

# Channel Quality Estimation Index (CQEI): A Long-Term Performance Metric for Fading Channels and an Application in EGC Receivers

Athanasios S. Lioumpas, *Student Member, IEEE*, George K. Karagiannidis, *Senior Member, IEEE*,  
and Athanassios C. Iossifides, *Member, IEEE*

**Abstract**—We introduce the Channel Quality Estimation Index (CQEI) as an alternative improved long-term performance criterion for wireless communications systems operating over fading channels. CQEI is simple to evaluate, since it depends on the channel statistics which vary much more slowly than the channel state itself, and thus can be obtained at the initialization state using a long training sequence and afterwards continuously improved during the whole communication period. Considering generalized fading channels (Nakagami- $m$ , Nakagami- $n$ , Nakagami- $q$ ) and a variety of modulations, we show that CQEI assesses the average error performance of a communication system more efficiently, compared to other long-term performance criteria such as the average signal to noise ratio (ASNR) and the amount of fading (AoF). As an application, we utilize CQEI to present a new class of equal gain combining (EGC) receivers operating over non-identical independent fading channels, called selection-EGC (S-EGC). This kind of receivers reduces or eliminates the combining loss by rejecting the weak branches that contribute more to increasing the noise than the signal during the combining stage. S-EGC could be efficiently applied in the emerging broadband communication systems (e.g., ultra-wideband), where the number of diversity paths can be considerably large due to the strong multipath effects. Numerical results show that the new proposed selection-EGC (S-EGC) receiver outperforms the classical one, when fading with non-identically distributed diversity branches is assumed.

**Index Terms**—Amount of fading (AoF), Channel Quality Estimation Index (CQEI), diversity, equal-gain combining (EGC), Nakagami- $m$  fading, Nakagami- $n$  fading, Nakagami- $q$  fading, S-EGC receivers.

## I. INTRODUCTION

WELL known performance criteria, used to describe the behavior of digital communications systems over fading channels, are the average bit error probability (ABEP), the outage probability, the average signal-to-noise ratio (ASNR) and the amount of fading (AoF) [1]. The ABEP serves as an excellent indicator of the overall fidelity of the system, but it is the most difficult to estimate prior to complete detection

in practical communication systems. The ASNR is the most simple to evaluate, since only the first statistical moment of the instantaneous SNR is needed. However, it cannot be efficiently associated with the error performance, especially in diversity systems operating over fading channels, because the variance of the instantaneous SNR is totally ignored. Thus, in order to describe the error performance more effectively, higher moments of the instantaneous SNR have to be considered. Ensuing this concept, AoF was introduced in [2], [3], as a measure of the severity of the fading channel. However, AoF expresses the relative variance of the signal envelope and it ignores the ASNR itself, which is sensibly related to the error performance. This means that two channels with almost the same AoF, may have different ASNRs, and consequently different error performance.

In this paper, we introduce the Channel Quality Estimation Index (CQEI) as a novel improved long-term performance criterion for wireless systems operating over fading channels. CQEI is defined as the ratio of the variance of the SNR to the cubed average SNR and it takes into account both the mean and the variance of the instantaneous SNR. Considering generalized fading channels (Nakagami- $m$ , Nakagami- $n$ , Nakagami- $q$ ) and a variety of modulations, we show that CQEI assesses the error performance of a communication system more efficiently, compared to the AoF and ASNR.

The applicability of CQEI is demonstrated through an application in equal-gain combining (EGC) receivers operating over non-identical independent fading channels. Among popular techniques such as maximal ratio combining (MRC), selection combining (SC) and switch combining (SWC) [1], EGC receivers appear to be an attractive diversity technique due to its simple implementation and its improved error performance, which is comparable to the optimal (MRC) technique. In pre-detection EGC, the received signals are co-phased, equally weighted, and then summed to form the resultant signal [4]–[9]. However, unlike MRC receivers, increment of the diversity branches may result in performance degradation instead of improvement, due to the so called *combining loss*. The latter may contribute more in increasing the noise than the signal power during the combination stage, because strong and weak branches are equally weighted. The optimum performance would be realized by using those branches that minimize the ABEP, but such a technique would be either impossible due to the nonexistence of simple closed form formulas

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A. S. Lioumpas and G. K. Karagiannidis are with the Department of Electrical and Computer Engineering, Aristotle University of Thessaloniki, 54124 Thessaloniki, Greece (e-mail: {alioumpa; geokarag}@auth.gr).

A. C. Iossifides is with Cosmote Mobile Telecommunications S.A., Thessaloniki, Greece (e-mail: iossifides@the.forthnet.gr).

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for the ABEP, or too complicated, because a continuous estimation of the ABEP would be required. Alternatively, a sub-optimum performance could be achieved, employing long term channel statistics, which can be easily realized in practical systems. Towards this concept, we utilize CQEI and propose an alternative EGC receiver, called selective-EGC (S-EGC), which determines the diversity branches to be used at the combination stage, resulting in rejection of the weak branches and thus in reduction of the combining loss. S-EGC could be efficiently applied in the emerging broadband communication systems (e.g., ultra-wideband- UWB), where the number of diversity paths can be considerably large due to the strong multipath effects and power delay profiles (pdp) that characterize their operating environments [10]. Moreover, in a UWB channel, the propagation environment varies widely from strong line-of-sight to extreme non-line-of sight, where the multipaths may be non identically distributed, concerning not only their average powers, but also their fading parameters [11], [12].

The outline of the paper is as follows. In Section II, we introduce the CQEI and prove that the error performance of a communications system, which operates over generalized fading channels, can be better estimated by the CQEI compared to the AoF or the ASNR. In Section III, we present the S-EGC receiver and numerical examples, while in Section V we present conclusions and final comments.

## II. CHANNEL QUALITY ESTIMATION INDEX (CQEI)

*Definition 1:* For a given fading distribution of a received signal, the Channel Quality Estimation Index (CQEI),  $\zeta$ , is defined by the ratio of the variance of the instantaneous received SNR to the cubed mean of the received SNR,  $\gamma$ , i.e.,

$$\zeta = \frac{\text{Var}\{\gamma\}}{[E\{\gamma\}]^3} = \frac{\text{AoF}}{E\{\gamma\}} \quad (1)$$

where  $\gamma$  is the instantaneous SNR of the received signal, defined as  $\gamma = r^2 \frac{E_b}{N_0}$ , with  $r$  being the fading envelope,  $E_b$  the energy per symbol,  $N_0$  the Gaussian noise spectral density and  $E\{\cdot\}$  denotes expectation.

