$N*$Nakagami: A Novel Stochastic Model for Cascaded Fading Channels

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***Abstract*— A generic and novel distribution, referred to as $N*$Nakagami, constructed as the product of N statistically independent, but not necessarily identically distributed, Nakagami-m random variables (RVs), is introduced and analyzed. The proposed distribution turns out to be a very convenient tool for modelling cascaded Nakagami-m fading channels and analyzing the performance of digital communications systems operating over such channels. The moments-generating, probability density, cumulative distribution, and moments functions of the $N*$Nakagami distribution are developed in closed form using the Meijer's G-function. Using these formulae generic closed-form expressions for the outage probability, amount of fading, and average error probabilities for several binary and multilevel modulation signals of digital communication systems operating over the $N*$Nakagami fading and the additive white Gaussian noise channel are presented. Complementary numerical and computer simulation performance evaluation results verify the correctness of the proposed formulation. The suitability of the $N*$Nakagami fading distribution to approximate the lognormal distribution is also being investigated. Using Kolmogorov-Smirnov tests, the rate of convergence of the central limit theorem as pertaining to the multiplication of Nakagami-m RVs is quantified.**

***Index Terms*— Cascaded fading, bit error rate (BER), outage probability (OP), Nakagami-m, lognormal fading, central limit theorem (CLT), multiple-input multiple-output (MIMO), keyhole channels, Kolmogorov-Smirnov test, applied stochastic models.**

I. INTRODUCTION

RADIO signals generally propagate according to the mechanisms of reflection, diffraction, and scattering, which roughly characterize the radio propagation by three nearly independent phenomena: Path loss variance with distance, shadowing (or long-term fading), and multipath (or short-term) fading [1]. Except path loss, which is only distance dependent, the other two phenomena can be statistically described by fading models with parameters determined by using experimental radio propagation measurements. These channel models find use in the design and pretest evaluation of wireless communications systems in general and of fading mitigation techniques in particular. As expectations for the performance and reliability of wireless systems become more demanding, the significance of accurate channel modelling in system design, evaluation, and deployment will continue [2].

Due to the existence of a great variety of fading environments, several statistical distributions have been proposed for channel modelling of fading envelopes under short-term, long-term, and mixed fading conditions. Short-term fading models include the well-known Rayleigh, Rice [3], Hoyt [4], and Nakagami-m [5] distributions. Recently, some other statistical distributions such as Weibull [6]–[8] and spherically invariant random process (SIRP) models [9, pp. 315–322], have been found to fit well with experimental short-term fading for mobile communications. For long-term fading conditions, it is widely accepted that the probability density function (PDF) of the fading envelopes can be modelled by the well-known lognormal distribution [10], [11]. When short- and long-term fading conditions coexist in wireless mobile channels, several mixed models have been proposed in order to simultaneously take into account both types of fading impairments. The most popular among these fading models encompass the Suzuki [12], Nakagami-lognormal [11], and Rice-lognormal [13] models. A comprehensive summary for the above fading statistical models can be found in [14, Section 2]. Recently, attention has been given to the so-called "multiplicative" fading models [15]. Such models do not separate the fading in several parts but rather study the phenomenon as whole. A physical interpretation for these models is justified by considering received signals generated by the product of a large number of rays reflected via N statistically independent scatterers [15]. For $N = 2$, the so-called double Rayleigh (i.e., Rayleigh$*$Rayleigh) fading model has been found to be suitable when both transmitter and receiver are moving [16]. For higher values of N, Coulson *et al.* have studied the distribution of the product of N correlated Rayleigh distributed random variables (RVs) via computer simulations [17]. It is interesting to note that the double Rayleigh model has been recently used for keyhole channel modelling of multiple-input multiple-output (MIMO) systems [18], [19]. Extending this model by characterizing the fading between each pair of the transmit and receive antennas in the presence of the keyhole as Nakagami-m, the double Nakagami-m (i.e., Nakagami-$m*$Nakagami-m) fading model has also been considered [20].

