

BER Performance of FSO Links over Strong Atmospheric Turbulence Channels with Pointing Errors

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Abstract—In this letter, we investigate the error rate performance of free-space optical (FSO) links over strong turbulence fading channels together with misalignment (pointing error) effects. First, we present a novel closed-form expression for the distribution of a stochastic FSO channel model which takes into account both atmospheric turbulence-induced fading and misalignment-induced fading. Then, we evaluate the average bit-error rate in closed form of a FSO system operating in this channel environment, assuming intensity modulation/direct detection with on-off keying. Numerical examples are further provided to collaborate on the derived analytical expressions.

Index Terms—Bit-error rate (BER), free-space optical (FSO) communications, K atmospheric turbulence channel, misalignment fading, pointing error.

I. INTRODUCTION

LONG distance outdoor free-space optical (FSO) links are highly vulnerable due to the degrading effects of atmospheric turbulence and pointing errors. As a result of variations in the refractive index, atmospheric turbulence causes irradiance fluctuations in the received signals (known as fading or scintillation) propagating along a horizontal path near ground. Turbulence-induced fading degrades the link performance particularly for distances of 1 km and above [1]. Another concern in FSO links is the pointing errors due to building sway. Thermal expansion, dynamic wind loads and weak earthquakes result in the sway of high-rise buildings. This causes vibrations of the transmitter beam and, therefore, misalignment between the transmitter and receiver [2].

Various statistical models have been proposed over the years to describe atmospheric turbulence channels for varying degrees of strength [1]. The K distribution has been found to be a suitable model for strong turbulence channels since it provides an excellent agreement between theoretical and experimental data [3]. In [4], Uysal and Li have used this turbulence model to evaluate the performance of coded FSO systems in terms of the pairwise error probability and bit-error rate (BER). In [5], Kiasaleh has studied the BER performance

of a FSO heterodyne communication system in the presence of K turbulence fading. In [6], Karagiannidis *et al.* have examined the overall performance of a relayed (multi-hop) FSO system.

The effects of misalignment and atmospheric turbulence on terrestrial FSO link performance have been investigated together for first time in [7] and [8]. In these works has been assumed negligible detector aperture size with respect to the beamwidth. In [9], Farid and Hranilovic have provided a FSO channel model which models the fading due to atmospheric turbulence and misalignment considering beamwidth, pointing error variance and detector size. Particularly, they have considered log-normal distributed and gamma-gamma distributed turbulence and have examined the system performance in terms of capacity and outage probability.

In this letter, we present a statistical FSO channel model considering the joint effects of K distributed strong turbulence fading and pointing errors. First, we obtain a closed-form probability density function (pdf) for the statistical description of this channel. Then, we derive the average BER expressions for an intensity-modulation/direct detection (IM/DD) FSO system with on-off keying (OOK). Selected numerical results are further provided in order to verify the accuracy of our mathematical analysis.

II. SYSTEM AND CHANNEL MODEL

A. System Model Assumptions

We consider a FSO system using IM/DD with OOK, which is widely deployed in commercial systems. The laser beams propagate along a horizontal path through a K turbulence channel with additive white Gaussian noise (AWGN) in the presence of pointing errors. The receiver integrates the photocurrent signal which is related to the incident optical power by the detector responsivity for each bit period. A unity responsivity coefficient is assumed.

The received electrical signal is given by

$$y = hx + n \quad (1)$$

where x is the binary transmitted signal, h is the normalized channel fading coefficient considered to be constant over a large number of transmitted bits, and n is AWGN with variance σ_n^2 . Since OOK modulation scheme is used, x is either 0 or $2P_t$ where P_t is the average transmitted optical power. The channel state is considered to be a product of two random factors i.e., $h = h_a h_p$ where h_a is the attenuation due to atmospheric turbulence and h_p the attenuation due to geometric spread and pointing errors.

