Capacity Performance Analysis of M-ary PPM TH-UWB Systems in the Presence of Narrowband Interference

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Abstract: **The capacity of ultra-wideband (UWB) systems in presence of narrowband interference (NBI) is studied. By appropriately modifying the Shannon capacity formula, an analytical expression for the capacity of M-ary pulse position modulation (PPM) signals in the presence of NBI is obtained. Performance evaluation results for the capacity of such signals as a function of the NBI carrier frequency and power are also presented.**

Index Terms: **Capacity, narrowband interference (NBI), pulse position modulation (PPM), spread spectrum communication, time hopping, ultra-wideband (UWB).**

I. INTRODUCTION

Ultra-wideband (UWB) radio system technology plays an important role in current research for wireless personal area networks [1]. One of the most widely studied schemes for such systems utilizes M-ary pulse position modulation (PPM) combined with time-hopping (TH) as a multiple access technique [2], [3]. The multiple access performance of this scheme has been extensively studied in the past [4]–[8]. Its performance in a multipath fading environment has also been addressed [9], [10].

The performance of UWB systems in channels with narrowband interference (NBI) has been studied in [12] and [13] by evaluating the processing gain of the single user correlator receiver, which is optimal in an additive white Gaussian noise (AWGN) environment. Further interference studies have shown that the single user receiver does not effectively combat strong NBI, especially for high data rate UWB systems where the spreading factor is relatively small [10]. In the same reference, a minimum mean square error (MMSE) receiver which improves the performance of such systems has been proposed and analyzed. These UWB-NBI studies employ as performance measures the processing gain [12], [13] and bit error rate [10].

The problem of estimating the capacity of UWB systems has received relatively less attention. It has been studied for the AWGN channel in [5] and [6]. A more recent study [11] evaluates the cut-off rate, also known as the practical capacity, of such systems in the presence of multiple access interference. Our current contribution focuses on studying the Shannon capacity of such systems in the presence of NBI. As well known, capacity is considered as a more fundamental performance measure since it provides limits on achievable performance. Like other capacity studies [6]–[11], the effect of multipath fading is not considered in this paper.

The remainder of this paper is organized as follows. In Section II the UWB and NBI signal models are presented and in Section III a method for evaluating the capacity is proposed and analyzed. Various performance evaluation results are presented and discussed in Section IV.

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II. SYSTEM MODEL

A typical M-ary PPM TH-UWB transmitted signal is mathematically represented as [2]:

$$s(t) = \sum_i \sqrt{E_p} q(t - iT_f - c_i T_c - \delta_m), 1 \le m \le M \quad (1)$$

where $q(t)$ is the UWB pulse, a Gaussian monocycle with duration T_p. In the above equation E_p is the received energy per pulse, T_f is the pulse repetition interval, usually referred to as frame, c_i is the TH sequence, T_c the TH slot duration, and δ_m is the modulation shift for M-ary orthogonal PPM ($\delta_m - \delta_{m-1} \ge T_p$). The pulse power is $P_p = E_p/T_p$. The TH sequence is modeled as a series of independent uniformly distributed random numbers taking integer values in the range $[0, T_f/T_c]$. Assuming N_p pulses per symbol are transmitted, the symbol rate is $R_s = 1/(N_p T_f)$ and thus the symbol duration is $T_s = N_p T_f$. The spreading factor is $\beta = T_f/T_p$, the pulse rate, $\lambda = 1/T_f$, is the average rate of the transmitted pulses and the effective bandwidth (3 dB bandwidth) is $W_p = 1.297/T_p$ [12].

The NBI waveform, $j(t)$, is modeled as a phase and/or amplitude modulated signal, e.g., phase shift keying (PSK) or quadrature amplitude modulation (QAM), with an ideal block spectrum of bandwidth W_J, i.e.,

$$j(t) = \sum_i w_n(t - iT_J) = \sqrt{J_0} \sum_i \frac{\sin[\pi W_J (t - iT_J)]}{\pi(t - iT_J)} \times \{a_n \cos[2\pi f_c (t - iT_J)] + b_n \sin[2\pi f_c (t - iT_J)]\} \quad (2)$$

where J_0 is the power spectral density (PSD) of the interfering signal received by the UWB receiver, f_c its carrier frequency, T_J the symbol duration, and $w_n(t)$ and $\{a_n, b_n\}$ are the waveform and the in-phase and quadrature components of the nth signal taking values according to the specific modulation used with $1 \le n \le M_J$ where M_J is the number of modulated signals. The NBI power is $P_J = J_0 W_J$.