In an effort to generalize all previously mentioned re-

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search works, in this paper, we introduce and analyze the $N*$Nakagami distribution constructed as the product of N statistically independent, but not necessarily identically distributed, Nakagami-m RVs. In this context, the statistics of this distribution is studied deriving its moments-generating function (MGF), PDF, CDF, and moments in closed form using Meijer's G-function[1]. Considering the proposed distribution as the fading channel model of a digital communication system, closed-form expressions are derived for the amount of fading (AoF), outage probability (OP), and average symbol error probability (ASEP) for several families of binary and multilevel modulation formats. Moreover, with the aid of the central limit theorem (CLT), the convergence rate of the proposed model towards the lognormal distribution as a function of N is investigated.

The rest of the paper is organized as follows. In Section II, the statistics of the $N*$Nakagami distribution is introduced and analyzed. Section III provides the performance of digital receivers operating in $N*$Nakagami fading channel model, while in Section IV, the convergence rate of the $N*$Nakagami distribution towards the lognormal distribution is examined. Concluding remarks are provided in Section V.

II. Definition and Statistical Characteristics

Let us consider $N \geq 1$ independent Nakagami-m[2] distributed RVs $\{R_\ell\}_{\ell=1}^{N}$, each with PDF

$$f_{R_\ell}(r) = \frac{2\, m_\ell^{m_\ell}}{\Omega_\ell^{m_\ell}\,\Gamma(m_\ell)}\, r^{2\, m_\ell - 1}\, \exp\left(-\frac{m_\ell}{\Omega_\ell}\, r^2\right) \qquad (1)$$

where $\Gamma(\cdot)$ is the Gamma function [22, eq. (8.310/1)], $\Omega_\ell = \mathbb{E}\langle R_\ell^2\rangle$, $m_\ell = \Omega_\ell^2/\mathbb{E}\langle (R_\ell^2 - \Omega_\ell)^2\rangle \geq 1/2$, and $\mathbb{E}\langle\cdot\rangle$ denotes expectation. As well known, the PDF in (1) includes the cases of Rayleigh ($m_\ell = 1$) and one-sided Gaussian ($m_\ell = 1/2$) distributions as special cases.

Definition 1 (The $N*$Nakagami distribution): We define the distribution of the product, Y, of N independent, but not necessarily identically distributed, RVs R_ℓ, i.e.,

$$Y \triangleq \prod_{i=1}^{N} R_i \qquad (2)$$

as the $N*$Nakagami distribution.

Theorem 1 (Moments-generating function): The MGF of Y is given by

$$\mathcal{M}_Y(s) = \frac{1/\sqrt{\pi}}{\prod_{i=1}^{N}\Gamma(m_i)}\, G_{2,N}^{N,2}\left[\frac{4}{s^2}\prod_{i=1}^{N}\left(\frac{m_i}{\Omega_i}\right)\Big|_{m_1,\, m_2, \ldots,\, m_N}^{1/2\,,\,1}\right] \qquad (3)$$

where $G[\cdot]$ is the Meijer's G-function [22, eq. (9.301)].

Proof: See [23, Appendix]. ∎

[1]Similarly to other authors (e.g., [14], [21]) and since the Meijer's G-function is tabulated, expressions containing this special function are considered to be in closed form.

[2]For the conciseness of the presentation and for the rest of the paper the term 'Nakagami' will be used instead of the actual 'Nakagami-m'.

By applying the transformation given by [22, eq. (9.303)] in (3), $G[\cdot]$ can be expressed using more widely used functions, such as the generalized Hypergeometric[3] [22, eq. (9.14/1)] .