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B. Turbulence and Misalignment Fading Model

In the K turbulence model the pdf of the irradiance is given by [4]

$$f_{h_a}(h_a) = \frac{2\alpha^{\frac{\alpha+1}{2}}}{\Gamma(\alpha)} h_a^{\frac{\alpha-1}{2}} K_{\alpha-1}\left(2\sqrt{\alpha h_a}\right), \quad h_a > 0 \quad (2)$$

where α is a channel parameter related to the effective number of discrete scatterers, $\Gamma(\cdot)$ is the well-known Gamma function [10, eq. (8.310.1)], and $K_\nu(\cdot)$ is the ν th-order modified Bessel function of the second kind [10, eq. (8.432.2)]. When $\alpha \rightarrow \infty$, (2) approaches the negative exponential (NE) distribution.

By considering a circular detection aperture of radius r and a Gaussian beam, the pdf of h_p can be derived using the assumptions and methodology described in [9] as

$$f_{h_p}(h_p) = \frac{\gamma^2}{A_0 \gamma^2} h_p^{\gamma^2-1}, \quad 0 \leq h_p \leq A_0 \quad (3)$$

where $\gamma = w_{z_{eq}}/2\sigma_s$ is the ratio between the equivalent beam radius at the receiver and the pointing error displacement standard deviation (jitter) at the receiver. The parameter $w_{z_{eq}}$ can be calculated using the relations $v = \sqrt{\pi}r/\sqrt{2}w_z$, $A_0 = [\text{erf}(v)]^2$ and $w_{z_{eq}}^2 = w_z^2 \sqrt{\pi} \text{erf}(v)/2v \exp(-v^2)$ where $\text{erf}(\cdot)$ is the error function and w_z is the beam waist (radius calculated at e^{-2}) at distance z (see [9] for more details).

Using the previous pdfs for turbulence and misalignment fading the combined pdf of h is given as

$$f_h(h) = \int f_{h|h_a}(h|h_a) f_{h_a}(h_a) dh_a \quad (4)$$

where $f_{h|h_a}(h|h_a)$ is the conditional probability given h_a state and is expressed by

$$\begin{aligned} f_{h|h_a}(h|h_a) &= \frac{1}{h_a} f_{h_p}\left(\frac{h}{h_a}\right) \\ &= \frac{\gamma^2}{A_0 \gamma^2 h_a} \left(\frac{h}{h_a}\right)^{\gamma^2-1}, \quad 0 \leq h \leq A_0 h_a. \end{aligned} \quad (5)$$

By substituting (2) and (5), (4) results in

$$\begin{aligned} f_h(h) &= \frac{2\gamma^2 \alpha^{\frac{\alpha+1}{2}}}{A_0 \gamma^2 \Gamma(\alpha)} h^{\gamma^2-1} \times \\ &\int_{h/A_0}^{\infty} h_a^{\frac{\alpha+1}{2}-1-\gamma^2} K_{\alpha-1}\left(2\sqrt{\alpha h_a}\right) dh_a. \end{aligned} \quad (6)$$

In order to solve the integral in (6), we express the $K_\nu(\cdot)$ in terms of the Meijer's G-function¹ [11, eq. (14)], i.e., $K_\nu(x) = \frac{1}{2} G_{0,2}^{2,0} \left[x^2/4 \middle| \frac{\nu}{2}, -\frac{\nu}{2} \right]$. Then, using [12, eq. (07.34.21.0085.01)], a closed-form expression is derived as

$$\begin{aligned} f_h(h) &= \frac{\alpha \gamma^2}{A_0 \Gamma(\alpha)} \left(\frac{\alpha h}{A_0}\right)^{\frac{\alpha-1}{2}} \times \\ &G_{1,3}^{3,0} \left[\frac{\alpha}{A_0} h \middle| -\frac{\alpha+1}{2} + \gamma^2, \frac{\alpha-1}{2}, \frac{1-\alpha}{2} \right]. \end{aligned} \quad (7)$$

¹Note that Meijers G-function [10, eq. (9.301)] is a standard built-in function in most of the well-known mathematical software packages. Additionally, using [10, eq. (9.303)], the Meijers G-function can be written in terms of the more familiar generalized hypergeometric functions [10, eq. (9.14.1)] but is omitted here due to space limitations.

