of this distribution in terms of tabulated functions is a very difficult task [5]. Concerning this well-recognized but cumbersome statistical problem, several approaches aiming at providing possible solutions have been published in the open technical literature. In possibly one of the earliest works, Stacy in his original GG paper developed an infinite series approach for determining the cumulative density function (cdf) of the sum of GG distributed random variables (RV)s [1]. Many years later, in an approach independent from that in [1], Beaulieu derived an infinite series for determining the cdf of the sum of Rayleigh distributed RVs [10]. Helstrom has computed the distribution of such a sum using saddle-point integration for uniformly weighted RVs [11], as well as for arbitrary weights [12]. Filho and Yacoub in [13] have derived an approximate probability density function (pdf) expression for the sum of Nakagami-m RVs. Very recently, Hu and Beaulieu have presented accurate and simple closed-form approximations to the cdf and pdf of the sum of independent and identically distributed (i.i.d.) Rayleigh RVs [14], while

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[1]After our paper has been accepted for publication, we became aware of another independent from our work contribution [9], which also address the problem of analyzing the error rate performance of EGC receivers over generalized Gamma fading channels. However, in [9] the solutions proposed are presented in integral form, whereas our approach involves closed-forn bounds.

Karagiannidis *et al.* have presented a closed-form union upper bound for the cdf of the weighted sum of N independent Rayleigh RVs [15]. All in all, although the problem of finding the distribution of the sum of fading envelopes has been extensively studied in the past for various distributions, the majority of the published methods being approximate solutions usually involving a truncation error. Hence, the derivation of an exact solution, in terms of tabulated functions, even for the simplest Rayleigh distribution, when $N > 2$ and with nonidentical statistical parameters, still remains an open research problem.

In this paper, in an effort to provide a solution to this problem and within the framework of studying the performance of EGC receivers over GG fading channels, another approach is proposed. Since an analytical solution for the distribution of the sum of RVs is very difficult to derive, the use of union bounds is proposed. In particular, by deriving a useful expression for the cdf of the *product* of N GG RVs and based on a well-known inequality between arithmetic and geometric means, closed-form union upper bounds for the cdf of the sum of GG distributed RVs are obtained. These bounds, which turn out to be quite tight, are used to analyze the ABEP and outage performance of N-branch EGC receivers operating over GG fading channels.

The remainder of the paper is organized as follows. In Section II, various statistical characteristics of the GG distribution are provided, while formulae for the distribution of the product as well as an upper bound for the cdf of the sum of N GG fading envelopes are presented. In Section III, various performance criteria of EGC receivers operating over GG fading channels are obtained, while in Section IV, numerical and computer simulation results are presented and compared. Finally, useful concluding remarks are provided in Section V.

II. Statistics of the GG Distribution

Let us consider $N \geq 1$ independent three-parameters' GG distributed RVs $\{R_\ell\}_{\ell=1}^{N}$ with pdf given by [1, eq. (1)]

$$f_{R_\ell}(r) = \frac{\beta_\ell\, r^{m_\ell \beta_\ell - 1}}{(\Omega_\ell/m_\ell)^{m_\ell}\, \Gamma(m_\ell)} \exp\left(-\frac{m_\ell}{\Omega_\ell} r^{\beta_\ell}\right) \tag{1}$$

where $\beta_\ell > 0$ and $m_\ell \geq 1/2$ are two parameters related to the fading severity, Ω_ℓ is related to the average fading power as $\mathcal{E}\langle R_\ell^2\rangle = (\Omega_\ell/m_\ell)^{2/\beta_\ell}\, \Gamma(m_\ell + 2/\beta_\ell)/\Gamma(m_\ell)$, with $\mathcal{E}\langle\cdot\rangle$ denoting expectation, and $\Gamma(\cdot)$ being the Gamma function [16, eq. (8.310/1)]. The distribution in (1) is very generic[2] since it includes commonly used fading models such as Rayleigh (for $\beta_\ell = 2$ and $m_\ell = 1$), Nakagami-m (for $\beta_\ell = 2$), and Weibull (for $m_\ell = 1$) as special cases. Moreover, for the limiting case of $\beta_\ell \to 0$ and $m_\ell \to \infty$, (1) becomes the well-known Lognormal pdf. The cdf and the nth-order moment of R_ℓ can be expressed as

$$F_{R_\ell}(r) = 1 - \frac{1}{\Gamma(m_\ell)} \Gamma\left(m_\ell, \frac{m_\ell}{\Omega_\ell} r^{\beta_\ell}\right) \tag{2}$$

and

$$\mathcal{E}\langle R_\ell^n\rangle = \left(\frac{\Omega_\ell}{m_\ell}\right)^{n/\beta_\ell} \frac{\Gamma(m_\ell + n/\beta_\ell)}{\Gamma(m_\ell)} \tag{3}$$

respectively, where n is a positive number and $\Gamma(\cdot,\cdot)$ is the upper incomplete Gamma function [16, eq. (8.350/2)].

A. Distribution of the Product of GG Variates

Let us define another RV, Y, as the product of the N GG distributed RVs R_ℓ, i.e.,

$$Y \triangleq \prod_{i=1}^{N} R_i \tag{4}$$

with β_ℓ's belong to rationals.

Theorem 1 (Moments-generating function): The moments-generating function (mgf) of Y is given by

$$\mathcal{M}_Y(s) = V\, G_{p,n}^{n,p}\left[W\, s^n \,\middle|\, \begin{matrix} I_n(\beta_\ell; 1-m_\ell) \\ \Delta(n\,;\,0) \end{matrix}\right] \tag{5}$$

where $G[\cdot]$ is the Meijer's G-function[3] [16, eq. (9.301)], $I_n(\beta_\ell; 1-m_\ell) \triangleq \Delta(n/\beta_1\,;\,1-m_1),\ \Delta(n/\beta_2\,;\,1-m_2), \ldots, \Delta(n/\beta_N\,;\,1-m_N)$, and $\Delta(k;x)$ is defined as $\Delta(k;x) \triangleq x/k,\ (x+1)/k, \ldots,\ (x+k-1)/k$, with x being an arbitrary real value and k a positive integer,

$$V = \sqrt{n}\left(\sqrt{2\pi}\right)^{N+1-n-p} \prod_{i=1}^{N} \frac{(n/\beta_i)^{m_i - 1/2}}{\Gamma(m_i)} \tag{6a}$$

and

$$W = \frac{1}{n^n} \prod_{i=1}^{N} \left(\frac{n\,\Omega_i}{m_i\,\beta_i}\right)^{n/\beta_i}. \tag{6b}$$

Moreover, n and p are two positive integers defined as

$$n \triangleq \prod_{i=1}^{N} k_i \tag{6c}$$

and

$$p \triangleq n \sum_{i=1}^{N} \frac{1}{\beta_i} \tag{6d}$$

under the constraint that

$$l_\ell = \frac{1}{\beta_\ell} \prod_{i=1}^{\ell} k_i \tag{6e}$$

is a positive integer, with k_ℓ and l_ℓ being also positive integers.

