Another Look at Multibranch Switched Diversity Systems

Athanasios S. Lioumpas, *Student Member, IEEE*, Aimilia P. Doukeli, and George K. Karagiannidis, *Senior Member, IEEE*

***Abstract*— We propose an alternative simpler implementation of generalized switch and stay combining (GSSC) receiver, which utilizes only one switching circuit and includes the classic dual SSC as a special case. Its performance is evaluated over Nakagami-m fading channels leading in closed-form expressions for the average symbol error probability. Moreover, for the case of non identical branches, the optimum threshold is accurately approximated in closed-form, avoiding the use of complicated numerical methods.**

***Index Terms*— Diversity techniques, fading channels, switch and stay combining (SSC).**

I. Introduction

It is well known that the classic dual switch and stay combining (SSC) [1]-[3] is less complex than maximal ratio combining (MRC), equal gain combining (EGC) or selection combining (SC) [4], since the number of the branches that have to be monitored is reduced. Nevertheless, in the emerging wireless communication systems operating in diversity-rich environments, i.e., with a large number of diversity paths, the combining of more branches is necessary and therefore SC or SSC may be insufficient techniques. Ko *et al* in [5] introduced the generalized switch and stay combining (GSSC), in order to take advantage of more than two diversity branches, while keeping the complexity lower than the one in MRC or generalized selection combining (GSC). However, the number of the required switching circuits is the same with the combined branches, resulting in transient phenomena and synchronization problems at the combination stage [6]. Moreover, for the case of independent but non identically distributed (i.n.i.d) branches, no method has been proposed for determining which branches must be involved at each switching circuit.

In this paper, we propose an alternative simpler implementation of a SSC scheme with more than two diversity branches, which utilizes only one switching circuit and includes the classic dual SSC as a special case. The error performance of the proposed system is evaluated over Nakagami-m fading channels assuming both independently identically distributed (i.i.d) and i.n.i.d branches, leading in closed-form formulas for the average symbol error probability (ASEP). Moreover, for the case of i.n.i.d branches, the optimum thresholds are accurately approximated by closed-form formulas, avoiding complicated numerical methods that may result in non practical system's implementations.

Manuscript received December 21, 2006. The associate editor coordinating the review of this letter and approving it for publication was Prof. Gianluca Mazzini.This research has been supported in part by the General Secretariat for Research and Technology (GSRT) of the Hellenic Ministry of Development under a grant for the Bilateral S&T Cooperation between the Hellenic Republic and Canada.

A. S. Lioumpas and G. K. Karagiannidis are with the Electrical and Computer Engineering Department, Aristotle University of Thessaloniki, 54124 Thessaloniki, Greece (e-mail: {alioumpa, geokarag}@auth.gr).

A. P. Doukeli is with the Electrical and Computer Engineering Department, National Technical University of Athens, Greece (e-mail: doukeli@mail.ntua.gr).

Digital Object Identifier 10.1109/LCOMM.2007.062084.

II. System Model and Mode of Operation

Consider a multichannel diversity reception system with $2L$ branches operating over flat fading channels where the receiver employs symbol-by-symbol detection.

The mode of operation of the proposed receiver is as follows:

1) The $2L$ branches are separated into two groups, each involving L branches. The receiver first estimates the output SNR of the first group (e.g. $\gamma_A = \gamma_1 + \gamma_2 + \ldots + \gamma_L$, where γ_k denotes the SNR of the kth branch) and compares it with a predetermined threshold γ_{Th}.
2) If the current group is acceptable (i.e. $\gamma_A > \gamma_{Th}$), the receiver combines the branches of this group using MRC. Otherwise, the receiver estimates the branches of the other group and combines them using MRC.
3) In the next time interval the receiver estimates the output SNR of the second group and the same procedure is followed.

We have to mention here that the receiver employs one combiner.

