Letter

Outage Probability Analysis for a Nakagami Signal in L Nakagami Interferers

GEORGE KARAGIANNIDIS, CHRIS GEORGOPOULOS, STAVROS KOTSOPOULOS
Uninersity of Patras, Patras, 26500, Greece
roikos@otenet.gr, georgopoulos@ee.upatras.gr, kotsop@ee.upatras.gr

Abstract. Outage analysis is one of the primary objectives in the design and operation of the 3rd Generation (3G) micro- and pico- cellular mobile radio systems in order to increase spectrum efficiency and to control the Quality of Service (QoS) and Grade of Service (GoS) demands. This paper presents an alternative direct formulation of the outage probability for Nakagami fading channels in the presence of multiple L Nakagami interferers with arbitrary parameters. This formulation can be used with and without a constraint of a minimum signal power. Computer simulation results are also presented to illustrate the proposed formulation.

1 INTRODUCTION

In the early days of cellular system deployment, good geographical coverage was of high importance. Now, several other requirements have been added. For example the 3rd Generation (3G) mobile radio systems will concentrate on the service quality, system capacity and personal and terminal mobility issues.

Outage probability is a measure of controlling the unsatisfactory reception's level, helping the designers to readjust the system's operating parameters. The term 'outage' is related to the criterion used for the assessment of satisfactory reception. Generally, there are two criteria in common use from the designers of cellular systems. According to the first criterion outage probability is defined as the probability that the undesired signal's power exceeds the desired power, by a protection factor-ratio denoted as β. In the second criterion, a requirement of a minimum signal power, λ, is needed for satisfactory reception in addition to the first criterion.

A variety of methods have been developed to determine the outage probability for several mobile radio environments such as Log-normal, Rayleigh, Rice, e.t.c [1][2][3][4][5][6][7][8][9]. The calculation of the outage probability in a Nakagami mobile environment is particularly important, since Nakagami fading is today one of the most appropriate frequency non-selective models in many mobile communication practical applications, especially in micro-cellular networks. Nakagami distribution (also called m-distribution) contains a set of other distributions as special cases (Rayleigh, e.t.c) and provides the optimum fits to collected data in outdoor and indoor environments, in the frequency range from 800 MHz to 4 GHz [10][11].

Several techniques have been developed to determine the outage probability for Nakagami channels. In [12] F-distribution can be used to evaluate the outage probability in a case with a single interferer whereas in [13] a closed form is extracted, for use in case of multiple Nakagami interferers with the restriction of integer fading parameters. But, in a real mobile radio application this is not true and the fading parameters take arbitrary values. The latest and most general approach has been presented in [14][15] where the outage probability is determined –with and without a minimum power constraint- from the characteristic function of the quadratic form, through a special lemma on inverse Fourier transform.

In this paper, an alternative approach is presented to solve the outage problem of a Nakagami desired signal in the presence of L Nakagami interferers with arbitrary parameters. This problem is solved both for the first and second criterion and the outage probability is derived in a closed form, which permits its calculation with the desired accuracy and speed using Gauss-Laguerre integration method. In Section 2, the outage problem is formulated and the final expressions for the outage probability are presented. The necessary mathematical analysis can be seen in Appendices A and B while in Section 3 computer results illustrate the proposed formulation and a comparison is made with the technique described in [15]. Finally, Section 4 includes the paper's concluding remarks.

2 MATHEMATICAL ANALYSIS OF THE PROPOSED METHOD

If r_k is a Nakagami variable, the corresponding power is Gamma distributed with Probability Density Function (PDF) given by [16]

$$f(\xi_k) = \left(\frac{m_k}{\Omega_k}\right)^{m_k} \cdot \frac{\xi_k^{m_k-1}}{\Gamma(m_k)} \cdot \exp\left(-\frac{m_k}{\Omega_k}\cdot \xi_k\right), \xi_k \geq 0 \tag{1}$$

where m_k is an arbitrary fading parameter and Ω_k is the average power.