Next, we show that the CQEI indicates significantly better the average error performance of a wireless system operating over most of the well known generalized fading channels compared to other long-term criteria, namely, the AoF and the ASNR.

### A. Nakagami- $m$ fading

Let the fading envelope  $r$  be described by the Nakagami- $m$  probability density function (PDF), which is given by [13]

$$p_r(r) = \frac{2m^m r^{2m-1}}{\Omega^m \Gamma(m)} e^{-\frac{mr^2}{\Omega}}, \quad r \geq 0 \quad (2)$$

where  $m$  is a fading parameter ranging from 0.5 to infinity,  $\Omega$  is the mean power of the fading envelope and  $\Gamma(\cdot)$  the Gamma function. Then, the SNR per symbol,  $\gamma$ , is distributed according to the gamma distribution as [1]

$$p_\gamma(\gamma) = \frac{m^m \gamma^{m-1}}{\bar{\gamma}^m \Gamma(m)} e^{-\frac{m\gamma}{\bar{\gamma}}}, \quad \gamma \geq 0 \quad (3)$$

where  $\bar{\gamma} = \Omega E_s / N_0$  is the average SNR per symbol. Taking into account that the moment generating function (MGF) is

$$M_\gamma(s) = \left(1 - \frac{s\bar{\gamma}}{m}\right)^{-m} \quad (4)$$

the AoF and the CQEI are given by

$$\text{AoF} = \frac{1}{m} \quad (5)$$

and

$$\zeta = \frac{1}{m\bar{\gamma}} \quad (6)$$

respectively.

*Theorem 1:* Let us assume an  $M$ -PSK, or an  $M$ -QAM or a DPSK communications system, operating over the non-identical Nakagami- $m$  fading channels:  $H_a \rightarrow (m_a, \bar{\gamma}_a)$  and  $H_b \rightarrow (m_b, \bar{\gamma}_b)$ . If  $m_a > m_b$  and  $\bar{\gamma}_a > \bar{\gamma}_b$ , then  $P_{e_a} < P_{e_b}$ , where  $P_{e_a}$ ,  $P_{e_b}$  are the average symbol error probabilities (ASEP) of the corresponding channels.

*Proof:* Using (4) and Appendix, in order to prove that  $P_{e_a} < P_{e_b}$ , we have to prove that

$$\frac{1}{\pi} \left(1 + \frac{\bar{\gamma}_a g}{m_a}\right)^{-m_a} < \frac{1}{\pi} \left(1 + \frac{\bar{\gamma}_b g}{m_b}\right)^{-m_b} \quad (7)$$

where  $g = \{g_{M\text{-PSK}}, g_{M\text{-QAM}}, g_{\text{DPSK}}\}$ , depend on the modulation scheme, with  $g_{M\text{-PSK}} = \sin^2(\pi/M)$ ,  $g_{M\text{-QAM}} = 3/[2(M-1)]$  and  $g_{\text{DPSK}} = -1$ . The above is directly proved, employing the results deduced in the Appendix, considering  $\bar{\gamma}g$  as the variable  $x$  and  $m$  as the variable  $y$ . ■

*Theorem 2:* Let us assume an  $M$ -PSK, or an  $M$ -QAM or a DPSK communications system, operating over the non-identical fading channels:  $H_a \rightarrow (m_a, \bar{\gamma}_a)$  and  $H_b \rightarrow (m_b, \bar{\gamma}_b)$ . The uncertainty region of the error performance estimation significantly decreases when CQEI is used, compared to AoF and ASNR.

*Proof:*

*Case I:*  $m_a > m_b$  and  $\bar{\gamma}_a > \bar{\gamma}_b$ .

In this case it will be  $\text{AoF}_a < \text{AoF}_b$  and  $\zeta_a < \zeta_b$  and as mentioned in Theorem 1, it will be  $P_{e_a} < P_{e_b}$ .

*Case II:*  $m_a > m_b$  and  $\bar{\gamma}_a < \bar{\gamma}_b$ .

In this case it will be  $\text{AoF}_a < \text{AoF}_b$ , while  $\zeta_a$  may be greater or less than  $\zeta_b$ , depending on the values of  $m_a$ ,  $m_b$  and  $\bar{\gamma}_a$ ,  $\bar{\gamma}_b$ . Let us assume that

$$\frac{1}{\pi} \left(1 + \frac{\bar{\gamma}_a g}{m_a}\right)^{-m_a} > \frac{1}{\pi} \left(1 + \frac{\bar{\gamma}_b g}{m_b}\right)^{-m_b} \quad (8)$$

which equivalently gives

$$\bar{\gamma}_b > \frac{m_b}{g} \left[ \left(1 + \frac{\bar{\gamma}_a g}{m_a}\right)^{\frac{m_a}{m_b}} - 1 \right]. \quad (9)$$

Since both parts of (8) are monotonic functions of constant sign, the inequality will also hold after integration of the two parts over the same integration interval, i.e.

$$\frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left(1 + \frac{\bar{\gamma}_a \sin^2(\frac{\pi}{M})}{m_a \sin^2(\varphi)}\right)^{-m_a} d\varphi > \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left(1 + \frac{\bar{\gamma}_b \sin^2(\frac{\pi}{M})}{m_b \sin^2(\varphi)}\right)^{-m_b} d\varphi \quad (10)$$

or  $P_{e_a} > P_{e_b}$ . Thus, the AoF will fail to correctly estimate the error performance of the two channels for those  $\bar{\gamma}_b, \bar{\gamma}_a$  satisfying (9).

On the contrary if  $\zeta_a < \zeta_b$ , or equivalently  $\bar{\gamma}_a m_a > \bar{\gamma}_b m_b$ , it will be  $P_{e_a} > P_{e_b}$ , only for those  $\bar{\gamma}_b, \bar{\gamma}_a$ , satisfying

$$\frac{m_a \bar{\gamma}_a}{m_b} > \bar{\gamma}_b > \frac{m_b}{g} \left[ \left( 1 + \frac{\bar{\gamma}_a g}{m_a} \right)^{\frac{m_a}{m_b}} - 1 \right] \quad (11)$$

and if  $\zeta_a > \zeta_b$ , it will be  $P_{e_a} < P_{e_b}$ , only for those  $\bar{\gamma}_b, \bar{\gamma}_a$ , satisfying

$$\frac{m_a \bar{\gamma}_a}{m_b} < \bar{\gamma}_b < \frac{m_b}{g} \left[ \left( 1 + \frac{\bar{\gamma}_a g}{m_a} \right)^{\frac{m_a}{m_b}} - 1 \right]. \quad (12)$$

Obviously, identical results could be derived assuming  $m_a < m_b$  and  $\bar{\gamma}_a > \bar{\gamma}_b$ . ■

Note that the ASNR is not a reliable criterion for the error performance of a wireless system operating over non-identical fading channels, since it totally ignores the fading severity. Therefore, comparing the ASNRs of channels with different  $m$  parameters, is the same as comparing the ASNRs of different distributions, which does not lead in safe results or decisions. For example, a channel with higher ASNR but with more severe fading conditions than another channel, may result in worse error performance.