III. Performance over Nakagami-m Fading

A. Output Statistics

Assuming that the SNR at the kth diversity branch is a Gamma distributed random variable (RV) with mean value $\overline{\gamma}_k$ and scale parameter m_k, then the probability density function (pdf) of the proposed receiver's output SNR is given by [7]

$$f_z(z) = \begin{cases} A(\gamma_{Th})F_{z_1}(\gamma_{Th})f_{z_2}(z) \\ +(1-A(\gamma_{Th}))F_{z_2}(\gamma_{Th})f_{z_1}(z), \; z < \gamma_{Th} \\ A(\gamma_{Th})\left(f_{z_1}(z) + F_{z_1}(\gamma_{Th})f_{z_2}(z)\right) + (1-A(\gamma_{Th})) \\ \times\left(f_{z_2}(z) + F_{z_2}(\gamma_{Th})f_{z_1}(z)\right), \; z \geq \gamma_{Th} \end{cases} \tag{1}$$

where $f_{z_n}(\cdot)$, $F_{z_n}(\cdot)$ are the pdf and cumulative density function (cdf) of the combined SNR of the nth group ($n = \{1,2\}$) respectively and

$$A(z) = F_{z_2}(z)/(F_{z_2}(z) + F_{z_1}(z)). \tag{3}$$

The cdf of the receiver's output SNR is given by [7]

$$F_z(z) = \begin{cases} A(\gamma_{Th})F_{z_1}(\gamma_{Th})\left(F_{z_1}(z) + F_{z_2}(z)\right), \quad z < \gamma_{Th} \\ A(\gamma_{Th})\left(F_{z_1}(z) + F_{z_2}(z) - 2\right) \\ +A(\gamma_{Th})F_{z_1}(z) + (1-A(\gamma_{Th}))F_{z_2}(z), z \geq \gamma_{Th} \end{cases} \tag{4}$$

$$P_e=\frac{b\Xi}{2}A(\gamma_{Th})F_{z_1}(\gamma_{Th})(I(k,\sqrt{a\gamma_{Th}},\eta_i)-I(k,0,\eta))+\frac{b\Xi}{2}(1-A(\gamma_{Th}))F_{z_2}(\gamma_{Th})\left(I\left(k,\sqrt{a\gamma_{Th}},\eta_i\right)-I\left(k,0,\eta_i\right)\right)$$
$$-\frac{b\Xi}{2}\left(A(\gamma_{Th})+(1-A(\gamma_{Th}))F_{z_2}(\gamma_{Th})\right)I(k,\sqrt{a\gamma_{Th}},\eta_i)-\frac{b\Xi}{2}\left(1-A(\gamma_{Th})+A(\gamma_{Th})F_{z_1}(\gamma_{Th})\right)I(k,\sqrt{a\gamma_{Th}},\eta_i) \quad (2)$$

For convenience, the fading parameters and the average SNRs of the branches of the nth group are denoted as $\left\{m_q^n\right\}_{q=1}^L$ and $\left\{\overline{\gamma}_q^n\right\}_{q=1}^L$ respectively. Moreover, $f_{z_n}(z)$ and $F_{z_n}(z)$ are given by [8]

$$f_{z_n}(z)=\sum_{i=0}^{L}\sum_{k=0}^{m_i^n}\Xi_L\left(i,k,\left\{m_q^n\right\}_{q=1}^L,\left\{\eta_q^n\right\}_{q=1}^L,\left\{l_q\right\}_{q=1}^{L-2}\right)\times f_{Y_i}\left(z;k;\eta_i^n\right)$$

and

$$F_{z_n}(z)=\sum_{i=0}^{L}\sum_{k=0}^{m_i^n}\Xi_L\left(i,k,\left\{m_q^n\right\}_{q=1}^L,\left\{\eta_q^n\right\}_{q=1}^L,\left\{l_q\right\}_{q=1}^{L-2}\right)\times F_{Y_i}\left(z;k;\eta_i^n\right)$$

where

$$f_{Y_i}\left(z;m_i^n;\eta_i^n\right)=\frac{z^{m_i^n-1}}{\eta_i^n(m_i^n-1)!}\exp\left(-\frac{z}{\eta_i^n}\right)$$