Assuming an AWGN channel and a single NBI source the received signal can be expressed as:

$$r(t) = s(t - \tau_s) + j(t - \tau_j) + n(t) \quad (3)$$

where $n(t)$ is the AWGN waveform with zero mean and one sided PSD N_0 and the delays τ_s and τ_j are due to the asynchronous transmission.

A typical UWB receiver consists of M correlators matched to the template functions $u_i(t) = q(t-\delta_i)$, $1 \le i \le M$. Assuming that the receiver is perfectly synchronized with the UWB signal, the output signal of the νth correlator at the kth symbol can be expressed as:

$$\begin{aligned} y_{k,\nu} = &\sum_{\kappa=kN_p}^{(k+1)N_p-1} \left\{ \int_{\kappa T_f}^{(\kappa+1)T_f} s(t) u_\nu(t-\kappa T_f - c_\kappa T_c)dt + \right. \\ &\int_{\kappa T_f}^{(\kappa+1)T_f} j(t-\tau_j+\tau_s) u_\nu(t-\kappa T_f - c_\kappa T_c)dt + \\ &\left. \int_{\kappa T_f}^{(\kappa+1)T_f} n(t+\tau_s) u_\nu(t-\kappa T_f - c_\kappa T_c)dt \right\} \\ = &\sum_{\kappa=kN_p}^{(k+1)N_p-1} [X_{\kappa,\nu} + I_{\kappa,\nu} + N] \end{aligned} \quad (4)$$

where $X_{\kappa,\nu}$, $I_{\kappa,\nu}$, and N are the contributions of the UWB, NBI and AWGN signals, respectively. The UWB signal term is $X_{\kappa,\nu} = \sqrt{E_p}$ if $\nu = m$ and zero otherwise, while the noise term remains Gaussian with variance $\sigma_N^2 = N_0$. Since the template functions in (4) are non-zero for a small fraction of the integration interval, for the convenience of the analysis this integration limits can be extended to $(-\infty, +\infty)$. It is also convenient to express $I_{\kappa,\nu}$ as:

$$I_{\kappa,\nu} = \sum_i I_{\kappa,\nu}^{i,w_n}, \quad (5a)$$

$$I_{\kappa,\nu}^{i,w_n} = \int_{-\infty}^{+\infty} w_n(t - iT_J - \tau_j + \tau_s) u_\nu(t - \kappa T_f - c_\kappa T_c)dt. \quad (5b)$$

Since $w_n(t)$ has a block spectrum with bandwidth W_J, by using the Fourier transform, $I_{\kappa,\nu}^{i,w_n}$ can be expressed as:

$$I_{\kappa,\nu}^{i,w_n} = \sqrt{J_0} r_n \int_{f_c - W_J/2}^{f_c + W_J/2} Q(f) \exp[j(2\pi f \tau + \theta_n)]\, df \quad (6)$$

where $\tau = \kappa T_f + c_\kappa T_c - iT_J - \tau_j + \tau_s$, $Q(f)$ is the Fourier transform of $q(t)$, and $\{r_n, \theta_n\}$ is the polar form of the coefficients $\{a_n, b_n\}$. Since $W_J \ll W_p$, (6) simplifies to:

$$I_{\kappa,\nu}^{i,w_n} = \sqrt{J_0} r_n Q(f_c) \frac{\sin(\pi W_J \tau)}{\pi \tau} \cos(2\pi f_c \pi W_J \tau + \theta_n). \quad (7)$$

III. CAPACITY EVALUATION

It is mathematically convenient to express the transmitted and received signals on a certain time interval as vectors. Thus, a single M-ary PPM UWB pulse can be represented as a vector, $\mathbf{x_{k,m}} = [x_{k,1}, x_{k,2}, \cdots, x_{k,M}]^{\mathrm{T}}$, $1 \le m \le M$, where m is the transmitted symbol in the kth frame and $x_{k,i} = X_{k,i}$ indicates the presence of the pulse at the ith PPM slot. Equivalently, the received signal vector, $\mathbf{y_k} = [y_{k,1}, y_{k,2}, \cdots, y_{k,M}]^{\mathrm{T}}$, is the output of the M correlators at the receiver.