Lemma 1 (Probability density function): The PDF of Y is given by

$$f_Y(y) = \frac{2}{y\,\prod_{i=1}^{N}\Gamma(m_i)}\, G_{0,N}^{N,0}\left[y^2\prod_{i=1}^{N}\left(\frac{m_i}{\Omega_i}\right)\Big|_{m_1,\, m_2, \ldots,\, m_N}^{-}\right]. \qquad (4)$$

Proof: By applying the inverse Laplace transform $\mathcal{L}^{-1}(\cdot;\cdot)$ to (3), the PDF of Y [22, Section 17.11], [24, eq. (3.38.1)]

$$f_Y(y) = \mathcal{L}^{-1}\{\mathcal{M}_Y(s); y\} \qquad (5)$$

can be expressed in closed form using [25, eq. (21)] as shown in (4). ∎

It is noted that for $N = 1$ and by using [25, eq. (11)], (4) simplifies to (1). Furthermore, for $N = 2$ and by using [22, eq. (9.34/3)], (4) simplifies to

$$f_Y(y) = \frac{4\, y^{m_1+m_2-1}}{\prod_{i=1}^{2}\Gamma(m_i)\,(\Omega_i/m_i)^{(m_1+m_2)/2}} \times K_{m_1-m_2}\left(2y\prod_{i=1}^{2}\sqrt{\frac{m_i}{\Omega_i}}\right) \qquad (6)$$

where $K_\nu(\cdot)$ is the νth order modified Bessel function of the second kind [22, eq. (8.432/1)] and ν is an arbitrary constant value. The same expression as in (6) can be also found in the original Nakagami paper [5, eq. (90)]. For $m_1 = m_2 = 1$, (6) becomes the so-called double Rayleigh fading model [16].

Lemma 2 (Cumulative distribution function): The CDF of Y is given by

$$F_Y(y) = \left[\prod_{i=1}^{N}\Gamma^{-1}(m_i)\right] \times G_{1,N+1}^{N,\,1}\left[y^2\prod_{i=1}^{N}\left(\frac{m_i}{\Omega_i}\right)\Big|_{m_1,\, m_2, \ldots,\, m_N,\, 0}^{1}\right]. \qquad (7)$$

Proof: Since the CDF of Y is given by

$$F_Y(y) = \int_0^y f_Y(x)\, dx \qquad (8)$$

by substituting (4) and using [25, eq. (26)], (7) can be obtained. ∎

Lemma 3 (Moments): The nth order moment of Y is given by

$$\mathbb{E}\langle Y^n\rangle = \prod_{i=1}^{N}\frac{\Gamma(m_i + n/2)}{\Gamma(m_i)}\left(\frac{\Omega_i}{m_i}\right)^{n/2}. \qquad (9)$$

Proof: Using (2), the nth order moment of Y can be expressed as

$$\mathbb{E}\langle Y^n\rangle = \mathbb{E}\left\langle\prod_{i=1}^{N} R_i^n\right\rangle. \qquad (10)$$

[3]It is noted that, both Meijer's G-function and generalized hypergeometric function are built-in functions in the most popular mathematical software packages such as Maple[TM].

Since the RVs R_ℓ are mutual independent, the above equation can be expressed as the product of the nth order moment of each RV as shown in (9). ■

III. Performance Analysis and Evaluation

Let us consider a digital communication system operating over the previously described $N*$ Nakagami fading channel and in the presence of additive white Gaussian noise (AWGN). The instantaneous SNR per symbol at the input of its receiver is given by [14]

$$\gamma = \frac{E_s}{N_0} Y^2 \tag{11}$$

where E_s is the transmitted symbol's average energy and N_0 is the single-sided AWGN power spectral density. The corresponding average SNR is

$$\overline{\gamma} = \mathbb{E}\left\langle Y^2 \right\rangle \frac{E_s}{N_0} = \frac{E_s}{N_0} \prod_{i=1}^{N} \Omega_i. \tag{12}$$

Dividing (11) and (12) by parts and using (7), the CDF of γ can be derived as

$$\begin{aligned} F_\gamma(\gamma) &= F_Y\left(\sqrt{\frac{\gamma}{\overline{\gamma}} \prod_{i=1}^{N} \Omega_i}\right) \\ &= \frac{1}{\prod_{i=1}^{N} \Gamma(m_i)} G_{1,N+1}^{N,\,1}\left[\frac{\gamma}{\overline{\gamma}} \prod_{i=1}^{N} m_i \,\middle|\, {}^{1}_{m_1,\, m_2, \ldots,\, m_N,\, 0}\right]. \end{aligned} \tag{13}$$