Proof: The proof is given in the Appendix. ■

The values for k_ℓ and l_ℓ can be found as follows: Depending upon the value β_1 taken in (6e), k_1 and l_1 are two minimum positive integers such that $l_1 = k_1/\beta_1$ holds (e.g. for $\beta_1 = 3.6$, $k_1 = 18$ and $l_1 = 5$). Similarly, k_ℓ and l_ℓ are two minimum positive integers such that (6e) holds. Note that for identical and integer order fading parameters (5) significantly simplifies.

[2]As pointed out in [4], the pdf of (1), introduced by Stacy in [1], is different from the type of generalization for the Gamma distribution presented in [17] which models the power of Rice-faded envelopes.

[3]Note that $G[\cdot]$ can be expressed in terms of more familiar generalized hypergeometric functions ${}_pF_q(\cdot;\cdot;\cdot)$ [16, eq. (9.14/1)] using the transformation presented in [16, eq. (9.303)], with p and q being positive integers. In addition, both $G[\cdot]$ and ${}_pF_q(\cdot;\cdot;\cdot)$ are included as built-in functions in most of the popular mathematical software packages such as Maple™ or Mathematica™.

Thus, for $\beta_\ell = \beta \; \forall \ell$, with $\beta \in \mathcal{N}$ being integer, then $n = \beta$, $p = N$, and $l_\ell = 1$, and hence, (5) reduces to

$$\mathcal{M}_Y(s) = \frac{\sqrt{\beta}\left(\sqrt{2\,\pi}\right)^{1-\beta}}{\prod_{i=1}^{N}\Gamma(m_i)} \times G_{N,\beta}^{\beta,N}\left[\left(\frac{s}{\beta}\right)^{\beta}\prod_{i=1}^{N}\frac{\Omega_i}{m_i}\,\middle|\,\begin{matrix}1-m_1,\,1-m_2,\ldots,\,1-m_N\\ \Delta(\beta\,;\,0)\end{matrix}\right] \tag{7}$$

which for $m_\ell = 1$ and $N = 1$ reduces to a known result [7, eq. (5)].

Lemma 1 (Probability density function): The pdf of Y is given by

$$f_Y(y) = \frac{\sqrt{n}\,V\,\left(\sqrt{2\,\pi}\right)^{n-1}}{y}\,G_{p,0}^{0,p}\left[\frac{W n^n}{y^n}\,\middle|\,\begin{matrix}I_n(\beta_\ell;1-m_\ell)\\ -\end{matrix}\right]. \tag{8}$$

Proof: Applying the inverse Laplace transform $\mathcal{L}^{-1}(\cdot;\cdot)$ [16, Sec. 17.11] in (5), the pdf of Y [16, Sec. 17.11]

$$f_Y(y) = \mathcal{L}^{-1}\left\{\mathcal{M}_Y(s); y\right\} \tag{9}$$

can be obtained in closed form using [18, eq. (21)] as in (8). ■

Lemma 2 (Cumulative distribution function): The cdf of Y is given by

$$F_Y(y) = \frac{V\left(\sqrt{2\,\pi}\right)^{n-1}}{\sqrt{n}}\,G_{1,\,p+1}^{p\,,\,1}\left[\frac{y^n}{W\,n^n}\,\middle|\,\begin{matrix}1\\ I_n(\beta_\ell;m_\ell),\,0\end{matrix}\right]. \tag{10}$$

Proof: Since the cdf of Y is given by

$$F_Y(y) = \int_0^y f_Y(x)\,dx \tag{11}$$

by using (8) and [18, eq. (26)], (10) can be easily obtained. ■

Note that for $m_\ell = 1$ and $\beta_\ell = 2 \; \forall \ell$, (10) can be reduced to [15, eq. (19)].

B. Distribution of the Sum of GG Variates

Let us define S to be the sum of N GG RVs, i.e.,

$$S \triangleq \sum_{i=1}^{N} R_i. \tag{12}$$

Theorem 2 (A cdf bound of the sum of GG RVs): The cdf of S is upper bounded as

$$F_S(y) \leq F_Y\left[\left(\frac{y}{N}\right)^N\right]. \tag{13}$$

Proof: Using the well-known inequality for the arithmetic and geometric means [16, Sec. 11.116]

$$\mathcal{A}_N \geq \mathcal{G}_N \tag{14}$$

with

$$\mathcal{A}_N \triangleq \frac{1}{N}\sum_{i=1}^{N} R_i \tag{15}$$

and

$$\mathcal{G}_N \triangleq \prod_{i=1}^{N} R_i^{1/N} \tag{16}$$

being the arithmetic and geometric means, respectively, S can be lower bounded as

$$S \geq N \prod_{i=1}^{N} R_i^{1/N}. \tag{17}$$

Using (4), (10), and (17), it can be easily seen that the cdf of S can be upper bounded as in (13). ■

It is interesting to note that the problem of obtaining an upper bound for the cdf of S with nonidentically distributed RVs (Ω_ℓ: average powers) may be equivalently stated as finding an upper bound for the cdf of a weighted sum of N i.i.d. RVs having equal average powers each, Ω, with weights $w_\ell = \sqrt{\Omega_\ell/\Omega}$.

III. Performance Analysis of Equal-Gain Diversity Receivers

Let us consider an N-branch EGC receiver operating over independent, but not necessarily identically distributed, GG fading channels. The baseband received signal in the ℓth ($\ell = 1, 2, \ldots, N$) antenna is $\zeta_\ell = w\,R_\ell\,\exp(\jmath\,\psi_\ell) + n_\ell$, where w is the complex transmitted symbol, with $E_s = \mathcal{E}\left\langle |w|^2\right\rangle$ being the transmitted average symbols' energy, R_ℓ is the instantaneous fading envelope being modeled as a GG distributed RV, ψ_ℓ is the instantaneous phase of the channel, and n_ℓ is the instantaneous additive white Gaussian noise (AWGN) sample with single-sided power spectral density N_0 identical for all channels. The usual assumption is made that the ψ_ℓ's are known to the receiver.