and

$$F_{Y_i}\left(z;m_i^n;\eta_i^n\right)=1-\frac{\Gamma\left(m_i^n,z/\eta_i^n\right)}{(m_i^n-1)!}$$

are the pdf and the cdf of the SNR at the ith branch of the nth group respectively, $\eta_i^n=\frac{\overline{\gamma}_i^n}{m_i^n}$, Ξ_L is given by [8, Eq. 7] and $R_L=\sum_{i=0}^{L}m_i^n$.

B. Average Symbol Error Probability

It is well known, that for several signaling constellations, the ASEP can be written as follows [9]

$$P_e=b\int_0^\infty Q(\sqrt{2a\gamma})f_\gamma(\gamma)d\gamma \quad (5)$$

where $Q(\cdot)$ is the Gaussian Q-function, which has a one-to-one mapping with the complementary error function [i.e., $Q(x)=0.5\ \mathrm{erfc}(x/\sqrt{2})$] commonly found in standard mathematical tabulations and $f_\gamma(\gamma)$ is the pdf of the SNR per symbol γ. For binary phase-shift keying (BPSK) $a=b=1$ and for M-ary pulse amplitude modulation (M-PAM) $a=3/(M^2-1)$, $b=2(1-1/M)$. Moreover, for high values of average input SNR and M-PSK, $a=\sin^2(\pi/2M)$, $b=2$, while for M-ary quadrature amplitude modulation (M-QAM), $a=3/2(M^2-1)$, $b=4(1-1/\sqrt{M})$. Replacing (1) in (5) and after manipulations the ASEP can be expressed in closed-form as (2) (see the top of the page), where

$$I\left(x,y,z\right)=\int Erfc(\sqrt{ay})y^{x-1}exp(-\frac{y}{z})dy=$$
$$=-\frac{Erfc[y]}{(z)^{-x}}\Gamma\left(x,\frac{y^2}{az}\right)+\frac{(x-1)!}{(z)^{-x}\sqrt{\pi}}\times\sum_{p=0}^{x-1}\frac{\left(1+\frac{1}{az}\right)^{-\frac{1}{2}-p}(az)^{-p}\Gamma\left(\frac{1}{2}+p,\frac{y^2(1+az)}{az}\right)}{p!} \quad (6)$$

and

$$\Xi=\sum_{i=0}^{L}\sum_{k=0}^{m_i}\Xi_L\left(i,k,\left\{m_q\right\}_{q=1}^L,\left\{\eta_q\right\}_{q=1}^L,\left\{l_q\right\}_{q=1}^{L-2}\right).$$

$\Gamma(\cdot)$ is the gamma function [10, (8.31)] and $\Gamma(\cdot,\cdot)$ is the incomplete gamma function [10, (8.35)]. The integral in (6), can be solved by applying integration by parts and using [10, (8.35), (2.326)]. At this point it should be noted that for the case of i.n.i.d branches with the same fading parameters, the maximum performance is achieved when the first group involves the L "strongest" branches and the second group the L "weakest" branches. The term strong is referred to the branches with greater average power. This observation was extracted through extensive simulations, but it's proof seems to be difficult and is out of the scope of this letter.

For the case of i.i.d branches (i.e. $\overline{\gamma}_k=\overline{\gamma}$ and $m_k=m$) the expression for the ASEP is reduced to

$$P_e=-\frac{b}{2}\left(1-\Gamma\left(m_T,\frac{\gamma_{Th}m_T}{\gamma_T}\right)/\Gamma\left(m_T\right)\right)\times I\left(m_T,0,\frac{\gamma_T}{m_T}\right)-\frac{b}{2}I\left(m_T,\sqrt{a\gamma_{Th}},\frac{\gamma_T}{m_T}\right) \quad (7)$$

where $m_T=\sum_{k=1}^{L}m_k$ and $\overline{\gamma}_T=\sum_{k=1}^{L}\overline{\gamma}_k$.