By taking the first derivative of (13) with respect to γ [26], the corresponding PDF can be obtained as

$$f_\gamma(\gamma) = \frac{1}{\gamma \prod_{i=1}^{N} \Gamma(m_i)} G_{0,N}^{N,0}\left[\frac{\gamma}{\overline{\gamma}} \prod_{i=1}^{N} m_i \,\middle|\, {}^{-}_{m_1,\, m_2, \ldots,\, m_N}\right]. \tag{14}$$

Note that for $N = 1$ and by using [25, eq. (11)], (14) simplifies to [14, eq. (2.7)], while for $N = 2$ and by using [22, eq. (9.34/3)], (14) also simplifies to a previously known result [20, eq. (31)]. With the aid of (11) and (12), the nth moment of γ can be easily derived as

$$\mathbb{E}\left\langle \gamma^n \right\rangle = \overline{\gamma}^n \prod_{i=1}^{N} \frac{\Gamma(m_i + n)}{\Gamma(m_i)\, m_i^n}. \tag{15}$$

A. Amount of Fading

The AoF is defined as the ratio of the variance to the square average SNR per symbol, i.e., $A_F \stackrel{\triangle}{=} \mathrm{var}(\gamma)/\overline{\gamma}^2$. Using (15), A_F can be easily expressed by a simple formula

$$A_F = \prod_{i=1}^{N} \left(1 + \frac{1}{m_i}\right) - 1. \tag{16}$$

From the above equation, it may be concluded that, since $m_\ell \geq 1/2$, then $0 < A_F \leq 3^N - 1$.

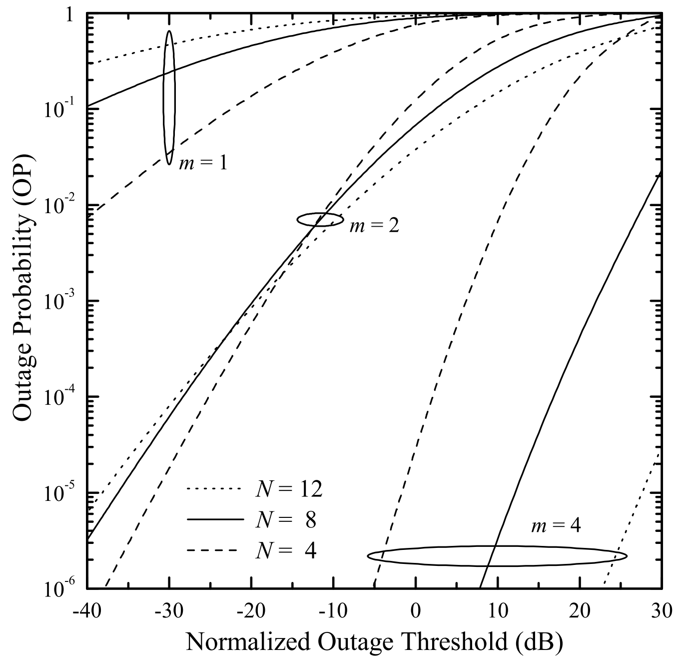

Fig. 1. OP performance as a function of the normalized outage threshold for the $N*$ Nakagami fading channel and for different values of N and m.

B. Outage Probability

The outage probability, P_{out}, is defined as the probability that the received SNR per symbol falls below a given threshold γ_{th}. Using (13), this probability can be obtained as

$$P_{\text{out}}(\gamma_{\text{th}}) = F_\gamma(\gamma_{\text{th}}). \tag{17}$$

It is noted that for low values of $\gamma_{\text{th}}/\overline{\gamma}$ (i.e., high $\overline{\gamma}$) and by using the asymptotic expansion for the Meijer's G-function presented in Appendix , an accurate and simple approximation of (17) is obtained.