The instantaneous SNR per symbol of the ℓth diversity channel can be expressed as

$$\gamma_\ell = R_\ell^2\,\frac{E_s}{N_0} \tag{18}$$

with its corresponding average SNR being

$$\overline{\gamma}_\ell = \mathcal{E}\left\langle R_\ell^2\right\rangle\frac{E_s}{N_0} = (m_\ell)_{2/\beta_\ell}\left(\frac{\Omega_\ell}{m_\ell}\right)^{2/\beta_\ell}\frac{E_s}{N_0} \tag{19}$$

where $(\xi)_u$ is the Pochhammer symbol defined as $(\xi)_u = \Gamma(\xi + u)/\Gamma(\xi)$. Based on an interesting property of the GG distribution, that the nth power of a GG distributed RV with parameters $(m_\ell, \beta_\ell, \Omega_\ell)$ is another GG distributed RV with parameters $(m_\ell, \beta_\ell/n, \Omega_\ell)$, it can be easily concluded that γ_ℓ is also a GG distributed RV with parameters $\left(m_\ell, \beta_\ell/2, (\Xi_\ell\,\overline{\gamma}_\ell)^{\beta_\ell/2}\right)$ where $\Xi_\ell = 1/(m_\ell)_{2/\beta_\ell}$. Hence, by using the formulae for $\{R_\ell\}$ given by (1)–(3), the corresponding expressions for $\{\gamma_\ell\}$ can be easily derived, replacing β_ℓ with $\beta_\ell/2$ and Ω_ℓ/m_ℓ with $(\Xi_\ell\,\overline{\gamma}_\ell)^{\beta_\ell/2}$ helping us to study the performance of multi-branch diversity receivers operating over GG fading channels. For example, from (2), the cdf of γ_ℓ can be derived as

$$F_{\gamma_\ell}(\gamma) = 1 - \frac{1}{\Gamma(m_\ell)}\,\Gamma\left[m_\ell, \left(\frac{\gamma}{\Xi_\ell\,\overline{\gamma}_\ell}\right)^{\beta_\ell/2}\right]. \tag{20}$$

According to the above mentioned property, hereafter in this paper, V and W are modified as

$$V = \sqrt{n}\,\left(\sqrt{2\,\pi}\right)^{N+1-n-p}\prod_{i=1}^{N}\frac{(2n/\beta_i)^{m_i-1/2}}{\Gamma(m_i)} \tag{21a}$$

and

$$W = \frac{1}{n^n} \prod_{i=1}^{N} \left(\Xi_i \, \overline{\gamma}_i\right)^n \left(\frac{2n}{\beta_i}\right)^{2n/\beta_i} \tag{21b}$$

with $p = 2n \sum_{i=1}^{N} (1/\beta_i)$ and $l_\ell = 2\,(\prod_{i=1}^{\ell} k_i)/\beta_\ell$.

A. Moments of the Output SNR

The instantaneous EGC output SNR per symbol is given by

$$\gamma_{\text{egc}} = \frac{1}{N} \left(\sum_{i=1}^{N} \sqrt{\gamma_i}\right)^2. \tag{22}$$

Using the multinomial identity [19, eq. (24.1.2)], the nth-order moment of γ_{egc}, $\mu_n = \mathcal{E}\left\langle \gamma_{\text{egc}}^n \right\rangle$, can be derived as

$$\begin{aligned} \mu_n &= \frac{1}{N^n} \, \mathcal{E} \left\langle \left(\sum_{i=1}^{N} \sqrt{\gamma_i}\right)^2 \right\rangle \\ &= \frac{(2n)!}{N^n} \sum_{\substack{k_1, k_2, \ldots, k_N = 0 \\ k_1 + k_2 + \cdots + k_N = 2n}}^{2n} \mathcal{E} \left\langle \prod_{i=1}^{N} \gamma_i^{k_i/2} / k_i! \right\rangle. \end{aligned} \tag{23}$$

Since the diversity input branches are uncorrelated, the mean product term in the above equation can be expressed as

$$\mathcal{E} \left\langle \prod_{i=1}^{N} \gamma_i^{k_i/2} \right\rangle = \prod_{i=1}^{N} \mathcal{E} \left\langle \gamma_i^{k_i/2} \right\rangle. \tag{24}$$

By substituting (3) and (24) in (23), the moments of the EGC output SNR for independent but not necessarily identically distributed input branches can be expressed in closed form as

$$\mu_n = \frac{(2n)!}{N^n} \sum_{\substack{k_1, k_2, \ldots, k_N = 0 \\ k_1 + k_2 + \cdots + k_N = 2n}}^{2n} \prod_{i=1}^{N} \frac{\left(\Xi_i \, \overline{\gamma}_i\right)^{k_i/2}}{k_i! \, \Gamma\left(m_i\right)} \Gamma\left(m_i + \frac{k_i}{\beta_i}\right). \tag{25}$$

1) Average output SNR: By setting $n = 1$ in (25), the EGC average output SNR per symbol, $\overline{\gamma}_{\text{egc}} = \mu_1$, can be obtained in closed form as

$$\begin{aligned} \overline{\gamma}_{\text{egc}} = \frac{1}{N} \Bigg[& \sum_{i=1}^{N} \overline{\gamma}_i + 2 \sum_{i=2}^{N} \sum_{j=1}^{i-i} \frac{\sqrt{\Xi_i \, \Xi_j \, \overline{\gamma}_i \, \overline{\gamma}_j}}{\Gamma\left(m_i\right) \Gamma\left(m_j\right)} \\ & \times \Gamma\left(m_i + \frac{1}{\beta_i}\right) \Gamma\left(m_j + \frac{1}{\beta_j}\right) \Bigg]. \end{aligned} \tag{26}$$

For $m_\ell = 1$ and $\beta_\ell = \beta \;\forall \ell$, (26) reduces to a known expression [8, eq. (18)] for Weibull fading channels. Moreover, for $m_\ell = m$ and $\beta_\ell = 2 \;\forall \ell$, (26) reduces to another known expression [20, eq. (19)] for Nakagami-m fading channels.