If $\sum_{i=1,\dots,L}\xi_i$ is the sum of the powers of L mutually independent Nakagami interferers and ξ_0 the LMP of the desired signal, then the outage probability denoted as q_C, in the case of non-existence of a minimum signal power, is given by

$$q_C = A \cdot \sum_{i=1}^{v} w_i \cdot \sum_{j=1}^{v} w_j \cdots \sum_{n=1}^{v} w_n \cdot \left(\prod_{t=i,j,\dots,n} x_t^{m_t-1}\right) \cdot F_{\xi_0}\left(\sum_{t=i,j,\dots,n} \frac{\beta\cdot\Omega_t}{m_t}\cdot x_t\right) \tag{2}$$

where A is a constant given by

$$A = \frac{1}{\prod_{i=1}^{L}\Gamma(m_i)} \tag{3}$$

$\Gamma(x)$ the Gamma function,

$F_{\xi_0}(x)$ the Nakagami Cumulative Distribution Function (CDF) defined as

$$F_{\xi_0}(x) = P(m_0, \frac{m_0}{\Omega_0}\cdot x) \tag{4}$$

with P being the Incomplete Gamma Function,

w_i; x_i are the weights and the abscissas of Gauss-Laguerre numerical integration, given by special tables [17] and

v denotes the accuracy of the computation.

The procedure for the derivation of Equation (2) is outlined in Appendix A.

In the case where a constraint of a minimum signal power λ exists, the outage probability denoted as q_{CC} is given by

$$q_{CC} = F_{\xi_0}(\lambda) + q_C - q_C \cdot F_{\xi_0}(\lambda) \tag{5}$$

The procedure for the derivation of Equation (5) is outlined in Appendix B.

3 NUMERICAL RESULTS

As an example, we consider a mobile environment with two Nakagami interferers as in [15]. Using equation (2) with v=8, outage probability- in the case of no minimum power constraint- is depicted in Figure 1 as a function to SIR/β. The values of m_0 are [0.5,1, 1.5,2].

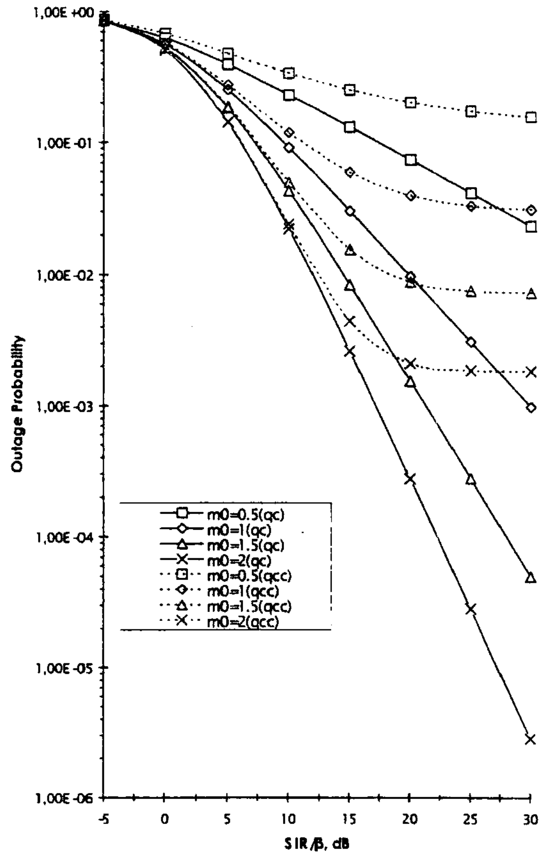

Figure 1: The outage probability (1st and 2nd outage criterion) versus the SIR/β for two Nakagami interferers with m=[1.44, 0.85], Ω=[5.5, 3.2] and protection ratio β=18 dB.

The outage probability q_{CC} for the same fading parameters and a minimum signal power constraint $\lambda = 0.031\cdot\Omega_0$ or $\lambda/\Omega_0 = -15dB$ is also calculated and the results are depicted in Figure 1 with the dot lines.

The Signal-to-Interference Ratio (SIR) is defined here as

$$SIR = 10\cdot\log_{10}\frac{\Omega_0}{\sum_{i=1}^{L}\Omega_i} \tag{6}$$

Moreover, in Figure 2 outage probability curves are shown (according to the first and second outage criterion with $\lambda/\Omega_0 = -15dB$) in a fully hexagonal shaped cellular system with six interferers.

In Figure 3, the outage probability for two Nakagami interferers with the parameters of Figure 1 is depicted versus the protection ratio β in dB. These are very useful curves since the protection ratio is related to the QoS demands of the system.