Having numerically evaluated (17), in Fig. 1, P_{out} is plotted as a function of the normalized outage threshold, $\gamma_{\text{th}}/\overline{\gamma}$, for the $N*$ Nakagami channel with $N = 4$, 8, and 12, and for different values of $m = m_\ell$. These results clearly show that for a fixed value of N, P_{out} improves as m increases. This occurs because by increasing m the fading severity of the cascaded channels decreases, and hence the deep fades generated by the product of Nakagami fading envelopes also decreases. The obtained results further indicate that for a given value of m, a threshold for $\gamma_{\text{th}}/\overline{\gamma}$ exists above (below) which P_{out} improves (degrades) with increasing N. For example, for $m = 2$ this threshold is around -15 dB. Although not shown in Fig. 1 similar thresholds have been found for other values of m.

C. Average Symbol Error Probability

The most straightforward approach to obtain the ASEP, $\overline{P}_{\text{se}}$, is to average the conditional symbol error probability $P_{\text{se}}(\gamma)$ over the PDF of γ, i.e.,

$$\overline{P}_{\text{se}} = \int_0^\infty P_{\text{se}}(\gamma)\, f_\gamma(\gamma)\, \mathrm{d}\gamma. \tag{18}$$

For $P_{\text{se}}(\gamma)$ there are well-known generic expressions for different sets of modulation schemes, including:

i) Binary phase/frequency shift keying (BPSK/BFSK), and for higher values of the average input SNR, differentially encoded BPSK (DEBPSK), quadrature phase shift keying (QPSK), minimum shift keying (MSK), and square M-ary quadrature amplitude modulation (M-QAM), with $M \geq 4$, in the form of

$$P_{\text{se}}(\gamma) = A \, \text{erfc}\left(\sqrt{B\,\gamma}\right) \tag{19}$$

where $\text{erfc}(\cdot)$ is the well-known complementary error function [22, eq. (8.250/4)];

ii) Non-coherent BFSK (NBFSK) and binary differential phase shift keying (BDPSK), in the form of

$$P_{\text{se}}(\gamma) = A \, \exp(-B\,\gamma); \tag{20}$$

iii) $\pi/4$-differential QPSK ($\pi/4$-DQPSK) with Gray encoding, M-ary phase shift keying (M-PSK), and M-ary differential phase shift keying (M-DPSK), with $M \geq 4$, in the form of

$$P_{\text{se}}(\gamma) = A \int_0^{\Lambda} \exp[-B(\theta)\,\gamma] \, \mathrm{d}\theta. \tag{21}$$

In the above $P_{\text{se}}(\gamma)$ expressions the particular values of A, B, and Λ depend on the specific modulation scheme employed and can be found in [27, Table 1]. In the following, (18) is solved in closed form using the Meijer's G-function for each one of the above sets of signals for the $N*$Nakagami fading channel.

Using (14), (18), and (19), it can be easily recognized that for the first set of modulating schemes (i.e., BPSK, BFSK, DEBPSK, QPSK, MSK, and square M-QAM), it is necessary to evaluate definite integrals, which include Meijer's, power, and exponential functions. Since such integrals are not tabulated, the solution can be found with the aid of [25, eq. (21)], so that the ASEP can be expressed as

$$\begin{aligned}\overline{P}_{\text{se}}\left(\overline{\gamma}\right) =& A\,\pi^{-1/2}\left[\prod_{i=1}^{N}\Gamma^{-1}\left(m_i\right)\right] \\ & \times G_{2,\,N+1}^{N\,,\,2}\left[\frac{1}{B\,\overline{\gamma}}\prod_{i=1}^{N} m_i \,\Big|_{m_1,\,m_2,\ldots,\,m_N\,,\,0}^{1/2\,,\,1}\right].\end{aligned} \tag{22}$$