2) Amount of fading: By using (25), the amount of fading (AoF), defined as the ratio of the variance to the square average SNR per symbol, i.e., $A_F \triangleq \text{var}(\gamma_{\text{egc}})/\overline{\gamma}_{\text{egc}}^2$, can be expressed in a simple closed form as

$$A_F \triangleq \frac{\mu_2}{\mu_1^2} - 1. \tag{27}$$

B. Outage Probability

By using (12), (17), (18), and (22), a lower bound for γ_{egc} can be expressed as $\gamma_{\text{egc}} \geq \gamma_{\text{egc}}^* = N \prod_{i=1}^{N} \gamma_i^{1/N}$. The cdf of γ_{egc}^* can be derived by substituting $y = (\gamma/N)^N$ in (10), i.e.,

$$F_{\gamma_{\text{egc}}^*}(\gamma) = \frac{V \left(\sqrt{2\,\pi}\right)^{n-1}}{\sqrt{n}} \, G_{1,p+1}^{\,p,1} \left[\frac{(\gamma/N)^{nN}}{W n^n} \,\Bigg|_{I_{2n}(\beta_\ell; m_\ell), 0}^{1} \right]. \tag{28}$$

If γ_{th} is a certain specified threshold, then the outage probability is defined as the probability that γ_{egc} falls below γ_{th}. An upper bound for this probability can be obtained by replacing γ with γ_{th} in (28) as

$$P_{\text{out}}\left(\gamma_{\text{th}}\right) \leq F_{\gamma_{\text{egc}}^*}\left(\gamma_{\text{th}}\right). \tag{29}$$

C. Average Bit Error Probability

One straightforward approach to obtain a bound for the ABEP, $\overline{P}_{\text{be}}$, is to average the conditional bit error probability, $P_{\text{be}}(\gamma)$, over the pdf of γ_{egc}^*, i.e.,

$$\overline{P}_{\text{be}} \leq \int_0^\infty P_{\text{be}}(\gamma) \, f_{\gamma_{\text{egc}}^*}(\gamma) \, d\gamma. \tag{30}$$

By taking the first derivative of $F_{\gamma_{\text{egc}}^*}(\gamma)$ in (28) with respect to γ, the corresponding pdf can be obtained as

$$f_{\gamma_{\text{egc}}^*}(\gamma) = \frac{N \sqrt{n} \, V}{\gamma \left(\sqrt{2\,\pi}\right)^{1-n}} \, G_{0,p}^{\,p,0} \left[\frac{(\gamma/N)^{n\,N}}{W \, n^n} \,\Bigg|_{I_{2n}(\beta_\ell; m_\ell)}^{-} \right]. \tag{31}$$

Moreover, for $P_{\text{be}}(\gamma)$ there are well-known generic expressions for two different sets of modulation schemes:

i) BPSK, BFSK, M-ary differentially encoded phase-shift keying (M-DEPSK), quadrature phase-shift keying (QPSK), M-ary phase-shift keying (M-PSK), M-ary frequency-shift keying (M-FSK), and M-ary differential PSK (M-DPSK) in the form of

$$P_{\text{be}}(\gamma) = A \, \text{erfc}\left(\sqrt{B\,\gamma}\right) \tag{32}$$

where $\text{erfc}(\cdot)$ is the well-known complementary error function [16, eq. (8.250/4)];

ii) Differential BPSK (DBPSK) and M-ary noncoherent FSK (M-NFSK), in the form of

$$P_{\text{be}}(\gamma) = A \, \exp(-B\,\gamma). \tag{33}$$

The particular values of A and B in (32) and (33) depend on the specific modulation scheme employed and can be found in [21]. Next, (30) is solved in closed form for each of the above two sets of signals.

1) M-PSK, M-FSK, M-DEPSK, and M-DPSK: Using (30), (31), and (32), it can be easily recognized that for this first set of modulation schemes, the evaluation of definite integrals, which include Meijer's, power, and exponential functions, is required. Since such integrals are not tabulated, the solution can be found with the aid of [18, eq. (21)], and thus the ABEP can be upper bounded as

$$\begin{aligned} \overline{P}_{\text{be}} \leq & \frac{A\,V}{\sqrt{\pi\,n}} \, \frac{1}{\left(\sqrt{2\,\pi}\right)^{n\,(N-1)}} \\ & \times G_{2n\,N, p+n\,N}^{\,p\;,\;2n\,N} \left[\frac{n^{n\,(N-1)}}{W\,B^{n\,N}} \,\Bigg|_{I_{2n}(\beta_\ell; m_\ell), \Delta(nN;0)}^{\Delta(nN;1)\,,\,\Delta(nN;1/2)} \right]. \end{aligned} \tag{34}$$

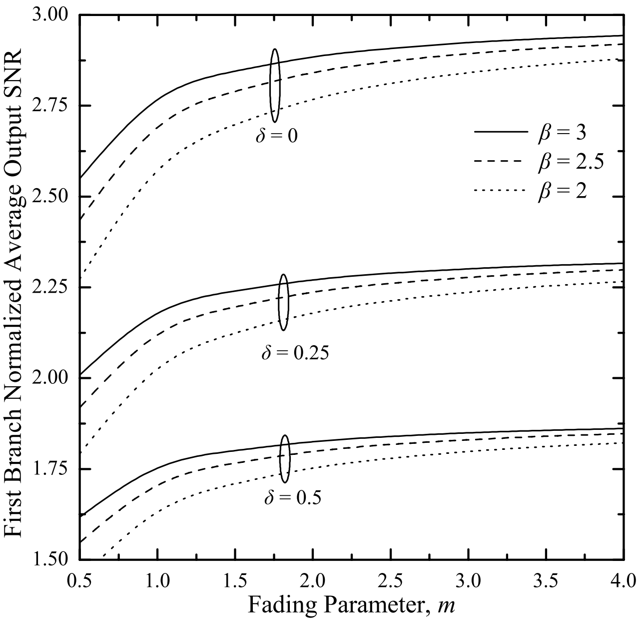

Fig. 1. First branch normalized EGC average output SNR as a function of m for $N = 3$.