Equation (7) can be further simplified using [12, eq. (07.34.16.0001.01)] as

$$f_h(h) = \frac{\alpha \gamma^2}{A_0 \Gamma(\alpha)} G_{1,3}^{3,0} \left[\frac{\alpha}{A_0} h \middle| -1 + \gamma^2, \alpha - 1, 0 \right]. \quad (8)$$

III. AVERAGE BER

The BER of IM/DD with OOK is given by $P_b(e) = p(1)p(e|1) + p(0)p(e|0)$ where $p(1)$ and $p(0)$ are the probabilities of sending 1 and 0 bits, respectively and $p(e|1)$ and $p(e|0)$ denote the conditional bit error probabilities when the transmitted bit is 1 and 0, respectively. Considering also that $p(1) = p(0) = \frac{1}{2}$ and $p(e|1) = p(e|0)$, it is easy to show that conditioned on h [13]

$$P_b(e|h) = p(e|1, h) = p(e|0, h) = Q\left(\frac{\sqrt{2}P_t h}{\sigma_N}\right) \quad (9)$$

where $Q(\cdot)$ is the Gaussian Q function which is related to the complementary error function $\text{erfc}(\cdot)$ by $\text{erfc}(x) = 2Q(\sqrt{2}x)$.

The average BER, $P_b(e)$ can be obtained by averaging (9) over the pdf of h i.e.,

$$P_b(e) = \int_0^{\infty} f_h(h) P_b(e|h) dh. \quad (10)$$

By substituting (8) in (10) and expressing the $K_\nu(\cdot)$ and $\text{erfc}(\cdot)$ integrands as Meijer's G-functions i.e., $\text{erfc}(\sqrt{x}) = \frac{1}{\sqrt{\pi}} G_{1,2}^{2,0} \left[x \middle| \frac{1}{0, 1/2} \right]$ [12, eq. (06.27.26.0006.01)], we obtain (11) which is shown at the top of next page. Thus, using [11, eq. (21)] and [10, eq. (9.31.1)], a closed-form solution for BER is derived as

$$P_b(e) = \frac{2^{\alpha-3} \gamma^2}{\sqrt{\pi^3} \Gamma(\alpha)} G_{6,3}^{2,5} \left[\frac{16P_t^2 A_0^2}{\sigma_N^2 \alpha^2} \middle| \frac{2-\gamma^2}{2}, \frac{1-\alpha}{2}, \frac{2-\alpha}{2}, 0, \frac{1}{2}, 1 \right]. \quad (12)$$

IV. NUMERICAL RESULTS

We consider a FSO link with normalized jitter, $\sigma_s/r = 0.1$, noise standard deviation, $\sigma_N = 10^{-7}$ A/Hz, and turbulence fading parameter, $\alpha = 2$. Fig. 1 shows the average BER in terms of the transmitted optical power in dBm for various values of the normalized beamwidth in steps of five ($w_z/r = 5, \dots, 25$). It is observed that using a narrow beamwidth a better BER performance is achieved because the received signal power is increased. However, as stated in [7], when the beamwidth is narrow, the transmitter may easily move from the line-of-sight (LOS) in the direction of the receiver resulting in cutoff in communications due to the increase of the misalignment effect.

In Figs. 2 and 3 the average BER versus turbulence fading parameter α (X-Axis) and normalized jitter σ_s/r (Y-Axis) is presented in 3D plots where $P_t = 20$ dBm and $w_z/r = 10$ or 20. From these figures it is shown that both building sway and turbulence degrade the BER performance of optical wireless links. However, this situation can be balanced by widening the beam at the expense of the received power in order to increase the robustness of the link in the presence of pointing errors. Therefore, a target BER can be achieved by selecting an optimum beamwidth which minimizes the

$$P_b(e) = \frac{\alpha\gamma^2}{2\sqrt{\pi}A_0\Gamma(\alpha)} \int_0^\infty G_{1,3}^{3,0} \left[\frac{\alpha}{A_0} h \left| -1 + \gamma^2, \alpha - 1, 0 \right. \right] G_{1,2}^{2,0} \left[\frac{P_t^2 h^2}{\sigma_N^2} \left| 0, \frac{1}{2} \right. \right] dh \quad (11)$$