An example for the above theoretic results is shown in Fig. 1, where a QPSK communication system is assumed, operating over two Nakagami- $m$  fading channels. In this figure we compare the failure regions of the AoF and the CQEI, which were found using (9)-(11) for  $m_a = 4$ ,  $m_b = 2$  and  $ASNR < 20$  dB. It is observed that the values of  $\bar{\gamma}_a$  and  $\bar{\gamma}_b$  for which is  $P_{e_a} > P_{e_b}$  and  $AoF_a < AoF_b$ , are significantly greater than the range of values of  $\bar{\gamma}_a$  and  $\bar{\gamma}_b$  for which  $P_{e_a} > P_{e_b}$  and  $\zeta_a < \zeta_b$  or  $P_{e_a} < P_{e_b}$  and  $\zeta_a > \zeta_b$ . Similar results can be obtained for any values of  $m_a$  and  $m_b$ , as shown in Fig. 2, which depicts the ratio of the CQEI failure region to the AoF failure region, versus the  $m$ -parameters of the Nakagami fading channels. We observe that the CQEI failure region is smaller than the corresponding region of the AoF, for any values of the  $m$  parameters. Moreover, for channels with comparable fading parameters, (true in most practical applications) and values larger than 2, the above ratio falls even below 0.1. We note that this comparison was performed by assuming  $\bar{\gamma}_a$  and  $\bar{\gamma}_b$  to be bounded by a limit of 25 dB, since it can be seen that the uncertainty regions of AoF and CQEI are infinite for unbounded  $\bar{\gamma}_a$  and  $\bar{\gamma}_b$ .

Similar results can be deduced for the Nakagami- $n$  and Nakagami- $q$  fading channels as follows.

### B. Nakagami- $n$ (Rician) fading

Let the fading envelope  $r$  be described by the Nakagami- $m$  PDF, described through [13]

$$p_r(r) = \frac{2(1+n^2)e^{-n^2}r}{\Omega} \exp \left[ -\frac{(1+n^2)r^2}{\Omega} \right] \times I_0 \left( 2nr \sqrt{\frac{1+n^2}{\Omega}} \right), \quad r \geq 0 \quad (13)$$

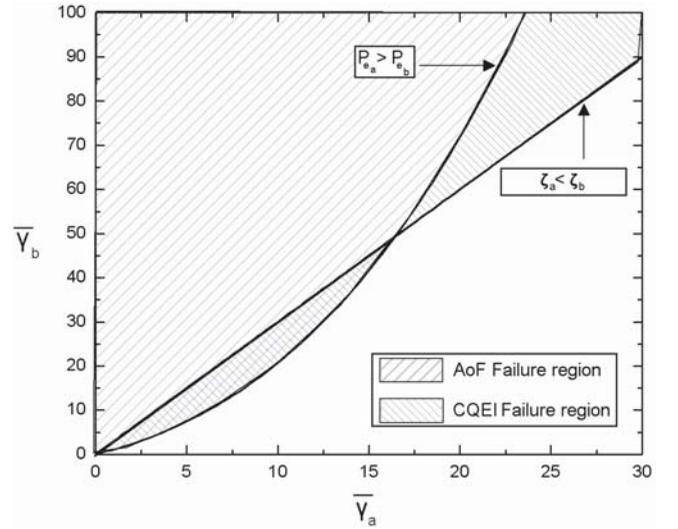


Fig. 1. The failure region of the AoF compared to the failure region of the CQEI, assuming two Nakagami- $m$  channels ( $m_a = 4$ ,  $m_b = 2$ ) and a QPSK communication system.

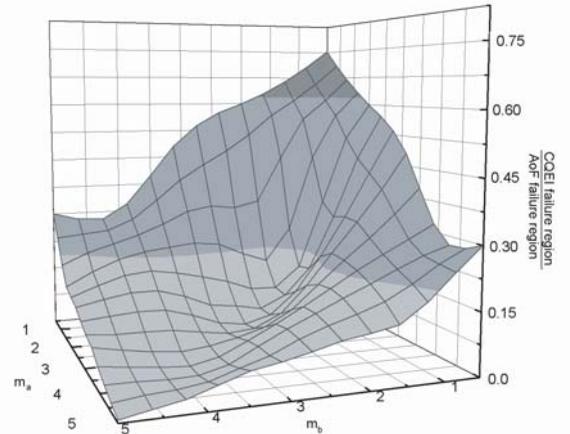


Fig. 2. The ratio of the AoF failure region to the CQEI failure region, as a function of the Nakagami- $m$  parameters, for a QPSK communication system.

where  $n$  is a fading parameter which ranges from 0 to infinity and which is related to the Rician  $K$  factor by  $K = n^2$  and  $I_0(\cdot)$  is the zeroth-order modified Bessel function of the first kind. The SNR per symbol,  $\gamma$ , is distributed according to the noncentral chi-square distribution given by [1]

$$p_\gamma(\gamma) = \frac{(1+n^2)e^{-n^2}}{\bar{\gamma}} \exp \left[ -\frac{(1+n^2)\gamma}{\bar{\gamma}} \right] \times I_0 \left( 2n \sqrt{\frac{(1+n^2)\gamma}{\bar{\gamma}}} \right), \quad \gamma \geq 0 \quad (14)$$

and the corresponding moment generating function (MGF) is

$$M_\gamma(s) = \frac{(1+n^2)}{(1+n^2) - s\bar{\gamma}} \exp \left[ \frac{n^2 s \bar{\gamma}}{(1+n^2) - s\bar{\gamma}} \right]. \quad (15)$$