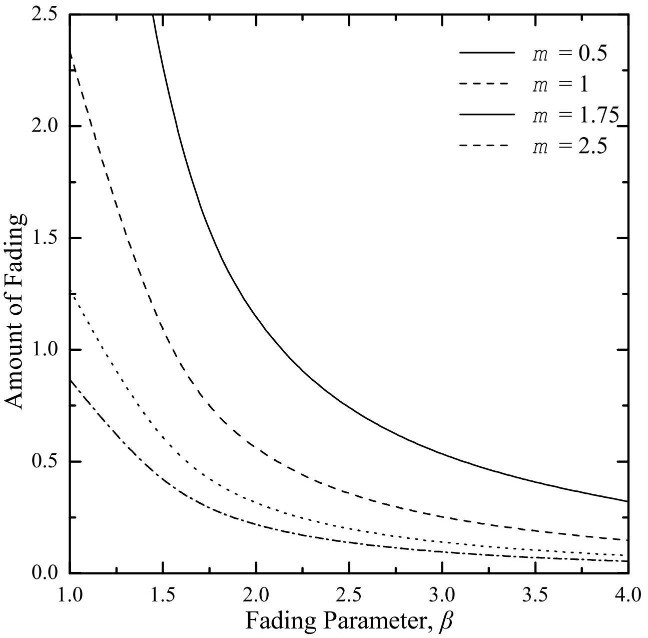

Fig. 2. Amount of fading at the output of the combiner as a function of β for $N = 2$ and i.i.d. branch SNRs.

2) DBPSK and M-NFSK: Similarly to the first set, for this second set (i.e., for DBPSK and M-NFSK), an upper bound for the ABEP can be derived as

$$\overline{P}_{\rm be} \leq \frac{A\,V\,\sqrt{N}}{\left(\sqrt{2\,\pi}\right)^{n\,(N-1)}}\, G^{\,p,n\,N}_{\,n\,N,p}\left[\frac{n^{n(N-1)}}{W B^{n\,N}}\,\middle|\,{}^{\Delta(nN;1)}_{I_{2n}(\beta_\ell;m_\ell)}\right]. \quad (35)$$

IV. Performance Evaluation and Discussion

In this section, using the previous mathematical analysis, numerical and simulation results are presented for the performance of EGC receivers operating over GG fading channels. For these performance evaluation results we consider the general case of not necessarily equal $\overline{\gamma}_\ell$'s. Particularly, an exponentially decaying power delay profile (PDP) is adopted

$$\overline{\gamma}_\ell = \overline{\gamma}_1 \exp[-\delta\,(\ell-1)] \quad (36)$$

with δ being the power decaying factor. For the convenience of the presentation of the performance evaluation results and without any loss of generality, it will be assumed that $m_\ell = m$ and $\beta_\ell = \beta\ \forall\ell$.

Using (26), Fig. 1 presents the first branch normalized average output SNR, $\gamma_{\rm egc}/\overline{\gamma}_1$, of a three-branch EGC receiver as a function of m, for several values of β and δ. It is noted that as m and/or β increases, $\gamma_{\rm egc}/\overline{\gamma}_1$ improves, while the combining loss of the receiver gets more accentuated as δ increases. Similar behavior has been also observed in [20] for Nakagami-m fading channels. By numerically evaluating (27), in Fig. 2, A_F is plotted as a function of β, for $N = 2$ and for several values of m. As expected, A_F decreases as β and/or m increases.

Having numerically evaluated (29), in Fig. 3, upper bounds for $P_{\rm out}$ are presented as a function of the normalized outage threshold, $\gamma_{\rm th}/\overline{\gamma}$, for $m = 2$, i.i.d. input branches (i.e., $\overline{\gamma}_\ell = \overline{\gamma}\ \forall\ell$), and different values of β and N. The obtained results clearly show that $P_{\rm out}$ improves with an increase of N and/or β. In order to verify the tightness of the bounds, curves obtained by means of computer simulations are also included for comparison purposes. By comparing the performances it is evident that the numerical results for the bounds (see (29)) are very close to the equivalent simulated ones which represent the exact $P_{\rm out}$ performance. This observation clearly demonstrates the accuracy of the proposed bounds. It is also noted that as β increases, the proposed bounds become even tighter. However, as N increases, the difference between the two performance results slightly increases. In Fig. 4, $P_{\rm out}$ is plotted as a function of the first branch normalized outage threshold, $\gamma_{\rm th}/\overline{\gamma}_1$, for $\beta = 2.5$, $N = 2$, and $\overline{\gamma}_1 = 0.5\,\overline{\gamma}_2$. These results suggest that the higher m is, the smaller are the differences between numerical and computer simulation results for $P_{\rm out}$. For example, at $P_{\rm out} = 10^{-3}$, the differences between them for $m = 1$, 2, and 4 are less than 2, 1, and 0.5 dB, respectively. The trend of the performance, as illustrated in Figs. 3 and 4, can be explained as follows. It is clear that the smaller the difference between the terms of the left hand side (LHS) and right hand side (RHS) of (14), the tighter are the bounds. In fact, the equality in (14) holds if and only if all R_ℓ's are equal with each other, i.e., $R_1 = R_2 = \cdots = R_N$. For relatively large values of m_ℓ's and/or β_ℓ's, all fading envelopes R_ℓ will be, with high probability, close to their average value, and thus, it is expected that R_ℓ's will take similar values. As for N, the smaller its value is the tighter are the bounds. This happens because both bounds and exact curves move towards the performance obtained for $N = 1$. In fact, from (14) it can be seen that for $N = 1$ these two curves coincide.

Using (34) and (35), the ABEP performance of an N-branch EGC receiver for several coherent and noncoherent binary and multilevel modulation schemes has been obtained. In Figs. 5 and 6, the ABEP, $\overline{P}_{\rm be}$, of DBPSK is plotted as a function of $\overline{\gamma}$ for i.i.d. fading statistics. In Fig. 5, $\overline{P}_{\rm be}$ is plotted as a function of $\overline{\gamma}$, for $\beta = 3$ and several values of m. As expected,

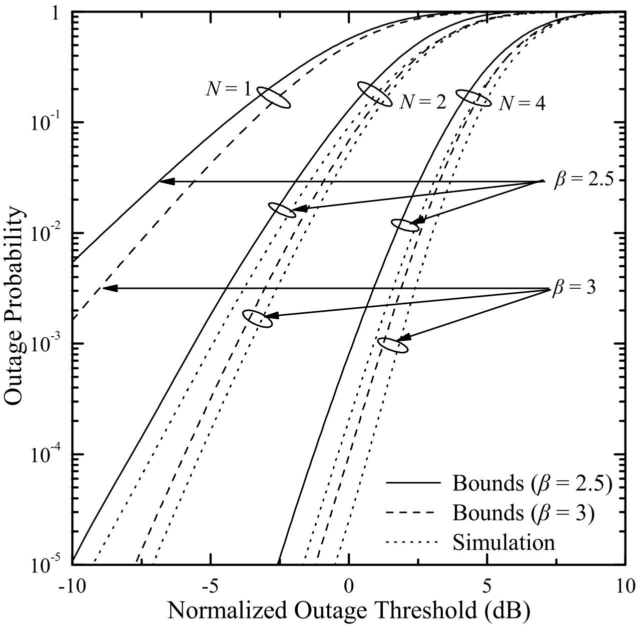

Fig. 3. Outage probability as a function of the average input SNR for $m = 2$ and i.i.d. branch SNRs.