Similarly, for the second set (i.e., NBFSK and BDPSK), the ASEP can be derived as

$$\overline{P}_{\text{se}}\left(\overline{\gamma}\right) = \frac{A}{\prod\limits_{i=1}^{N}\Gamma\left(m_i\right)} G_{1,N}^{N,1}\left[\frac{1}{B\,\overline{\gamma}}\prod_{i=1}^{N} m_i \,\Big|_{m_1,\,m_2,\ldots,\,m_N}^{1}\right] \tag{23}$$

while for the third set (i.e., $\pi/4$-DQPSK with Gray encoding, M-PSK, and M-DPSK), is given by

$$\begin{aligned}\overline{P}_{\text{se}}\left(\overline{\gamma}\right) =& A\left[\prod_{i=1}^{N}\Gamma^{-1}\left(m_i\right)\right] \\ & \times \int_0^{\Lambda} G_{1,N}^{N,1}\left[\frac{1}{B(\theta)\,\overline{\gamma}}\prod_{i=1}^{N} m_i \,\Big|_{m_1,\,m_2,\ldots,\,m_N}^{1}\right]\mathrm{d}\theta.\end{aligned} \tag{24}$$

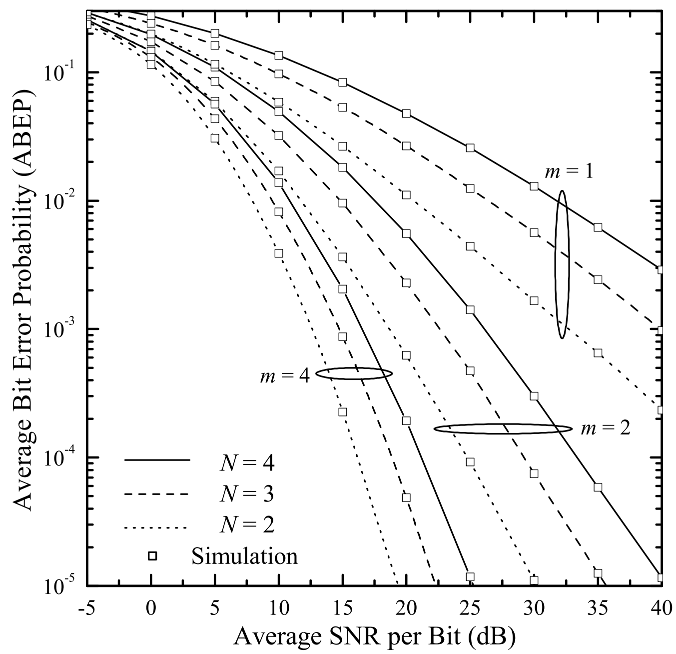

Fig. 2. ABEP performance of a Gray encoded QPSK modulation scheme received over the $N*$Nakagami fading channel for different values of N and m.

The integral in the last equation can be evaluated via numerical integration using any of the well-known mathematical software packages (e.g. available in Mathematica$^{\text{TM}}$).

Using (22)–(24), the ASEP of all these coherent and non-coherent binary and multilevel modulation schemes can be evaluated. As a typical example, in Fig. 2, the average bit error probability (ABEP) of Gray encoded QPSK $\overline{P}_{\text{be}} = \overline{P}_{\text{se}}/\log_2(M)$ ($M = 4$) is presented, as a function of the average SNR per bit, $\overline{\gamma}_b = \overline{\gamma}/\log_2(M)$, for several values of $m_\ell = m$ and N. As expected, these performance evaluation results show that $\overline{P}_{\text{be}}$ improves as m increases and/or N decreases. This happens because as m increases and/or N decreases, the probability that any of the cascaded fading channels is in deep fade increases significantly. Thus, the higher m and/or lower N are, the lower is the fading severity of the channel. As illustrated in Fig. 2, the above analytical performance evaluation results have also been verified by means of computer simulations. It is also noted that similar behavior has been also observed for P_{out} (see Fig. 1).

IV. Lognormal Distribution Approximation

In this section, using statistical tools and arguments, an accurate approximation for the lognormal distribution with the proposed $N*$Nakagami distribution is presented. In particular, by performing specific statistical tests, the convergence rate of CLT for the $N*$Nakagami towards the lognormal distribution is investigated.