The AoF and the CQEI are given by

$$AoF = \frac{1+2n^2}{(1+n^2)^2} \quad (16)$$

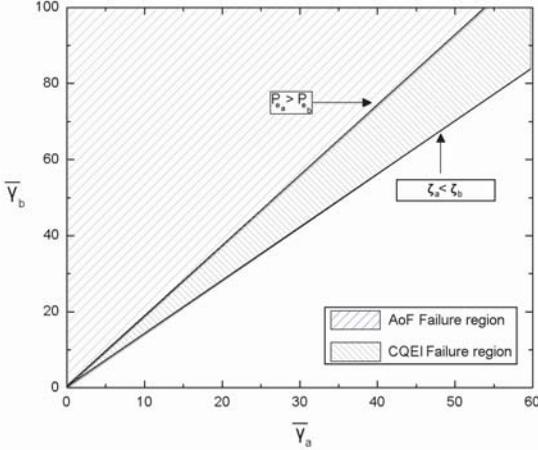


Fig. 3. The failure region of the AoF compared to the failure region of the CQEI, assuming two Nakagami- $n$  channels with  $K_a = 2.5$  dB,  $K_b = 1.2$  dB and a BPSK communication system.

and

$$\zeta = \frac{1 + 2n^2}{\bar{\gamma}(1 + n^2)^2} \quad (17)$$

respectively.

*Theorem 1* and *Theorem 2* can be also proved assuming two Nakagami- $n$  fading channels,  $H_a \rightarrow (n_a, \bar{\gamma}_a)$  and  $H_b \rightarrow (n_b, \bar{\gamma}_b)$ . Following the same procedure as in the Appendix, it is easily deduced that if  $n_a > n_b$  and  $\bar{\gamma}_a > \bar{\gamma}_b$ , then  $P_{e_a} < P_{e_b}$ . Similarly, for the case that  $n_a > n_b$  and  $\bar{\gamma}_a < \bar{\gamma}_b$ , in which an uncertainty raises, we conclude to the following results:

The AoF fails to correctly estimate the error performance of the two channels, for those  $\bar{\gamma}_b, \bar{\gamma}_a$  satisfying

$$\frac{1 + n_a^2}{1 + n_a^2 + g\bar{\gamma}_a} \exp\left(-\frac{n_a^2 g \bar{\gamma}_a}{1 + n_a^2 + g\bar{\gamma}_a}\right) > \frac{1 + n_b^2}{1 + n_b^2 + g\bar{\gamma}_b} \exp\left(-\frac{n_b^2 g \bar{\gamma}_b}{1 + n_b^2 + g\bar{\gamma}_b}\right). \quad (18)$$

On the contrary if  $\zeta_a < \zeta_b$ , it will be  $P_{e_a} > P_{e_b}$ , for those  $\bar{\gamma}_b, \bar{\gamma}_a$  satisfying (18) and

$$\bar{\gamma}_b < \frac{\bar{\gamma}_a(1 + n_a^2)^2(1 + 2n_b^2)}{(1 + n_b^2)^2(1 + 2n_a^2)}. \quad (19)$$

The above failure regions concerning Rician fading, are shown in Fig. 3. Similarly, as in the Nakagami- $m$  case the failure region of the AoF is significantly greater than the failure region of the CQEI.

### C. Nakagami- $q$ (Hoyt) fading

Let the fading envelope  $r$  be described by the Nakagami- $m$  PDF, described through [13]

$$p_r(r) = \frac{2(1 + q^2)r}{q\Omega} \exp\left[-\frac{(1 + q^2)r^2}{4q^2\Omega}\right] \times I_0\left(\frac{(1 - q^4)r^2}{4q^2\Omega}\right), \quad r \geq 0 \quad (20)$$

where  $q$  is a fading parameter which ranges from 0 to 1. The SNR per symbol,  $\gamma$ , is distributed according to [1]

$$p_\gamma(\gamma) = \frac{(1 + n^2)}{2q\bar{\gamma}} \exp\left[-\frac{(1 + q^2)^2\gamma}{4q^2\bar{\gamma}}\right] \times I_0\left(\frac{(1 - q^4)\gamma}{4q^2\bar{\gamma}}\right), \quad \gamma \geq 0 \quad (21)$$

and the corresponding moment generating function (MGF) is

$$M_\gamma(s) = \left[1 - 2s\bar{\gamma} + \frac{(2s\bar{\gamma})^2 q^2}{(1 + q^2)^2}\right]^{-\frac{1}{2}}. \quad (22)$$

The AoF and the CQEI are given by

$$AoF = \frac{2(1 + q^4)}{(1 + q^2)^2} \quad (23)$$

and

$$\zeta = \frac{2(1 + q^4)}{\bar{\gamma}(1 + q^2)^2}. \quad (24)$$

*Theorem 1* and *Theorem 2* can be also proved assuming the two Nakagami- $q$  fading channels  $H_a \rightarrow (q_a, \bar{\gamma}_a)$  and  $H_b \rightarrow (q_b, \bar{\gamma}_b)$ . Following the same procedure as in the Appendix, it is easily deduced that if  $q_a > q_b$  and  $\bar{\gamma}_a > \bar{\gamma}_b$ , then  $P_{e_a} < P_{e_b}$ . Similarly, for the case that  $q_a > q_b$  and  $\bar{\gamma}_a < \bar{\gamma}_b$ , in which an uncertainty raises, we conclude to the following results:

The AoF fails to correctly estimate the error performance of the two channels, for those  $\bar{\gamma}_b, \bar{\gamma}_a$  satisfying

$$\bar{\gamma}_b > r_1 \quad (25)$$

where  $r_1 = (-2g + \sqrt{4g^2 + 4b_2(b_1 + 2g\bar{\gamma}_a)})/2b_2$ , with  $b_1 = g^2 q_a^2 / ((1 + q_a^2)^2)$  and  $b_2 = g^2 q_b^2 / ((1 + q_b^2)^2)$ .

On the contrary, if  $\zeta_a < \zeta_b$ , it will be  $P_{e_a} > P_{e_b}$ , for those  $\bar{\gamma}_b, \bar{\gamma}_a$  satisfying

$$\frac{\bar{\gamma}_a(1 + q_b^4)(1 + q_a^2)^2}{(1 + q_a^4)(1 + q_b^2)^2} > \bar{\gamma}_b > r_1 \quad (26)$$

In Fig. 4, the AoF failure region is compared to the CQEI failure region for two Nakagami- $q$  fading channels. Similarly to the previous fading models, we observe that the CQEI estimates more efficiently the error performance compared to the AoF. Figs. 3 and 4 concern specific values of the fading parameters, but the results are comparable for any values of them.