C. Optimum Threshold

The optimum threshold is an additional important system design issue for SSC systems and affects the system error performance. This optimal value γ_{Th}^* is the solution of the equation $dP_e/d\gamma_{Th}=0$ [4], and can be obtained using numerical minimization methods. However it could increase the receiver's complexity and therefore extinguish its benefits compared to other diversity techniques such as MRC or GSC. Thus, closed-form expressions that approximate the optimum threshold would significantly decrease the receiver's complexity at the cost of minimal or null performance degradation, since the performance is not affected by a small declination of the calculated threshold from the optimum one. Following a similar procedure as in [5], an approximation of the optimum

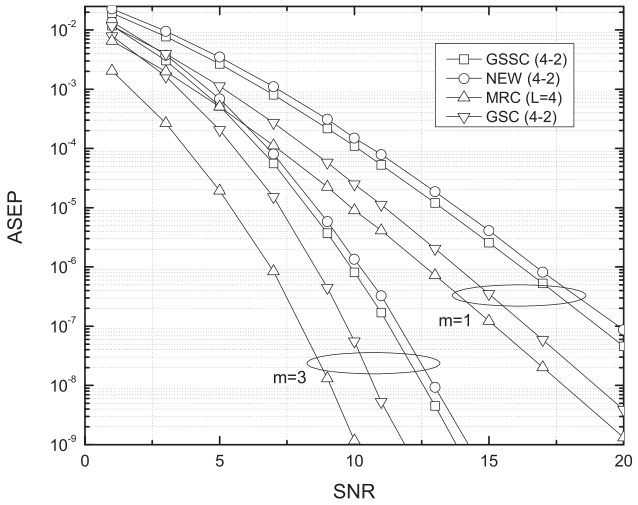

Fig. 1. The ASEP of the new scheme compared to GSSC, MRC and GSC, over i.i.d Nakagami-m fading with L=4.

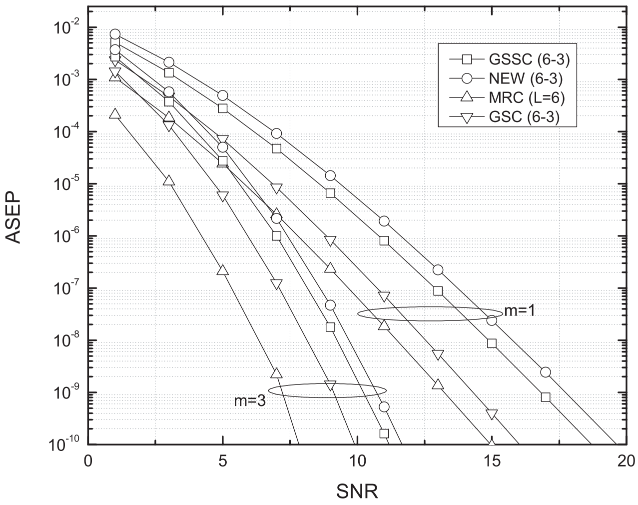

Fig. 2. The ASEP of the new scheme compared to GSSC, MRC and GSC, over i.i.d Nakagami-m with L=6.

threshold for BPSK and M-PAM modulation and Nakagami-m fading, can be expressed as

$$\gamma_{Th}^{*} = \frac{1}{a}\sum_{i=1}^{L} Q^{-1}\left(\frac{1}{2L}\frac{\sqrt{\frac{a\overline{\gamma}_i}{\pi m}}}{2\left(1+\frac{a\overline{\gamma}_i}{m}\right)^{m+\frac{1}{2}}}\frac{\Gamma\left(m+\frac{1}{2}\right)}{\Gamma\left(m+1\right)}\right.$$
$$\left.\times {}_2F_1\left(1, m+\frac{1}{2}; m+1; \frac{1}{1+\frac{a\overline{\gamma}_i}{m}}\right)^2\right), \quad (8)$$