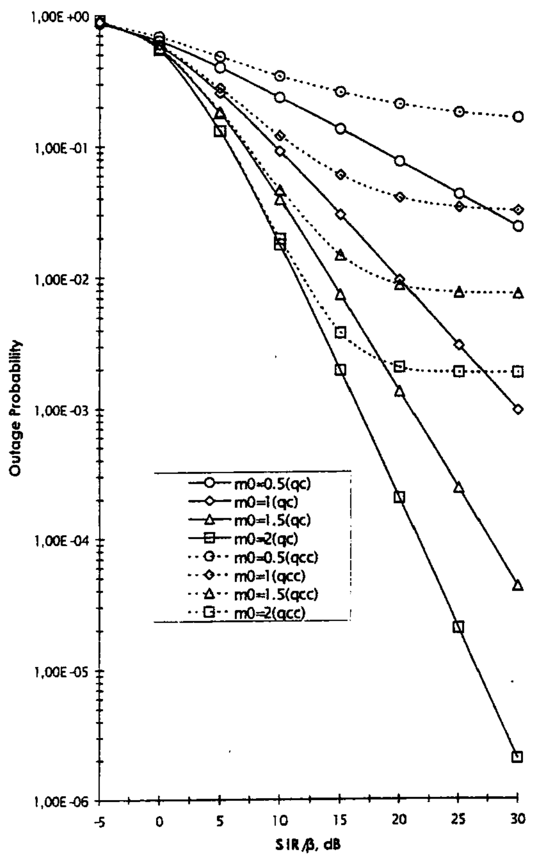

Figure 2: The outage probability (1st and 2nd outage criterion) versus the SIR/β for six Nakagami interferers with m=[0.7, 1.1, 1.7, 2.4, 3.5, 4.8], Ω=[2.4, 3.2, 3.6, 4.2, 5.2, 6.1] and protection ratio β=18 dB.

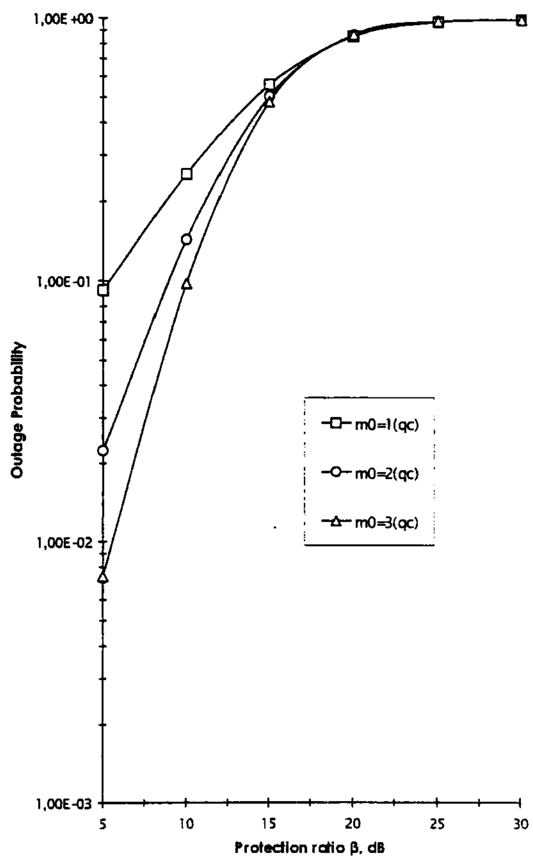

Figure 3: The outage probability (1st and 2nd outage criterion) versus the protection ratio β for two Nakagami interferers with m=[1.44, 0.85], Ω=[5.5, 3.2] and SIR=15 dB.