## III. SELECTIVE-EGC (S-EGC) RECEIVERS

### A. System and Channel Model

Consider a multichannel diversity reception system with  $L$  branches operating over a flat fading environment, in which the receiver employs symbol-by-symbol detection. The signal received over the  $k$ th diversity branch in a symbol interval of duration  $T_S$  can be expressed as

$$r_k(t) = \Re\left\{\left[a_k(t - \tau_k)e^{-j\varphi_k(t)}s(t) + n_k(t)\right]e^{j2\pi f_c t}\right\} = \Re\left\{R_k(t)e^{j2\pi f_c t}\right\}, \quad k = 1 \dots L \quad (27)$$

where  $s(t)$  is the complex baseband information-bearing signal with average symbol energy  $2E_s$ ,  $a_k(t)$  is the random magnitude,  $\varphi_k(t)$  and  $\tau_k$  are the random phase and delay of the  $k$ th

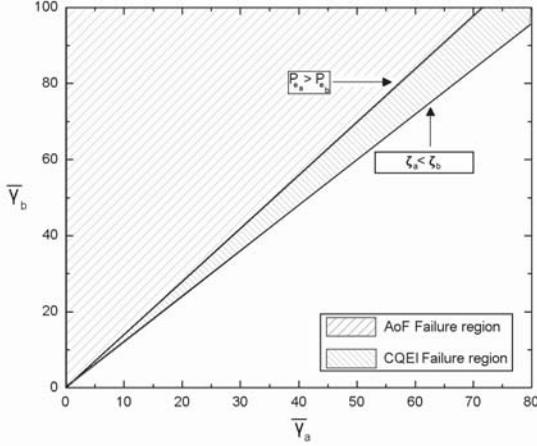


Fig. 4. The failure region of the AoF compared to the failure region of the CQEI, assuming two Nakagami- $q$  channels with  $q_a = 0.5$ ,  $q_b = 0.4$  and a BPSK communication system.

diversity branch,  $f_c$  is the carrier frequency and  $n_k(t)$ , representing the additive noise, is a zero-mean complex Gaussian random process with two-sided power spectral density  $2N_0$ .

Assuming that the random phase  $\varphi_k(t)$  and the delay  $\tau_k$  are known at the receiver, the received signals are co-phased and transferred to baseband so that the signal at the  $k$ th branch reduces to

$$z_k(t) = \Re \left\{ a_k(t)s(t) + n_k(t)e^{j\varphi_k(t)} \right\}, \quad k = 1, \dots, L. \quad (28)$$

Unlike the classical EGC, the S-EGC receiver does not combine all the input signals a priori. Instead, the set of the branches to be combined is determined by a selection algorithm presented in the next subsection. Let  $\mathcal{L} = \{z_1, z_2, \dots, z_L\}$  the set of the baseband signals available at the receiver, and  $\mathcal{L}_l$  the  $l$ th subset of  $\mathcal{L}$ ,  $l = 1, \dots, 2^L - 1$ , excluding the empty set. Moreover, let

$$\xi_l = \sum_{z_j \in \mathcal{L}_l} z_j \quad (29)$$

be the sum of the elements of  $\mathcal{L}_l$ , and  $\Xi = \{\xi_1, \dots, \xi_{2^L-1}\}$  the set of the  $\xi_l$ 's. For equally likely transmitted symbols with energy  $E_s$ , the instantaneous output SNR per symbol of the S-EGC receiver is given by

$$\gamma_{out} = \frac{E_s}{L_\zeta N_0} (\xi_l^*)^2 \quad (30)$$

where  $\xi_l^*$  is the sum of the elements of  $\mathcal{L}_l^*$  which includes the branches to be combined, and  $L_\zeta$  is the number of elements of  $\mathcal{L}_l^*$ .

### B. Mode of Operation and Performance Analysis

Next, we present a possible algorithm implementation for the proposed receiver. The S-EGC requires the mean value and the variance of the SNR of each diversity branch. These values could be estimated before the communication starts, using a long training sequence, and then continuously improved during the communication period. We note that no continuous

channel estimation is needed. On the contrary, the above estimation is performed in a long term mode, depending on the propagation environment. Thus, the steps of the proposed algorithm could be the following:

- 1) Estimate the ASNR  $\bar{\gamma}_i$  and the fading parameter (i.e.  $m_i$ ,  $n_i$  or  $q_i$ ) of each branch, for  $i = 1, \dots, L$ , using a long training sequence at the initialization stage.
- 2) Calculate the CQEI for each branch,  $\zeta_i$ , for  $i = 1, \dots, L$  and sort them in increasing order (i.e.,  $\zeta_{[1]} \leq \zeta_{[2]} \leq \dots \leq \zeta_{[L]}$ , where  $\zeta_{[i]}$  is the ordered  $\zeta_i$ ). At this step the set  $\mathcal{L}_l^*$  includes only the branch with the minimum  $\zeta$  (i.e.,  $\zeta_{[1]}$ ) and the output CQEI,  $\zeta_{out}$ , equals to the minimum  $\zeta$  (i.e.,  $\zeta_{out} = \zeta_{[1]}$ ).
- 3) Add the branch with the next smaller  $\zeta$  to the set  $\mathcal{L}_l^*$  and calculate the output CQEI,  $\zeta_{out}$  (i.e.,  $\zeta_{out} = \text{Var}\{\gamma_{\mathcal{L}_l^*}\} / [E\{\gamma_{\mathcal{L}_l^*}\}]^3$ , where  $\gamma_{\mathcal{L}_l^*}$  denotes the output SNR for the set  $\mathcal{L}_l^*$ ). If the resulted  $\zeta_{out}$  is greater than the previous one, exclude the last added branch and goto to step 5. The output CQEI can be evaluated using the closed form formulas presented in the next subsection.
- 4) Repeat step 3 until the maximum number of branches has been reached.
- 5) Equally weight and sum the signals from the branches in  $\mathcal{L}_l^*$ .
- 6) Repeat the steps 1-5 if the channels' long-term statistics change.

The maximum number of repetitions are  $L-1$  and it should be highlighted that they are not performed continuously, but at the initialization state, before the communication begins, or when the channel's statistics changes. Note that in practical wireless systems it remains unchanged or changes very slowly. Therefore, we could say that the S-EGC system's complexity is comparable to the classical EGC one.

After determining the set of branches to be combined, the ABEP performance of the S-EGC receiver can be evaluated, using one of the already published methods for the classical EGC receivers [5]- [9], [14], [15].