A. Problem Statement and Preliminaries

Let μ_Υ and σ_Υ^2 be the mean and the variance, respectively, of a lognormally distributed RV X, which has the following

CDF

$$F_X(x) = 1 - \frac{1}{2}\,\mathrm{erfc}\left[\frac{\ln(x) - \mu_\Upsilon}{\sqrt{2}\,\sigma_\Upsilon}\right]. \tag{25}$$

The average SNR per symbol, $\overline{\gamma} = \mathbb{E}\left\langle X^2\right\rangle E_s/N_0$, is given by $\overline{\gamma} = \exp\left(\mu_\Upsilon + \sigma^2_\Upsilon/2\right)$.

In order to identify the necessary conditions for the $N*$ Nakagami distribution, Y, to become a lognormal distribution, X, it is convenient to define another RV, $\Upsilon = \ln(Y) \triangleq \sum_{i=1}^{N} \ln(R_i)$. By applying the CLT and for large N, Υ tends towards the normal (Gaussian) distribution, so that Y tends to be lognormal distributed [28, pp. 220–221]. For the limiting case where $Y \equiv X$, both distributions have the same mean $\mu_\Upsilon = \mathbb{E}\langle\ln(Y)\rangle$ and variance $\sigma^2_\Upsilon = \mathbb{E}\langle\ln^2(Y)\rangle - \mathbb{E}\langle\ln(Y)\rangle^2$. Hence, using (2) and [22, eqs. (4.352/1) and (4.358/2)], these can be obtained as

$$\mu_\Upsilon = \frac{1}{2}\sum_{i=1}^{N}\left[\Psi\left(m_i\right) - \ln\left(\frac{m_i}{\Omega_i}\right)\right] \tag{26}$$

and

$$\sigma^2_\Upsilon = \frac{1}{4}\sum_{i=1}^{N}\Psi^{(1)}\left(m_i\right) \tag{27}$$

respectively, where $\Psi^{(1)}(\cdot)$ is the first derivative of the Digamma function $\Psi(\cdot)$ [22, eq. (8.360)].

B. K-S Goodness-of-Fit Tests

In order to measure the difference between two CDFs, a number of statistical tests, such as the absolute value of the area between them, or their integrated mean square difference, may be applied [29]. However, a particularly simple and computational efficient measure is the Kolmogorov-Smirnov (KS) statistical test defined as the maximum value of the absolute difference between the two CDFs of X and Y [28, pp. 272–273]. Thus, for comparing one data set from $F_Y(\cdot)$ to the known $F_X(\cdot)$, the K-S statistical test is defined as

$$\mathcal{T} \triangleq \max\left|F_X(x) - F_Y(x)\right|. \tag{28}$$

Definition 2 (Null hypothesis $\mathbf{H}_0$*):* We define $\mathbf{H}_0$ as the null hypothesis under which observed data of Y belong to the CDF of the lognormal distribution, $F_X(\cdot)$.

To test $\mathbf{H}_0$, the K-S goodness-of-fit test compares $\mathcal{T}$ to a critical level $\mathcal{T}_{\max}$, as a function of N and for a given significance level α. Any hypothesis for which $\mathcal{T} > \mathcal{T}_{\max}$, is rejected with significance $1-\alpha$, while any hypothesis for which $\mathcal{T} < \mathcal{T}_{\max}$ is accepted with the same level of significance.

Following [17], but without loss of generality, we consider that the RVs R_ℓ are independent and identically distributed Nakagami RVs ($\Omega_\ell = \Omega$ and $m_\ell = m$). For comparison purposes, the performance test results presented here have been obtained from (28) using (25)–(27), for $\mathcal{T}_{\max} = 0.09$ for at least 10^4 samples and a significance level of $\alpha = 5\%$ [17, Table I]. Fig. 3 presents performance evaluation results for the K-S goodness-of-fit test, $\mathcal{T}$, versus the number of RVs, N, with m as parameter. Similarly to [17], these comparisons have been obtained by averaging the results of 30 simulation runs, each for at least 10^4 samples. The performance results of Fig. 3 show that for $m \leq 2$ and $N < 30$, $\mathbf{H}_0$ is rejected with 95% significance although the distribution is clearly converging towards the lognormal distribution with increasing N, which agrees with the observations made in [15], [17]. However, as m and/or N increase, Y converges for relatively low values of N towards the lognormal distribution. For example, when $m \geq 10$ and $N \geq 7$, $\mathbf{H}_0$ is accepted with 95% significance.