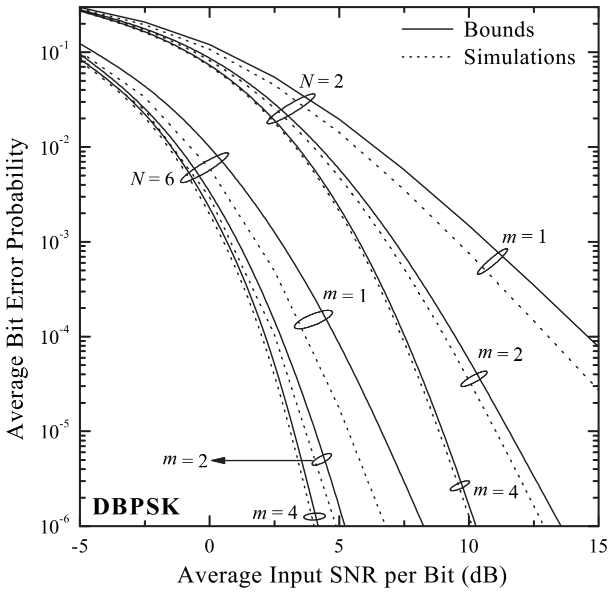

Fig. 5. ABEP of EGC with DBPSK modulation format as a function of the average input SNR per bit for $\beta = 3$ and i.i.d. branch SNRs.

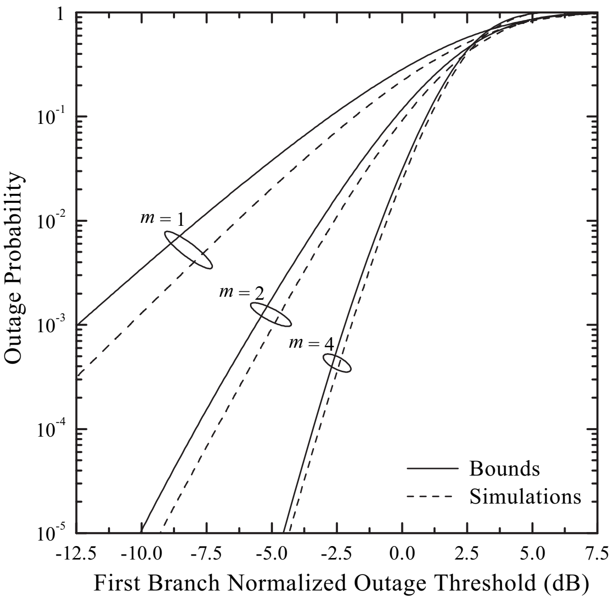

Fig. 4. Outage probability as a function of the first branch normalized average input SNR for $\beta = 2.5$ and $\overline{\gamma}_1 = 0.5\,\overline{\gamma}_2$.

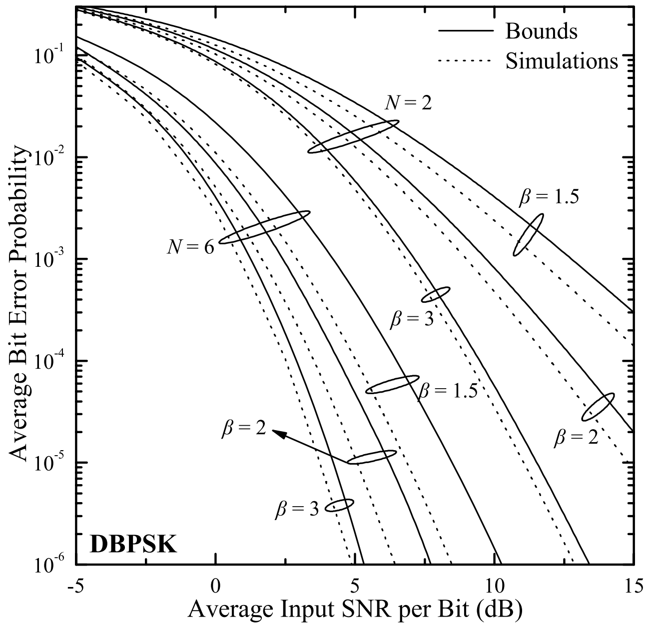

Fig. 6. ABEP of EGC with DBPSK modulation format as a function of the average input SNR per bit for $m = 2$ and i.i.d. branch SNRs.

the obtained performance evaluation results show that $\overline{P}_{\rm be}$ improves with an increase of $\overline{\gamma}$. For comparison purposes the curves for the corresponding exact $\overline{P}_{\rm be}$, obtained via computer simulations, are also included in the same figure. By comparing the numerically evaluated results with the computer simulated ones, we deduce a close match between them. For example in Fig. 5, for $\overline{P}_{\rm be} = 10^{-5}$, $N = 6$ and $m = 2$, their difference is less than 0.5 dB. In Fig. 6, $\overline{P}_{\rm be}$ is plotted as a function of $\overline{\gamma}$, for $m = 2$ and several values of β. As for m, β, and N, similar findings with those observed from Figs. 3 and 4 can also be extracted. Fig. 7 presents $\overline{P}_{\rm be}$ of M-PSK with Gray encoding as a function of $\overline{\gamma}$ for $N = 3$, $m = 2$, $\beta = 2.5$, i.i.d. input branches, and several values of M. As expected, for a fixed $\overline{\gamma}$, $\overline{P}_{\rm be}$ degrades with increasing M. Furthermore, the higher M, the tighter the bounds. Finally, in Fig. 8, $\overline{P}_{\rm be}$ of 8-PSK with Gray encoding is plotted as a function of $\overline{\gamma}_1$ for $\beta = 2.5$, $\delta = 0.2$, and $M = 8$. Again here we note the tightness of the proposed bounds.