### C. The CQEI of the S-EGC Output SNR

Let  $Q$  be the set of indexes of the fading envelopes, which belong to  $\mathcal{L}_l$  and satisfy that

$$\sum_{z_j \in \mathcal{L}_l} z_j = \xi_l^*. \quad (31)$$

Using (30), the  $n$ th moment of the S-EGC output SNR that combines the branches of the set  $\mathcal{L}_l^*$  is, by definition

$$\mu_n = E\langle \gamma_{out}^n \rangle = E\left\langle \left[ \frac{E_s}{L_\zeta N_0} (s_l^*)^n \right] \right\rangle = \left( \frac{E_s}{L_\zeta N_0} \right)^n E\langle (s_l^*)^{2n} \rangle. \quad (32)$$

After mathematical manipulations and following the same procedure as in [14], the  $n$ th moment can be expressed as

$$\mu_n = \frac{(2n)!}{L_\zeta^n \prod_{i \in Q} \Gamma(m_i)} \sum_{\substack{n_1, \dots, n_L=0 \\ n_1 + \dots + n_L = 2n}} \prod_{i \in Q} \frac{\Gamma(m_i + \frac{n_i}{2}) \bar{\gamma}_i^{n_i/2}}{n_i! m_i^{n_i/2}}. \quad (33)$$

When the receiver operates over Rician fading channels, the moments of the S-EGC output SNR can be written in a simple

and closed-form expression given by [15]

$$\mu_n = \frac{(2n)!}{L^n \zeta^n} \sum_{\substack{k_1, \dots, k_L=0 \\ k_1 + \dots + k_L = 2n}}^{2n} \left[ \prod_{i \in Q} \frac{\Gamma(1 + \frac{k_i}{2}) \bar{\gamma}_i^{k_i/2}}{k_i! (1 + K_i)^{k_i/2}} \right] \times {}_1F_1 \left( -\frac{k_i}{2}; 1; -K_i \right) \quad (34)$$

where  ${}_1F_1(\cdot, \cdot; \cdot)$  is the confluent hypergeometric function of the first kind and  $K_i$  is the Rice factor of the  $i$ th input path, defined as the ratio of the signal power of the dominant component over the scattered power and is related to the  $n_i$  parameter as  $K_i = n_i^2$ .

For the case of Hoyt-fading channels, the moments of the S-EGC output SNR can be written in a simple and closed-form expression given by [15].

$$\mu_n = \frac{(2n)!}{L^n \zeta^n} \sum_{\substack{k_1, \dots, k_L=0 \\ k_1 + \dots + k_L = 2n}}^{2n} \left[ \prod_{i \in Q} \frac{\Gamma(1 + \frac{k_i}{2}) \bar{\gamma}_i^{k_i/2}}{k_i!} \right] \times {}_2F_1 \left( -\frac{k_i - 2}{4}; -\frac{k_i}{4}, 1; \left( \frac{1 - q_i^2}{1 + q_i^2} \right)^2 \right) \quad (35)$$

where  ${}_2F_1(\cdot, \cdot, \cdot; \cdot)$  is the Gauss hypergeometric function and  $q_i$  is the Nakagami- $q$  fading parameter of the  $i$ th branch.

Given the raw moments by (33)-(35), the  $k$ -central moment of output SNR,  $E\langle(\gamma_{out} - \bar{\gamma}_{out})^k\rangle$  can be written after using the binomial theorem, as

$$E\langle(\gamma_{out} - \bar{\gamma}_{out})^k\rangle = \sum_{n=0}^k \frac{k! (-1)^{k-n} (\bar{\gamma}_{out})^{k-n}}{n! (k-n)!} \mu_n. \quad (36)$$

Using the above results and the definition of (1), the CQEI of the output SNR can be directly extracted. These equations are used in the proposed algorithm presented in the previous subsection to calculate the CQEI of the receiver's output.

The fading parameters  $m_i$ 's,  $n_i$ 's or  $q_i$ 's are assumed to be distinct, which is true in practical applications, and the power delay profile of the input paths is non-uniform, representing multipath diversity over frequency-non-selective fading channels. The estimation of the above fading parameters has been extensively studied in the literature. Available techniques for the estimation of the  $m$  parameter can be found in [16], [17], while the estimation of the  $n$  parameter has been treated in [18]. Moreover, the approximation of the Nakagami- $q$  model by a suitable Nakagami- $m$  model was proposed by Nakagami in [13].

#### D. Numerical Results and Discussion

In this subsection, we examine the performance of the proposed S-EGC receiver and compare it with the classical EGC receiver. We provide representative numerical results in form of curves and tables illustrating the performance of the S-EGC over Nakagami- $m$  and Nakagami- $n$  fading channels and the effectiveness of the ASNR, AoF and CQEI to estimate the optimum diversity branches. The maximum available branches are assumed to be  $L = 5$  with each branch carrying signal from a propagation path with different fading parameters and an exponentially pdp, i.e.  $\bar{\gamma}_i = \bar{\gamma} e^{-di}$ , where  $d$  is the decaying

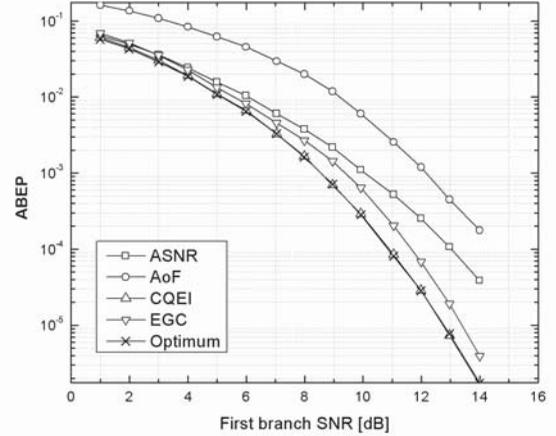


Fig. 5. The ABEP of the S-EGC receiver when various criteria employed, compared to an  $L = 5$  EGC receiver for a BPSK communication system, over Nakagami- $m$  fading ( $d=1$ ,  $m_1 = 1.5$ ,  $m_2 = 2$ ,  $m_3 = 2.5$ ,  $m_4 = 3.2$ ,  $m_5 = 1.3$ ).