As discussed in Section 1, Zhang has presented in [15] a closed formula for the calculation of the outage probability in Nakagami fading channels in the presence of L Nakagami interferers with arbitrary parameters. This is an important result in literature since it was the first time that the outage problem was solved for arbitrary parameters. According to the Zhang's formulation the probability of outage is given by

$$P_{OUT}^{I}=\frac{1}{2}+\frac{1}{\pi}\cdot\int_{0}^{\infty}\frac{\sin\left[\sum_{k=1}^{L}m_k\cdot\tan^{-1}\left(\frac{\beta\cdot t}{\frac{m_k}{\Omega_k}}\right)-m_0\cdot\tan^{-1}\left(\frac{t}{\frac{m_0}{\Omega_0}}\right)\right]}{t\cdot\left[1+\left(\frac{t}{\frac{m_0}{\Omega_0}}\right)^2\right]^{\frac{m_0}{2}}\cdot\prod_{k=1}^{L}\left[1+\left(\frac{\beta\cdot t}{\frac{m_k}{\Omega_k}}\right)^2\right]^{\frac{m_k}{2}}}dt \quad (7)$$

The key feature of Zhang's method is the numerical calculation of the integrand in Equation (7). He recommends the approximation of the function included in the integrand piece-wisely using a number of different polynomials (piece-wisely Gaussian quadrature technique).

Some comments on Zhang's technique are as follows:

1. Equation (7) is really a useful tool because it involves only one integral.
2. The piece-wise technique needs to find every time- when a parameter is changed- the new locations and the new limit t_{max} of the integrand. This procedure provides the same time consumption for small and large number of interferers.
3. The calculation time strongly depends on the number of found locations since the Gaussian quadrature method must be applied N times, with N the number of the locations found.
4. The accuracy of the final result is related to the selected integer NN (see Appendix B of [15]), which refers to the steps that should be followed in order to determine the locations. Hence, when the limit t_{max} is not small the NN should be selected with a higher value. Moreover, the accuracy is also related to the value at which the function entire the integral seems to be negligible.

Table 1: Numerical results for the outage probability P^{I}_{OUT} in the presence of six Nakagami interferers, using Zhang's method (β=18 dB).

ZHANG'S METHOD- SIX NAKAGAMI INTERFERERS				
M=[0.8, 1.2, 1.8, 2.2, 2.5, 4.9) Ω=[1.3, 1.8, 2.6, 3, 3.2, 6]				
	$\frac{SIR}{\beta}=15dB$		$\frac{SIR}{\beta}=25dB$	
m_0	T_C (sec)	P^{I}_{OUT}	T_C (sec)	P^{I}_{OUT}
1	3	0.031092	2	0.003156
2	3	0.00205	2	0.000021
3	2	0.000161	3	0.0000001
4	2	0.0000141	2	9×10^{-10}

Table 2: Numerical results for the outage probability P^{I}_{OUT} in the presence of three Nakagami interferers, using Zhang's method (β=18 dB).

ZHANG'S METHOD- THREE NAKAGAMI INTERFERERS				
M=[0.8, 1.2, 1.8) Ω=[1.3, 1.8, 2.6]				
	$\frac{SIR}{\beta}=15dB$		$\frac{SIR}{\beta}=25dB$	
m_0	T_C (sec)	P^{I}_{OUT}	T_C (sec)	P^{I}_{OUT}
1	2	0.031002	2	0.003156
2	3	0.00237	2	0.00025
3	2	0.000242	1	27×10^{-7}
4	2	0.00003	1	8×10^{-9}

Table 3: Numerical results for the outage probability in the presence of six Nakagami interferers, using the method presented in this paper (β=18 dB).

METHOD PRESENTED IN THIS PAPER- SIX NAKAGAMI INTERFERERS				
M=[0.8, 1.2, 1.8, 2.2, 2.5, 4.9) Ω=[1.3, 1.8, 2.6, 3, 3.2, 6]				
	$\frac{SIR}{\beta}=15dB$		$\frac{SIR}{\beta}=25dB$	
m_0	T_C (sec)	q_C	T_C (sec)	q_C
1	18	0.030338	18	0.003080
2	18	0.002002	18	0.00002
3	18	0.000158	18	0.0000001
4	18	0.000013	18	10^{-9}

Table 4: Numerical results for the outage probability in the presence of three Nakagami interferers, using the method presented in this paper (β=18 dB).