V. Conclusions

Capitalizing on the product of N GG RVs, its mgf, pdf, and cdf were obtained in closed form. These expressions were used to derive a closed-form union upper bound for the cdf

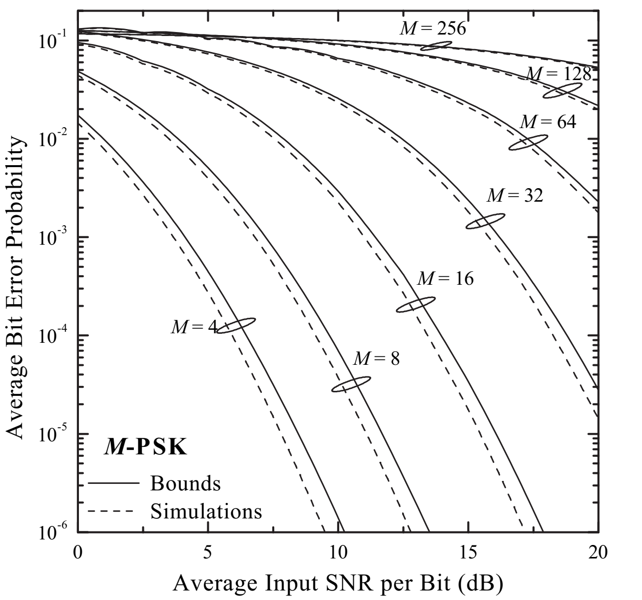

Fig. 7. ABEP of a triple-branch EGC with Gray encoded M-PSK modulation as a function of the average input SNR per bit for $m = 2$, $\beta = 2.5$, and i.i.d. branch SNRs.

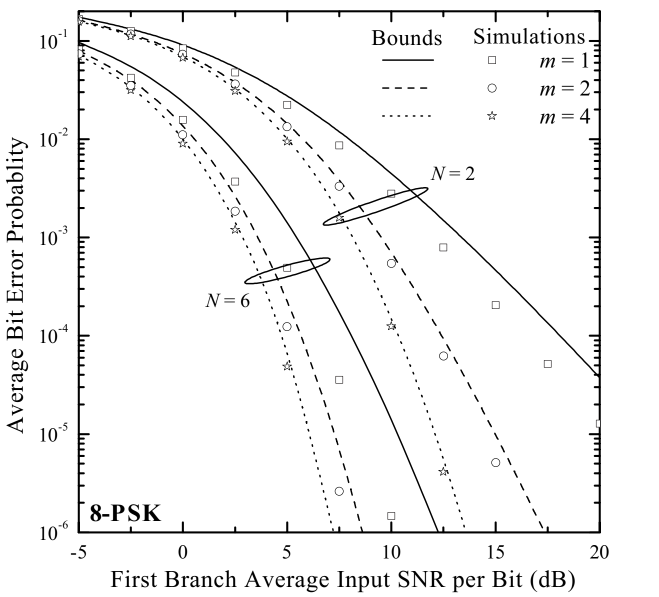

Fig. 8. ABEP of EGC with Gray encoded 8-PSK modulation as a function of the average SNR per bit of the first input branch for $\beta = 2.5$ and $\delta = 0.2$.

of the sum of GG distributed RVs. Based on this bound, the performance of N-branch EGC receivers operating over GG fading channels was studied and important performance measures such as the outage probability and the ABEP were obtained in closed form. By comparing numerical evaluation and computer simulation results, it was shown that the higher the value of m and/or β and/or the smaller the value of N, the tighter are the proposed bounds.

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APPENDIX
PROOF OF THEOREM 1

In order to prove Theorem 1, we follow a similar procedure as in [22]. Starting from the definition of the mgf of Y, i.e.,

$$\mathcal{M}_Y(s) = \mathcal{E}\left\langle \exp\left(-s\,Y\right)\right\rangle \tag{A–1}$$

and by using (1) and (4), $\mathcal{M}_Y(s)$ can be written as

$$\mathcal{M}_Y(s) = \left[\prod_{i=1}^{N}\left(\frac{m_i}{\Omega_i}\right)^{m_i}\frac{\beta_i}{\Gamma\left(m_i\right)}\right] \times \int_0^\infty \cdots \int_0^\infty \int_0^\infty \left(\prod_{i=1}^{N} r_i^{m_i\,\beta_i-1}\right)\exp\left(-s\prod_{i=1}^{N} r_i\right) \times \exp\left(-\sum_{i=1}^{N}\frac{m_i}{\Omega_i}\,r_i^{\beta_i}\right) dr_1\,dr_2\cdots dr_N \tag{A–2}$$

which after applying the transformation $z_\ell = r_\ell^{\beta_\ell}$ can be rewritten as

$$\mathcal{M}_Y(s) = \left[\prod_{i=1}^{N}\left(\frac{m_i}{\Omega_i}\right)^{m_i}\frac{1}{\Gamma\left(m_i\right)}\right] \times \int_0^\infty \cdots \int_0^\infty \int_0^\infty \left(\prod_{i=1}^{N} z_i^{m_i-1}\right)\exp\left(-s\prod_{i=1}^{N} z_i^{1/\beta_i}\right) \times \exp\left(-\sum_{i=1}^{N}\frac{m_i}{\Omega_i}\,z_i\right) dz_1\,dz_2\cdots dz_N. \tag{A–3}$$

With the aid of [18, eq. (11)], the above multiple integrals can be expressed in terms of Meijer's functions as

$$\mathcal{M}_Y(s) = \left[\prod_{i=1}^{N}\left(\frac{m_i}{\Omega_i}\right)^{m_i}\frac{1}{\Gamma\left(m_i\right)}\right] \times \int_0^\infty z_N^{m_N-1}\,G_{0,1}^{1,0}\left[\frac{m_N}{\Omega_N}\,z_N\,\middle|\,{}^{-}_{0}\right]\cdots\int_0^\infty z_2^{m_2-1} \times G_{0,1}^{1,0}\left[\frac{m_2}{\Omega_2}\,z_2\,\middle|\,{}^{-}_{0}\right]\int_0^\infty z_1^{m_1-1}G_{0,1}^{1,0}\left[s\prod_{i=1}^{N} z_i^{1/\beta_i}\,\middle|\,{}^{-}_{0}\right] \times G_{0,1}^{1,0}\left[\frac{m_1}{\Omega_1}\,z_1\,\middle|\,{}^{-}_{0}\right] dz_1\,dz_2\cdots dz_N. \tag{A–4}$$