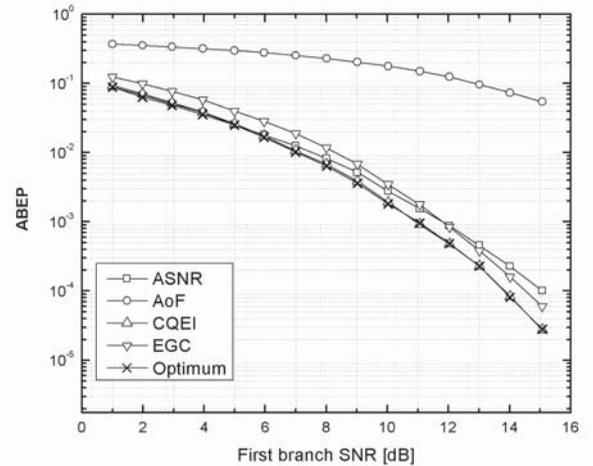


Fig. 6. The ABEP of the S-EGC receiver when various criteria employed, compared to an  $L = 5$  EGC receiver for a BPSK communication system, over Nakagami- $m$  fading ( $d=2$ ,  $m_1 = 1.2$ ,  $m_2 = 2$ ,  $m_3 = 3.1$ ,  $m_4 = 2.2$ ,  $m_5 = 3.0$ ).

factor. We note that the error performance of the S-EGC receiver is evaluated using the analytical analysis of classical EGC, once the set of combining branches is determined by applying the algorithm presented in the previous subsection.

The optimum criterion is considered to be the expected ABEP, which leads to the selection of those branches that minimize the total ABEP. The ASNR and the AoF are compared to the CQEI by applying the same algorithm described in the previous section, with a modification needed for the ASNR, since in this case we search for the branches that maximize the output's ASNR.

In Fig. 5 we compare the error performance of S-EGC and EGC receivers over Nakagami- $m$  fading, assuming a power decay factor  $d = 1$ . It is observed that the S-EGC engaging CQEI as a selection criterion, almost eliminates the

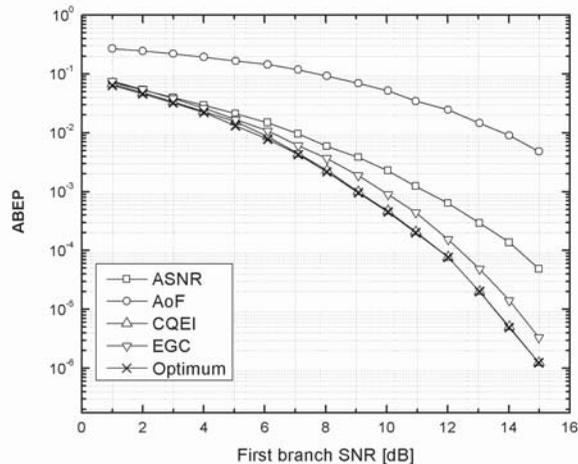


Fig. 7. The ABEP of the S-EGC receiver when various criteria employed, compared to an  $L = 5$  EGC receiver for a BPSK communication system, over Rician fading ( $d = 1$ ,  $K_1 = 2.3$  dB,  $K_2 = 1.0$  dB,  $K_3 = 4.7$  dB,  $K_4 = 2$  dB,  $K_5 = 1.7$  dB).

combining loss and outperforms the EGC receiver, while the other criteria, as the ASNR and the AoF, fail to estimate the optimum diversity branches. For a power decay factor of  $d = 2$  the above comparisons are shown in Fig. 6. Similar results can be derived, with the difference that the performance of the ASNR concerning the optimum branches estimation has improved while the AoF totally fails. This behavior is expected as high values of the decay factor result in an error performance which is determined only by the first one or two branches. The CQEI, in both cases, successfully determines the near optimum diversity branches.

In Table I, we present the efficiency of the ASNR, AoF and CQEI in estimating the optimum diversity branches for various values of the decay factor and for fading parameters that follow a truncated Gaussian distribution with mean,  $\mu = 2$  and variance,  $\sigma^2 = 1.5$ . The latter assumption has been adopted in the channel modeling standards for UWB [19]. We can see that the CQEI estimates the optimum diversity branches better than the other criteria for almost any case. In general, we observe that the AoF constitutes a reliable criterion only for small power decay factors, while the ASNR for high power delay profiles. We note, that a failure in estimating the optimum diversity branches does not always lead to dramatic performance decrement, as the difference from the optimum may not be significant.

S-EGC with CQEI outperforms classic EGC over Nakagami- $n$  fading as well, as shown in Figs. 7 and 8. In this case, due to the strong line of sight (LOS) path, the AoF totally fails to estimate the error performance of each diversity branch, in contrast to the ASNR, which for high power delay profiles ( $d = 1.8$ ) approaches the optimum performance. Table II presents the performance of the ASNR, AoF and CQEI over Rician fading, for various values of the decay factor. The CQEI, compared to the other criteria, is the most effective in the estimation of the optimum diversity branches in almost every case and the only one that can be employed in the

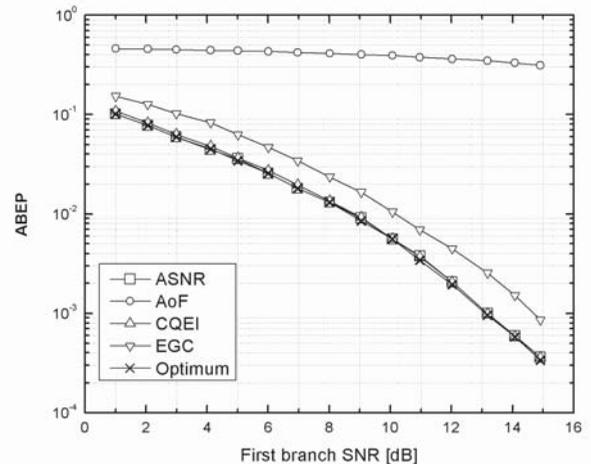


Fig. 8. The ABEP of the S-EGC receiver when various criteria employed, compared to an  $L = 5$  EGC receiver for a BPSK communication system, over Rician fading ( $d = 1.8$ ,  $K_1 = 1.7$  dB,  $K_2 = 1.5$  dB,  $K_3 = 1.6$  dB,  $K_4 = 2.3$  dB,  $K_5 = 1.5$  dB).

TABLE I

PERCENTAGE (%) OF SUCCESSFULLY SELECTING THE OPTIMUM NUMBER OF BRANCHES WHEN DIFFERENT CRITERIA APPLIED, FOR VARIOUS VALUES OF THE AVERAGE SNR AND THE POWER DECAY FACTOR AT THE RECEIVER, OVER NAKAGAMI- $m$  FADING.