METHOD PRESENTED IN THIS PAPER- THREE NAKAGAMI INTERFERERS				
M=[0.8, 1.2, 1.8) Ω=[1.3, 1.8, 2.6]				
	$\frac{SIR}{\beta}=15dB$		$\frac{SIR}{\beta}=25dB$	
m_0	T_C (sec)	q_C	T_C (sec)	q_C
1	<<1	0.030430	<<1	0.003098
2	<<1	0.002334	<<1	0.000024
3	<<1	0.000239	<<1	0.000000267
4	<<1	0.0000296	<<1	3.5×10^{-9}

In order to compare the method proposed in this paper with Zhang's as far as the consumption of time and the accuracy are concerned, several outage probability numerical results are tabulated in Tables 1 and 2 using Zhang's technique for six and three Nakagami interferers, respectively. In these Tables the observed calculation time T_C, is also depicted.

In Tables 3 and 4 the corresponding results using the method presented in this paper, are shown. The calculations depicted in all Tables were performed on a Pentium II (333 MHz) Personal Computer with the use of Mathcad 8.0 software.

As we can see above, the calculation time using the new proposed technique is higher than the mean time of Zhang's method for six interferers, while it is quicker in the case of three interferers. Moreover, the time consumed using Zhang's technique is about the same for small and large number of interferers. Using the new proposed method the calculation time for four interferers is 0.8 sec since the corresponding value for five interferers is 2.5 sec. Hence, it is obvious that the proposed method offers an advantage as far as the factor of calculation speed is concerned, especially for small numbers of interferers (less than four).

Taking into consideration the accuracy of computation, the two methods give slightly different results due to the alternative ways by which each of them approximates numerically the outage probability. However, this difference is negligible for practical mobile radio applications.

4 CONCLUSIONS

An alternative formula for the direct evaluation of the outage probability due to Nakagami signal and multiple Nakagami interferers with arbitrary parameters, was presented. This result is particularly important since such a mobile environment seems to be the most realistic scenario in a micro- or pico-cellular mobile radio communication system. The derived formula can be numerically calculated with the desired accuracy using the Gauss-Laguerre integration method. The new proposed formula offers advantage in controlling the accuracy of computation of existed techniques, computational speed (specially for small number of interferers) and results in reduced complexity using the existing computer tools.

APPENDIX A

If $\sum_{i=1}^{L} \xi_i$ is the sum of the powers of L mutually independent Nakagami interferers and ξ_0 the power of the desired signal, then the outage probability denoted as q_C, in the case of non-existence of a minimum signal power, is given by

$$q_C = \Pr ob\left(\frac{\xi_0}{\sum_{i=1}^{L} \xi_i} < \beta\right) \tag{8}$$

or

$$q_C = \Pr ob\left(\xi_0 - \beta \cdot \sum_{i=1}^{L} \xi_i < 0\right) \tag{9}$$

If $w = \xi_0 - \beta \cdot \sum_{i=1}^{L} \xi_i$, then (8) and (9) can be written as

$$q_C = \Pr ob(w < 0) \tag{10}$$

The PDF of the $\beta \cdot \xi_i$ is given by

$$\begin{aligned} f_{\beta \cdot \xi_i}(x_i) &= \frac{1}{\beta} \cdot f_{\xi_i}\left(\frac{x_i}{\beta}\right) = \\ &= \frac{1}{\beta} \cdot \left(\frac{m_k}{\Omega_k}\right)^{m_k} \cdot \frac{x_i^{m_k - 1}}{\beta^{m_k - 1} \cdot \Gamma(m_k)} \cdot \exp\left(-\frac{m_k}{\Omega_k} \cdot \frac{x_i}{\beta}\right) \end{aligned} \tag{11}$$

Let $\Phi_w(r)$, $\Phi_{\xi_0}(r)$, $\Phi_{\beta \cdot \xi_i}(r)$ be the characteristic functions of the variables w, ξ_0 and $\beta.\xi_i$ respectively. The $\Phi_w(r)$ can be written as:

$$\Phi_w(r) = \Phi_{\xi_0}(r) \cdot \prod_{i=1}^{L} \Phi_{\beta \cdot \xi_i}(-r) \tag{12}$$

Using the definition of the characteristic function, Equation (12) assumes the form:

$$\Phi_w(r) = \Phi_{\xi_0}(r) \cdot \prod_{i=1}^{L} \int_0^{\infty} \exp(-irx_i) \cdot f_{\beta \cdot \xi_i}(x_i) dx_i \tag{13}$$