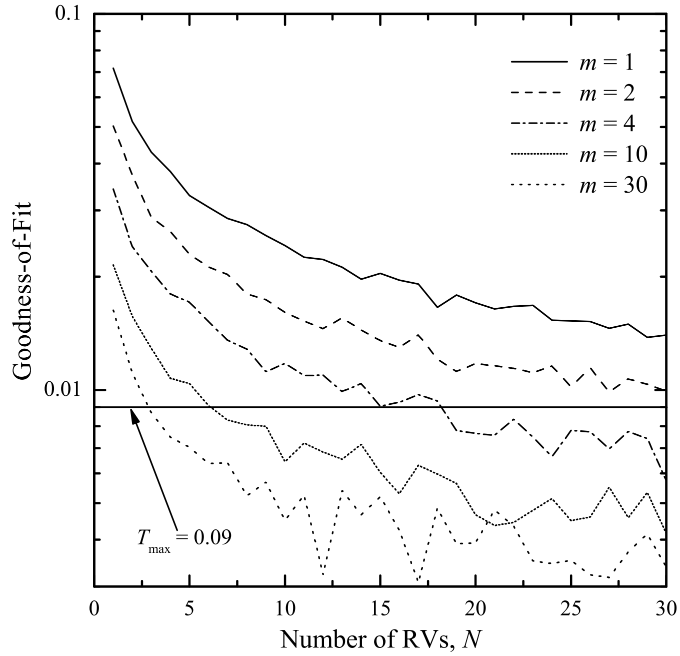

Fig. 3. Hypothesis testing distribution using the K-S Goodness-of-Fit test for the $N*$ Nakagami to approximate the lognormal distribution with 5% significance level.

V. Conclusions

A novel distribution, referred to as $N*$ Nakagami, constructed as the product of N statistically independent (but not necessarily identically distributed) Nakagami RVs, was introduced and analyzed. Basing on this generic distribution, a number of open research problems have been addressed. Firstly, it was used for performance studies of digital communication systems employing various families of modulation schemes operating over the $N*$ Nakagami fading channel model. Secondly, by performing K-S tests, a quantification of the convergence rate of the CLT demonstrated that even for small N, the proposed distribution accurately approximates the lognormal distribution and that interestingly, the convergence rate increases with an increase of m. It would be interesting to conduct experimental channel measurements which can verify the suitability of the proposed $N*$ Nakagami distribution to indeed model realistic wireless fading channels.

Acknowledgments

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APPENDIX

Let $z > 0$, $\{a_i\}_{i=1}^{p}$, $\{b_j\}_{j=1}^{q}$ arbitrary real number, and m, n, p, and q arbitrary positive integers. For $z \to 0$, the following asymptotic expansion of the Meijer's G-function [30, eq. (07.34.06.0006.01)]

$$G_{m,n}^{p,q}\left[z\left|\begin{smallmatrix}a_1,\ldots,a_n,\, a_{n+1},\ldots,a_p\\ b_1,\ldots,b_m,\, b_{m+1},\ldots,b_q\end{smallmatrix}\right.\right] = \sum_{k=1}^{m}\frac{\prod\limits_{\substack{j=1\\ j\neq k}}^{m}\Gamma\left(b_j-b_k\right)\prod\limits_{j=1}^{n}\Gamma\left(1-a_j+b_k\right)}{\prod\limits_{j=n+1}^{p}\Gamma\left(a_j-b_k\right)\prod\limits_{j=m+1}^{q}\Gamma\left(1-b_j+b_k\right)}\, z^{b_k} \tag{A.1}$$

holds, where the arguments in Gamma functions must not be negative integers.

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