		$d \setminus \bar{\gamma}$	0	5	10	15	20
ASNR	0.5	71	78	77	78	82	
	1	65	26	15	8	8	
	1.5	90	82	65	36	16	
	2	91	92	91	65	62	
AoF	0.5	92	91	89	91	90	
	1	23	24	38	60	65	
	1.5	2	2	3	2	8	
	2	6	7	5	2	2	
CQEI	0.5	95	93	94	95	92	
	1	68	71	92	84	78	
	1.5	80	79	88	85	79	
	2	94	96	96	94	86	

S-EGC receiver, since it is the only scheme of the three under discussion that always outperforms classical EGC. We note that the decision on the optimum number of branches is based on the CQEI of the output SNR, which distribution is unknown. Taking into account the above remarks and the efficiency of the CQEI in estimating the optimum number of branches, we could describe the CQEI as an error performance criterion for generalized fading channels, even with unknown statistics.

#### IV. CONCLUSIONS

A novel improved long-term performance criterion for wireless communications systems operating over fading channels, called CQEI, was presented. Considering generalized fading channels (Nakagami- $m$ , Nakagami- $n$ , Nakagami- $q$ ) and a variety of modulations, it was shown that CQEI assesses the average error performance of a communication system more effectively, compared to other long-term performance criteria

TABLE II

PERCENTAGE (%) OF SUCCESSFULLY SELECTING THE OPTIMUM NUMBER OF BRANCHES WHEN DIFFERENT CRITERIA APPLIED, FOR VARIOUS VALUES OF THE AVERAGE SNR AND THE POWER DECAY FACTOR AT THE RECEIVER, OVER RICIAN FADING.

	$d \setminus \bar{\gamma}$	0	5	10	15	20
ASNRR	0.5	79	78	79	77	71
	1	68	69	57	45	33
	1.5	98	97	25	16	14
	2	98	98	93	59	43
AoF	0.5	85	30	15	20	25
	1	2	12	60	91	71
	1.5	2	3	2	2	3
	2	3	2	4	2	3
CQEI	0.5	92	25	4	20	15
	1	85	91	95	74	65
	1.5	83	91	99	99	99
	2	99	99	98	65	52

such as the ASNRR and AoF. As an application, CQEI was utilized in the introduction of a new class of EGC receivers operating over non-identical independent fading channels, called selection-EGC (S-EGC). It was shown that this kind of receivers reduces or eliminates the combining loss by rejecting the weak branches that contribute more to increasing the noise than the signal during the combining stage. The proposed receiver could be further investigated, concerning important implementation issues, such as the impact of imperfect channel parameters estimation and the frequency of the the channel estimation updates, which depends on the propagation environment.

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#### APPENDIX

Let us consider the function

$$f(x, y) = \left(1 + \frac{x}{y}\right)^{-y}, \quad x, y > 0 \quad (\text{A1})$$

The partial derivative  $\frac{\partial}{\partial y} f(x, y)$  is given by

$$\frac{\partial}{\partial y} f(x, y) = \left(1 + \frac{x}{y}\right)^{-y} \left(\frac{x}{y+x} - \ln\left(1 + \frac{x}{y}\right)\right). \quad (\text{A2})$$

Using the well known inequality

$$\frac{x}{x+1} < \ln(1+x), \quad (\text{A3})$$

it is deduced that

$$\left(\frac{x}{y+x} - \ln\left(1 + \frac{x}{y}\right)\right) < 0 \quad (\text{A4})$$

and since

$$\left(1 + \frac{y}{x}\right)^{-x} \geq 0 \quad (\text{A5})$$

we derive that

$$\frac{\partial}{\partial y} f(x, y) \leq 0. \quad (\text{A6})$$

The partial derivative  $\frac{\partial}{\partial x} f(x, y)$  is given by

$$\frac{\partial}{\partial x} f(x, y) = -\left(1 + \frac{x}{y}\right)^{-1-y} \quad (\text{A7})$$

which is negative  $\forall y > 0, x > 0$ .

Thus the function  $f(x, y)$  is monotonically decreasing, which means that  $f(x_1, y_1) < f(x_2, y_2), \forall x_1 > x_2, y_1 > y_2$ .

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**Athanasios S. Lioumpas** (S'06) was born in Thessaloniki, Greece, in 1982. He received the Diploma in electrical and computer engineering in 2005, and continues his studies towards the Ph.D. degree from the Aristotle University of Thessaloniki, Greece. His research interests include digital communications over fading channels, diversity techniques and mobile radio communications.



**George K. Karagiannidis** was born in Pithagorion, Samos Island, Greece. He received his university degree in 1987 and his Ph.D degree in 1999, both in Electrical Engineering, from the University of Patras, Patras, Greece.

From 2000 to 2004 he was Researcher at the Institute for Space Applications and Remote Sensing, National Observatory of Athens, Greece. In June 2004, he joined the faculty of Aristotle University of Thessaloniki, Greece where he is currently an Assistant Professor at the Electrical and Computer

Engineering Department.

His major research interests include wireless communications theory, digital communications over fading channels, cooperative diversity systems, satellite communications and free-space optical communications. He has published and presented more than 70 technical papers in scientific journals and international conferences, he is co-author in two chapters in books and also co-author in a Greek Edition Book on Mobile Communications.

Dr. Karagiannidis acts as reviewer for several international journals and he served as Technical program Committee Member for several IEEE conferences. He is member of the editorial boards of *IEEE Transactions on Communications*, *IEEE Communications Letters* and *EURASIP Journal on Wireless Communications and Networking*.



**Athanassios C. Iossifides** (S'95, M'98) was born in Alexandroupolis, Greece, in 1969. He received the electrical and computer engineering degree in 1994, and the Ph.D degree in telecommunications, in 2000, both from the Faculty of Engineering of Aristotle University of Thessaloniki, Greece. During 1996-1998 he served as a laboratory assistant in the area of computer programming and during 1999-2000 as a scientific assistant in the area of telecommunication systems in Technological Institute of Thessaloniki, Greece. Since 1999, he is with COSMOTE Mobile

Telecommunications S.A. as a senior engineer, where he is currently in charge of North Greece Access and Transmission Network Management Team of Operation & Maintenance Dpt. He has participated in several research programs, such as "ATTACH" (1996-1998), COST Action 252 (1997-1998), Archimedes II (2006), dealing with mobile/satellite communications systems and techniques, as well as COSMOTE S.A. internal projects in the area of transmission systems and 3-3.5G technology. Dr. Iossifides main interests lie in the areas of modulation and channel coding techniques, multiple access techniques, wideband and spread spectrum communications, terrestrial and satellite mobile communications, modeling and simulation of telecommunication systems, applied transmission systems and optical communications. He has published and presented over 20 technical papers in scientific journals and international conferences. He serves as a reviewer for IEEE and EURASIP international journals.