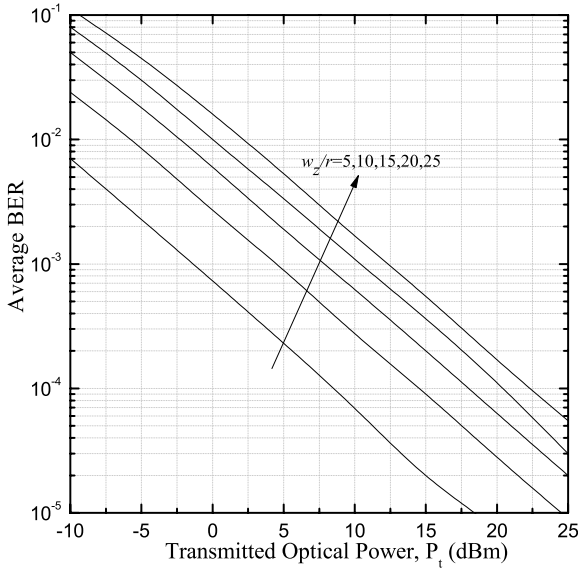


Fig. 1. BER versus transmitted power for several values of normalized beamwidth.

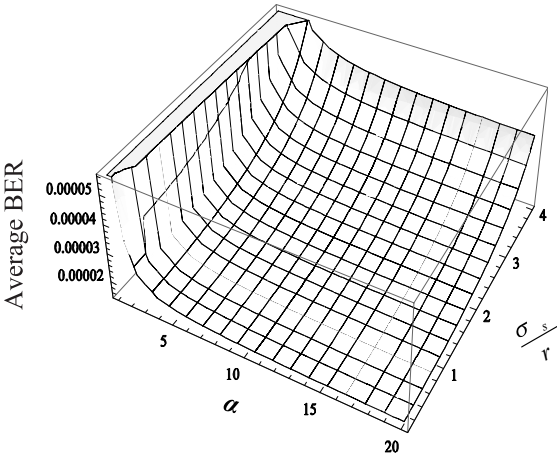


Fig. 2. BER versus α and σ_s/r assuming $P_t = 20$ dBm and $w_z/r = 10$.

required transmitting power. This optimization problem is very difficult due to the complex derived formulae of the average BER and special algorithms may be needed which is out of the scope of this letter.

V. CONCLUSION

The BER performance of a FSO system operating over K-distributed turbulence fading in the presence of misalignment fading, was studied. A useful multiplicative statistical channel model was derived including both fading conditions and also a closed-form BER expression was presented. It is clear that the designer of optical wireless systems should take into consideration the two dominant factors affecting the performance of optical wireless communications, the atmospheric turbulence

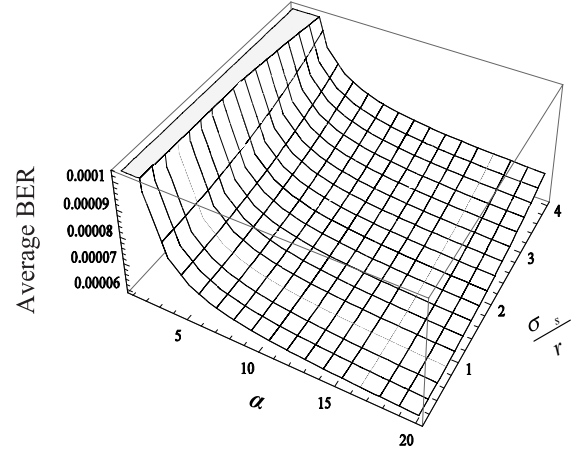


Fig. 3. BER versus α and σ_s/r assuming $P_t = 20$ dBm and $w_z/r = 20$.

and the building sway.

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