where ${}_2F_1\left(\cdot,\cdot;\cdot;\cdot\right)$ is the Gauss Hypergeometric function and Q^{-1} is the inverse Gaussian Q function. For the case of i.i.d branches and M-PAM an approximated threshold can be expressed as

$$\gamma_{Th}^{*} = \frac{1}{2a} Q^{-1}\left(\frac{\sqrt{\frac{a\overline{\gamma}_T}{\pi m_T}}}{2\left(1+\frac{a\overline{\gamma}_T}{m_T}\right)^{m_T+\frac{1}{2}}}\frac{\Gamma\left(m_T+\frac{1}{2}\right)}{\Gamma\left(m_T+1\right)}\right.$$
$$\left.\times\ {}_2F_1\left(1, m_T+\frac{1}{2}; m_T+1; \frac{1}{1+\frac{a\overline{\gamma}_T}{m_T}}\right)^2\right) \quad (9)$$

which for $a = 1$ results to the exact threshold for BPSK. Note, that the proposed receiver includes the classic dual SSC as a special case when L=1 and therefore, the performance analysis presented above can be also applied in SSC receivers. To the best of the authors' knowledge, the ASEP of SSC receivers has not been given in a closed-form for the case of i.n.i.d Nakagami-m fading channels.

In Fig. 1 we compare the ASEP of the new proposed scheme to that of GSSC, MRC and GSC assuming BPSK modulation. It is observed that the performance loss of the new scheme is about 0.5 dB compared to GSSC for two different fading scenarios. Similar results are obtained from Fig. 2, where six available branches are assumed. Specifically, for $m = 1$, the new scheme suffers 1 dB loss in performance, while for $m = 3$ only 0.5 dB. However, the new receiver utilizes always one switching circuit, while the number of switching circuits in GSSC equals the number of the combined branches. For the case of i.n.i.d fading, a comparison with the GSSC could not be done, since no method has been proposed for determining which branches must be involved at each switching circuit.

REFERENCES

[1] A. A. Abu-Dayya and N. C. Beaulieu, "Analysis of switched diversity systems on generalized-fading channels," I*EEE Trans. Commun.*, vol. 42, pp. 2959–2966, Nov. 1994.

[2] M. A. Blanco and K. J. Zdunek, "Performance and optimization of switched diversity systems for the detection of signals with Rayleigh fading," *IEEE Trans. Commun.*, vol. 27, pp. 1887–1895, Dec. 1979.

[3] N. C. Sagias and P. T. Mathiopoulos, "Switched diversity receivers over generalized Gamma fading channels," *IEEE Commun. Lett.*, vol. 9, no. 10, pp. 871–873, Oct. 2005.

[4] M. K. Simon and M. Alouini, *Digital Communication over Fading Channels, Second Ed.* John Wiley, 2005.

[5] Y.-C. Ko and M.-S. Alouini, "Analysis and optimization of switched diversity systems," *IEEE Trans. Veh. Technol.*, vol. 49, pp. 1813–1831, Sept. 2000.

[6] H. W. Arnold, "Switched-diversity FSK in frequency-selective Rayleigh fading," *IEEE J. Sel. Areas Commun.*, vol. 2, no. 4, pp. 540–547, July 1984.

[7] C. Tellambura, A. Annamalai, and V. K. Bhargava, "Unified analysis of switched diversity systems in independent and correlated fading channels," *IEEE Trans. Commun.*, vol. 49, no. 11, pp. 1955–1965, Nov. 2001.

[8] G. K. Karagiannidis, N. C. Sagias, and T. A. Tsiftsis, "Closed-form statistics for the sum of squared Nakagami-m variates and its applications," *IEEE Trans. Commun.*, vol. 54, no. 8, pp. 1353–1359, Aug. 2006.

[9] G. L. Stuber, *Principles of Mobile Communications*. Norwell, MA: Kluwer Academic Publishers, 1996.

[10] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products, Fifth Ed.* New York: Academic, 1994.