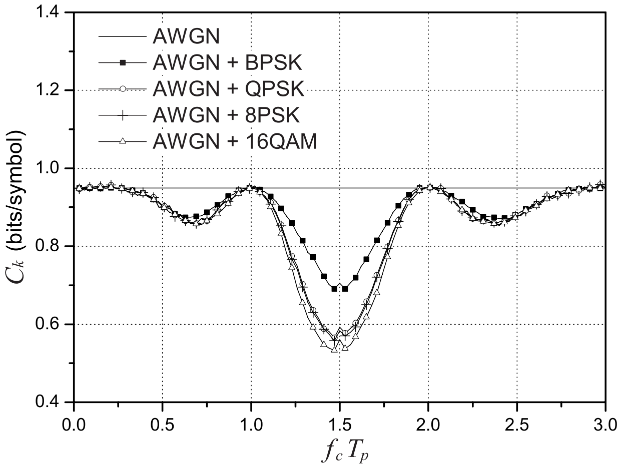

Fig. 1. Capacity of 2-PPM in AWGN channel with NBI vs. NBI frequency.

An expression of the Shannon capacity formula for M-ary PPM on Gaussian and Webb noise channels without interference has been presented in [14]. However, this expression cannot be used to directly compute the UWB capacity since the components of the vector $\mathbf{y_k}$ are correlated. In order to take into account this correlation, we propose the following generic expression which, in principle, calculates the mutual information in a time interval spanning several symbols.

$$\begin{aligned} C_K = &\log_2 M - \frac{1}{K} \int_{\mathbf{y_1 y_2 \cdots y_K}} p(\mathbf{y_1 y_2 \cdots y_K} | \mathbf{x_{1,1} x_{2,1} \cdots x_{K,1}}) \\ &\log_2 \left[\frac{\sum_{i_1, i_2, \cdots, i_K = 1}^{M} p(\mathbf{y_1 y_2 \cdots y_K} | \mathbf{x_{1,i_1} x_{2,i_2} \cdots x_{K,i_K}})}{p(\mathbf{y_1 y_2 \cdots y_K} | \mathbf{x_{1,1} x_{2,1} \cdots x_{K,1}})} \right] \\ &d\mathbf{y_1} d\mathbf{y_2} \cdots d\mathbf{y_K} \end{aligned} \quad (8)$$

where $\mathbf{y_1}, \mathbf{y_2}, \cdots, \mathbf{y_K}$ are the received signal vectors at K successive pulse intervals. Assuming perfect timing synchronization, i.e., τ_j and τ_s are known at the receiver, the conditional probability $p(\mathbf{y_1 y_2 \cdots y_K} | \mathbf{x_{1,i_1} x_{2,i_2} \cdots x_{K,i_K}})$ can be mathematically expressed as:

$$p(\mathbf{y_1 y_2 \cdots y_K} | \mathbf{x_{1,i_1} x_{2,i_2} \cdots x_{K,i_K}}) = \frac{1}{M_J^{K_J}} \sum_{n_1, n_2, \cdots, n_{K_J} = 1}^{M_J} \prod_{k=1}^{K} \prod_{m=1}^{M} N_{\mu,\sigma}(y_{k,m}) \quad (9)$$

where $N_{\mu,\sigma}$ is the Gaussian probability density function (PDF) with mean $\mu = x_{k,m} + \sum_{\kappa=1}^{K_J} I_{k,m}^{\kappa, w_{n_\kappa}}$ and variance $\sigma^2 = N_0$. K_J, the number of the NBI symbols included in the computation, has been chosen $K_J T_J \ge K T_f$ so that at least all the symbols that overlap with the K UWB pulses are taken into account.

It should be noted that (8) is equivalent to the original Shannon capacity formula, with the difference that it calculates the normalized mutual information over the interval of K symbols. Clearly, as $K \to \infty$, C_K becomes the exact capacity.

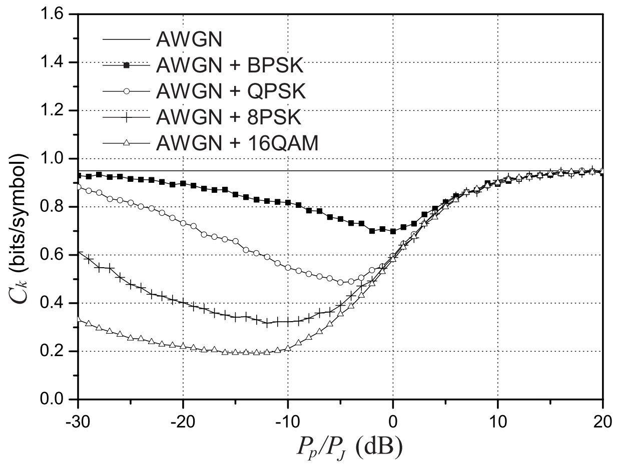

Fig. 2. Capacity of 2-PPM in AWGN channel with NBI vs. the pulse to NBI power ratio.