Setting $x_i = \frac{\beta \cdot \Omega_i}{m_i} \cdot r_i$ in (11), from (12) and (13) and making certain simplifications, we obtain:

$$\begin{aligned} \Phi_w(r) = A \cdot \Phi_{\xi_0}(r) \cdot \int_0^{\infty} \cdots \int_0^{\infty} \exp\left(-jr\left(\sum_{i=1}^{L} \frac{\Omega_i \cdot \beta}{m_i} \cdot r_i\right)\right) \cdot \\ \left(\prod_{i=1}^{L} r_i^{m_i - 1}\right) \cdot \exp\left(-\sum_{i=1}^{L} r_i\right) dr_1 \cdots dr_L \end{aligned} \tag{14}$$

where A is a constant given by

$$A = \frac{1}{\prod_{i=1}^{L} \Gamma(m_i)} \tag{15}$$

But, using (10) and by definition, we have:

$$q_C = \int_{-\infty}^{0} f_w(\tau) d\tau = \frac{1}{2\pi} \cdot \int_{-\infty}^{0} \int_0^{\infty} \Phi_w(r) \cdot \exp(-jr\tau) dr d\tau \tag{16}$$

Now, using Equations (14), (15) and (16) and taking into account the fact that by definition

$$\begin{aligned} \int_0^{\infty} \Phi_{\xi_0}(r) \cdot \exp\left[-jr\left(\tau + \sum_{i=1}^{L} \frac{\beta \cdot \Omega_i}{m_i} \cdot r_i\right)\right] dr = \\ = 2 \cdot \pi \cdot f_{\xi_0}\left(\tau + \sum_{i=1}^{L} \frac{\beta \cdot \Omega_i}{m_i} \cdot r_i\right), \end{aligned} \tag{17}$$

q_C can be written as

$$\begin{aligned} q_C = A \cdot \int_0^{\infty} \cdots \int_0^{\infty} \left(\prod_{i=1}^{L} r_i^{m_i - 1}\right) \cdot \exp\left(-\sum_{i=1}^{L} r_i\right) \cdot \\ \cdot F_{\xi_0}\left(\sum_{i=1}^{L} \frac{\beta \cdot \Omega_i}{m_i} \cdot r_i\right) dr_1 \cdots dr_L \end{aligned} \tag{18}$$

with $F_{\xi_0}(x)$ being the Nakagami CDF defined as [15]

$$F_{\xi_0}(x) = P\left(m_0, \frac{m_0}{\Omega_0} \cdot x\right) \tag{19}$$

with P being the Incomplete Gamma Function. Equation (18) involves L integrals for L Nakagami interferers and it's second part can be calculated numerically with high desired accuracy using the Gauss-Laguerre method. According to this method,

$$\int_0^\infty \exp[-x]\, g(x)dx = \sum_{i=0}^{v} w_i \cdot g(x_i) \tag{20}$$

with w_i, x_i are the weights and the abscissas of Gauss-Laguerre numerical integration, given by special tables [17] and v denotes the accuracy of the computation.

Hence, the final formula for the outage probability is given by

$$q_C = A \cdot \sum_{i=1}^{v} w_i \cdot \sum_{j=1}^{v} w_j \cdots \sum_{n=1}^{v} \begin{matrix} w_n \cdot \left(\prod_{t=i,j,\ldots,n} x_t^{m_t - 1} \right) \cdot \\ \cdot F_{\xi_0}\left(\sum_{t=i,j,\ldots,n} \frac{\beta \cdot \Omega_t}{m_t} \cdot x_t \right) \end{matrix} \tag{21}$$

APPENDIX B

If $\sum_{i=1}^{L} \xi_i$ is the sum of the L mutually independent Nakagami interferers and ξ_0 the power of the desired signal, then the outage probability denoted as q_{CC}, in the case of the existence of a minimum signal power λ, is given by

$$q_{CC} = 1 - \Pr ob\left[\left(\xi_0 - \beta \cdot \sum_{i=1}^{L} \xi_i > 0 \right) \cap (\xi_0 > \lambda) \right] \tag{22}$$

or

$$q_{CC} = 1 - (1 - q_C) \cdot (1 - \Pr ob(\xi_0 < \lambda)) \tag{23}$$

and finally

$$q_{CC} = F_{\xi_0}(\lambda) + q_C - q_C \cdot F_{\xi_0}(\lambda) \tag{24}$$

with q_C, being the outage probability with no-minimum power constraint given in Appendix A and $F_{\xi_0}(x)$ the Nakagami CDF

$$F_{\xi_0}(x) = P(m_0, \frac{m_0}{\Omega_0} \cdot x) \tag{25}$$

and P being the Incomplete Gamma Function.