In order to solve the above N-fold integral, the following properties for $\Delta\left(\cdot;\cdot\right)$ should be taken into consideration:

Property 1: For any real x and positive integers k and p, it holds that

$$\Delta[k;\,\Delta(p;\,x)] = \Delta(k\,p;\,x). \tag{A–5}$$

Property 2: For any real x and integer k, it holds that the sum of terms included in $\Delta(k;\,x)$, i.e., $\mathcal{S} = \sum_{i=0}^{N-1}(x+i)/k$, is given by

$$\mathcal{S} = x + \frac{k-1}{2}. \tag{A–6}$$

Starting from the inner integral (i.e., that on z_1), continuing towards the outer one (i.e., that on z_N), and using Properties 1 and 2, successive integrations of the form

$$\int_0^{\infty} z_\ell^{m_\ell - 1}\, G_{c,d}^{a,b}\left[\Psi\, z_\ell^{\varphi} \middle|_{J}^{H}\right]\, G_{0,1}^{1,0}\left[\frac{m_\ell}{\Omega_\ell}\, z_\ell \middle|_{0}^{-}\right]\, dz_\ell$$

arise due to the fact that each of these integrals can be solved in terms of a Meijer's function using [18, eq. (22)], with a, b, c, $d \in \mathcal{N}$, Ψ, $\varphi \in \Re^*$, and H, $J \in \Re$. Hence, after N successive integrations, (A–2) can be expressed in closed form as in (5).

References

[1] E. W. Stacy, "A generalization of the Gamma distribution," *Ann. Math. Stat.*, vol. 33, no. 3, pp. 1187–1192, Sept. 1962.

[2] A. J. Coulson, A. G. Williamson, and R. G. Vaughan, "Improved fading distribution for mobile radio," *IEE Proc. Commun.*, vol. 145, no. 3, pp. 197–202, June 1998.

[3] M. D. Yacoub, "The α-μ distribution: A general fading distribution," in *Proc. IEEE International Symposium on Personal, Indoor, Mobile Radio Commun.*, vol. 2, Lisbon, Sept. 2002, pp. 629–633.

[4] V. A. Aalo, T. Piboongungon, and C.-D. Iskander, "Bit-error rate of binary digital modulation schemes in generalized Gamma fading channels," *IEEE Commun. Lett.*, vol. 9, no. 2, pp. 139–141, Feb. 2005.

[5] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd ed. New York: Wiley, 2005.

[6] N. C. Beaulieu and A. A. Abu-Dayya, "Analysis of equal gain diversity on Nakagami fading channels," *IEEE Trans. Commun.*, vol. 39, no. 2, pp. 225–234, Feb. 1991.

[7] J. Cheng, C. Tellambura, and N. C. Beaulieu, "Performance of digital linear modulations on Weibull slow-fading channels," *IEEE Trans. Commun.*, vol. 52, no. 8, pp. 1265–1268, Aug. 2004.

[8] G. K. Karagiannidis, D. A. Zogas, N. C. Sagias, S. A. Kotsopoulos, and G. S. Tombras, "Equal-gain and maximal-ratio combining over nonidentical Weibull fading channels," *IEEE Trans. Wireless Commun.*, vol. 4, no. 3, pp. 841–846, May 2005.

[9] T. Piboongungon, V. A. Aalo, and C.-D. Iskander, "Average error rate of linear diversity reception schemes over generalized gamma fading channels," in *Proc. IEEE Southeastcon*, Ft. Lauderdale, FL, Apr. 2005, pp. 265–270.

[10] N. C. Beaulieu, "An infinite series for the computation of the complementary probability distribution function of a sum of independent random variables and its application to the sum of Rayleigh random variables," *IEEE Trans. Commun.*, vol. 38, no. 10, pp. 1463–1474, Sept. 1990.

[11] C. W. Helstrom, "Performance of receivers with linear detectors," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 26, no. 2, pp. 210–217, Mar. 1990.

[12] ——, "Computing the distribution of sums of random sine waves and of Rayleigh-distributed random variables by saddle-point integration," *IEEE Trans. Commun.*, vol. 45, no. 11, pp. 1487–1494, Nov. 1997.

[13] J. C. S. S. Filho and M. D. Yacoub, "Nakagami-m approximation to the sum of m non-identical independent Nakagami-m variates," *Electron. Lett.*, vol. 40, no. 15, pp. 951–952, July 2004.

[14] J. Hu and N. C. Beaulieu, "Accurate closed-form approximations to Rayleigh sum distributions and densities," *IEEE Commun. Lett.*, vol. 9, no. 2, pp. 109–111, Feb. 2005.

[15] G. K. Karagiannidis, T. A. Tsiftsis, and N. C. Sagias, "A closed-form upper-bound for the distribution of the weighted sum of Rayleigh variates," *IEEE Commun. Lett.*, vol. 9, no. 7, pp. 589–591, July 2005.

[16] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. New York: Academic, 2000.

[17] J. Cheng and T. Berger, "Performance analysis for MRC and postdetection EGC over generalized Gamma fading channels," in *Proc. IEEE Wireless Commun. and Networking Conf.*, vol. 1, New Orleans, LA, Mar. 2003, pp. 120–125.

[18] V. S. Adamchik and O. I. Marichev, "The algorithm for calculating integrals of hypergeometric type functions and its realization in REDUCE system," in *Proc. International Conf. on Symbolic and Algebraic Computation*, Tokyo, Japan, 1990, pp. 212–224.

[19] M. Abramovitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, 9th ed. New York: Dover, 1972.

[20] G. K. Karagiannidis, "Moments-based approach to the performance analysis of equal gain diversity in Nakagami-m fading," *IEEE Trans. Commun.*, vol. 52, no. 5, pp. 685–690, May 2004.

[21] S. Sampei, *Applications of Digital Wireless Technologies to Global Wireless Communications*. London: Prentice Hall, 1997.

[22] G. K. Karagiannidis, T. A. Tsiftsis, and R. K. Mallik, "Bounds of multihop relayed communications in Nakagami-m fading," *IEEE Trans. Commun.*, vol. 54, no. 1, pp. 18–22, Jan. 2006.

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