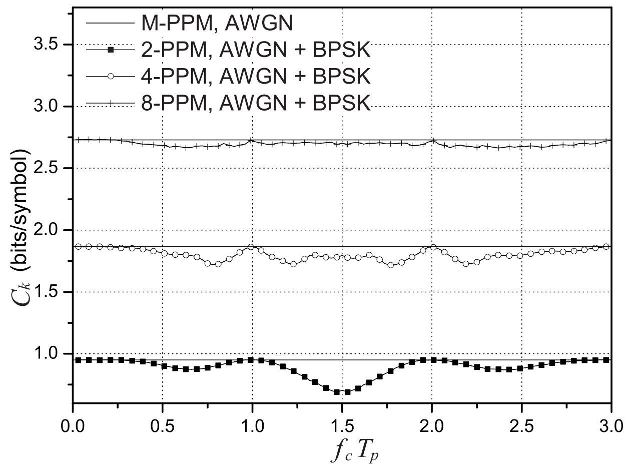

Fig. 3. Capacity of 2-, 4-, and 8-PPM in AWGN channel with NBI vs. NBI frequency.

IV. PERFORMANCE RESULTS AND DISCUSSION

The capacity formula (8) is evaluated with Monte-Carlo integration, as the M-dimensional integral makes exhaustive calculation a computationally very inefficient task [14]. For NBI, the modulations considered were binary PSK (BPSK), quadrature PSK (QPSK), 8-PSK, and 16-QAM and the UWB signal was M-ary PPM. Unless otherwise noted, the following parameters are used for the performance evaluation: $\beta = 100$, $\delta_2 - \delta_1 = T_p$, $N_p = 1$, $E_p/N_0 = 10$ dB, $P_p = P_J$, $W_J = \lambda = 0.0077 W_p$, and $f_c = 1.5/T_p$. W_J and f_c are expressed as a function of W_p and T_p, respectively, because it was observed that if W_J/W_p and $f_c T_p$ are constant, the capacity evaluation results remained unchanged.

Due to the inherent complexity of (8) it turns out that the exact evaluation of C_K for large values of K is computationally extremely complex. However, extensive numerical evaluation tests have shown that, for the proposed UWB system, C_K converges rapidly even for small values of K and in practical cases, i.e., when $W_J \leq \lambda$, the performance results were reasonably accurate for $K > T_J/T_f$. Thus for our performance evaluation results, $K = 3$ was chosen.

Fig. 1 illustrates the dependence of the UWB capacity on f_c. The not completely symmetric (with respect to f_c) shape of the capacity curves is due to the non-symmetric PSD of the employed UWB pulse, $q(t)$. When f_c becomes too small or too large, the NBI signal is attenuated by the matched filter of the receiver thus diminishing its impact on capacity. Furthermore, our results verify the observation made in [15], that when $f_c = k/\delta$, $k \in \mathbb{N}$, the interference is completely cancelled. The minor discontinuity observed at $f_c T_p = 1.5$ happens because at this frequency the interval of three frames used for the evaluation is an exact multiple of the NBI carrier period. It is also interesting to note that the QPSK and 8-PSK capacity curves in Fig. 1 are identical. This happens because for $P_p = P_J$ the UWB capacity for these two modulation schemes is equal (see Fig. 2).

Fig. 2 illustrates the performance of C_K versus P_p/P_J. It is noted that the impact on capacity becomes more severe when the bits per symbol of the narrowband signal are increased. Also, these performance results are somewhat surprising since it is usually expected that as P_J increases the capacity would decrease. However, in our case, when P_J becomes too high the degradation of the UWB capacity is reduced. This is due to the fact that the probability of incorrect decoding of the signal becomes smaller. To explain this let us consider as an example the sum of two baseband binary signals decoded by a single receiver. If both signals have similar power levels, it is reasonable to expect that the receiver cannot reliably decode them, while a significant difference in their power levels results in improved performance for both signals.

Fig. 3 presents the capacity performance of higher order M-ary PPM modulations, i.e., $M > 2$. In this case, a significant improvement on the overall capacity performance is observed as M becomes larger. Furthermore, as M increases, the dependence of the capacity degradation on f_c diminishes. Since the narrowband signal is observed by the receiver only during the reception of the UWB pulses, a higher M results in a larger observation window and thus in a reduced NBI effect.

V. CONCLUSIONS

In this paper we have derived an expression for the capacity of a TH-UWB system in the presence of NBI. Based on this expression, we have evaluated and discussed the capacity of a TH-UWB system under such interference conditions. The obtained performance evaluation results have indicated that the effect of NBI is more severe when the narrowband signal has similar power level to the received UWB signal. Furthermore, the effect of NBI is mitigated significantly with the use of higher order PPM for the UWB signal.

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