Manuscript received on February 2, 1999.

REFERENCES

[1] L.F. Fenton. The Sum of Log-normal Probability Distributions in Scatter Transmission Systems. *IRE Transactions on Communications Systems*, pages 57-67, March 1960.

[2] Y. Nagata and Y. Akaiwa. Analysis for Spectrum Efficiency in Single Cell Trunked and Cellular Mobile Radio. *IEEE Transactions on Vehicular Technology*, Vol. VT-36, pages 100-113, August 1987.

[3] R. Prasad and J. Arnbak. Comments on Analysis for Spectrum Efficiency in Single Cell Trunked and Cellular Mobile Radio. *IEEE Transactions on Vehicular Technology*, Vol. 37, No. 4, November 1987.

[4] Y.S. Yeh and S.C. Schwartz. Outage Probability in Mobile Telephony due to Multiple Log-normal Interferers. *IEEE Transactions on Communications*, Vol. COM-32, pages 380-388, April 1984.

[5] G. Karagiannidis, C. Georgopoulos, S. Kotsopoulos, D. Karaboulas and A. Doukas. High Capacity Cellular Mobile Systems: on the Increase of Co-Channel's Interference Evaluation Accuracy. In *International Workshop on Mobile Communications*, Thessaloniki, Greece, September 1996.

[6] G. Karagiannidis, C. Georgopoulos and S. Kotsopoulos. A new Mathematical Method for the Exact Evaluation of the Conditional Co-channel Probability in Cellular Mobile Radio Systems. *Journal of Electrical and Electronics Engineering*, Vol. 16, No. 3&4, pages 123-130, Australia, 1996.

[7] T. Tjhung, C. Chai and X. Dong. Outage Probability for a Rician signal in L Rician Interferers. *Electronics Letters*, Vol. 31, No. 7, pages 532-533, March 1995.

[8] J. Linnartz. Exact Analysis of the Outage Probability in Multiple-User Mobile Radio. *IEEE Transactions on Communications*, Vol. 40, No. 1, pages 20-23, January 1992

[9] T. Tjhung, C. Chai and X. Dong. Outage Probability for Log-normal Shadowed Rician Channels. *IEEE Transactions on Vehicular Technology*, Vol. 46, No. 2, pages 400-407, May 1997.

[10] H. Hashemi. The Indoor Propagation Channel. In *Proceedings of the IEEE*, Vol. 81, No. 7, pages 943-967, July 1993.

[11] P.J. Barry and A.G. Williamson. Radiowave propagation into and within buildings at 927 MHz. *Electronics Letters*, Vol. 23, No. 5, pages 248-249, Feb. 1987.

[12] E. Al-Hussaini. Effects of Nakagami fading on antijam performance requirements. *Electronics Letters*, Vol. 24, No. 4, pages 208-209, 1988.

[13] Abu-Dayya and N.C. Beaulieu. Outage probabilities of cellular mobile systems with multiple Nakagami interferers. *IEEE Transactions on Vehicular Technology*, Vol. VT-40, No. 6, pages 757-767, November 1991.

[14] Q.T. Zhang. Outage probabiliy of cellular mobile radio in the presence of multiple Nakagami interferers with arbitrary parameters. *IEEE Transactions on Vehicular Technology*, Vol. 44, No. 3, pages 661-667, August 1995.

[15] Q.T. Zhang. Outage probability in cellular mobile radio due to Nakagami signal and interferers with arbitrary parameters. *IEEE Transactions on Vehicular Technology*, Vol. VT-45, No. 2, pages 364-372, May 1996.

[16] M. Nakagami. The m-distribution: A general formula of intensity distribution of rapid fading. Statistical Methods in Radio Wave Propagation. W.G. Hoffman, Ed. Oxford, England: Pergamon, 1960.

[17] S. Gradshteyn and I.M. Ryzhik, Table of Integrals, Series and Products. New York, Academic Press, 1980.