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Journal of the Franklin Institute 345 (2008) 723–728
Journal of The Franklin Institute
www.elsevier.com/locate/jfranklin

Symbol error probability of decode and forward cooperative diversity in Nakagami-*m* fading channels

Diomidis S. Michalopoulos[a,*], George K. Karagiannidis[a], George S. Tombras[b]

[a]*Department of Electrical and Computer Engineering, Division of Telecommunications, Aristotle University of Thessaloniki, 54124 Thessaloniki, Greece*
[b]*Laboratory of Electronics, Department of Physics, University of Athens, Panepistimioupolis, 15784 Athens, Greece*

Received 7 December 2007; accepted 19 March 2008

Abstract

In this paper, we provide closed-form expressions for the symbol error probability of decode and forward cooperative diversity systems, when operating over independent but not necessarily identically distributed Nakagami-*m* fading channels. The results hold for arbitrary number of relays, and refer to the *M*-ary QAM and *M*-ary PSK modulations.

Keywords: Cooperative diversity; Decode and forward relaying; Nakagami-*m* fading; Symbol error probability

1. Introduction

Cooperative diversity is an emerging concept that serves as a substitute of the well-known conventional diversity methods, promising increased coverage and performance in future wireless communications, without the need for high transmitting powers. In particular, in cooperative diversity systems a set of spatially distributed terminals co-operate with each other in order to form virtual antenna arrays and hence achieve diversity without being necessary to pack multiple antennas per terminal. This allows for a direct

*Corresponding author.
E-mail addresses: dmixalo@auth.gr (D.S. Michalopoulos), geokarag@auth.gr (G.K. Karagiannidis), gtombras@cc.uoa.gr (G.S. Tombras).

0016-0032/$32.00
doi:10.1016/j.jfranklin.2008.03.006

implementation of this form of diversity in systems where small, hand-held devices with high processing capabilities need to be employed.

Recently published results have demonstrated that cooperative diversity systems offer remarkable advances in terms of outage and error probability, when the communication links involved suffer from small-scale and/or large-scale fading. Specifically, in [1,2] the authors conducted an exact outage and error analysis, respectively, for arbitrary number of decode-and-forward (DF) relays and Rayleigh fading channels. For the versatile case of Nakagami-m fading, an approximate outage and error analysis can be found in [3]; that analysis, however, considered only the case of amplify and forward relays.

In this paper, we extend the analysis conducted in the aforementioned works to the versatile case of Nakagami-m fading channels (where m takes any positive integer value), which allows for a better overview of the performance of such systems in more general fading scenarios. In particular, we provide a closed-form expression for the exact symbol error probability (SEP) of cooperative diversity with arbitrary number of DF relays, when operating over Nakagami-m fading channels.

The remainder of this paper is organized as follows: a brief description of the system under consideration is given in Section 2. In Section 3, the closed-form expression for the SEP of DF cooperative diversity over Nakagami-m fading channels is derived. Finally, in Section 4 we provide some numerical for several realizations regarding the average channel conditions.

2. System model

We consider a source node, S, communicating with a destination one, D, with the help of L relaying terminals denoted by R_i, $i = \{1, \ldots, L\}$, as shown in Fig. 1.

The relays are assumed half-duplex, which implies that the communication is performed in two phases: in the former, the source broadcasts the message to the relays, whilst in the latter, the relays that have successfully decoded the original signal re-encode and forward the received data to the destination, in some predetermined order. These relays (i.e., those with successful symbol detections) form the so-called decoding set, denoted here by Θ.

Henceforth, we assume that error checking codes are employed (e.g., cyclic redundancy check (CRC) codes), so that the relays are able to identify any possible erroneous detections. The destination is assumed to combine the multiple replicas of the original

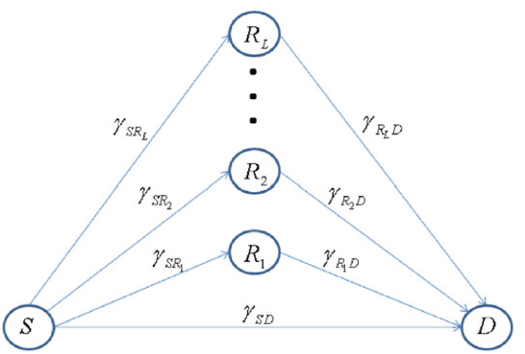

Fig. 1. The typical cooperative diversity set-up.

signal into a maximal-ratio combiner (MRC). Slow, flat Nakagami-m distributed fading between any nodes is assumed, which is independent but not necessarily identical with each other. The instantaneous signal-to-noise ratio (SNR) of the link between the nodes A, B is denoted by γ_{AB}, with $A, B \in \{S, R_1, \ldots, R_L, D\}$; the expected value of γ_{AB} is denoted by $\bar{\gamma}_{AB}$.

3. SEP analysis

Using the total probability law, the overall SEP can be evaluated as

$$P_e = \sum_{\Theta} \Pr\{\Theta\}\Pr\{e|\Theta\} \tag{1}$$

where the summation region comprises of all the possible subsets of available relays, including the empty set. The probability of each of such subsets being the decoding set is defined as

$$\Pr\{\Theta\} = \prod_{i:R_i\in\Theta}(1-\varepsilon_{SR_i})\prod_{j:R_j\notin\Theta}\varepsilon_{SR_j} \tag{2}$$

where ε_{AB} denotes the SEP of the $A \to B$ link. Using the results given in [4, Chapter 8.2], we may express $\Pr\{\Theta\}$ in closed-form for any of the M-QAM and M-PSK modulations used, by substituting ε_{SR_i} and $\varepsilon_{SR_j}$with [4, Eqs. (8.109), (8.115)] or [4, Eq. (8.122)]. For example, for the case of M-PSK modulation $\Pr\{\Theta\}$ can be written as

$$\begin{aligned}\Pr\{\Theta\} = &\prod_{i:R_i\in\Theta}\left[\frac{1}{M} + \left(\frac{1}{2}+\frac{\tan^{-1}a_i}{\pi}\right)\sqrt{\frac{B\bar{\gamma}_{SR_i}/m_{SR_i}}{1+B\bar{\gamma}_{SR_i}/m_{SR_i}}}\left(\sum_{k=0}^{m_{SR_i}-1}\binom{2k}{k}\right.\right.\\ &\times\left.\left.\frac{1}{[4(1+B\bar{\gamma}_{SR_i}/m_{SR_i})]^k} + \sin(\tan^{-1}a_i)\sum_{k=1}^{m_{SR_i}-1}\sum_{j=1}^{k}\frac{T_{jk}[\cos(\tan^{-1}a_i)]^{2(k-j)+1}}{(1+B\bar{\gamma}_{SR_j}/m_{SR_j})^k}\right)\right]\\ &\times\prod_{i:R_i\notin\Theta}\left[\frac{M-1}{M} - \left(\frac{1}{2}+\frac{\tan^{-1}a_i}{\pi}\right)\sqrt{\frac{B\bar{\gamma}_{SR_i}/m_{SR_i}}{1+B\bar{\gamma}_{SR_i}/m_{SR_i}}}\left(\sum_{k=0}^{m_{SR_i}-1}\binom{2k}{k}\right.\right.\\ &\times\left.\left.\frac{1}{[4(1+B\bar{\gamma}_{SR_i}/m_{SR_i})]^k} + \sin(\tan^{-1}a_i)\sum_{k=1}^{m_{SR_i}-1}\sum_{j=1}^{k}\frac{T_{jk}[\cos(\tan^{-1}a_i)]^{2(k-j)+1}}{(1+B\bar{\gamma}_{SR_j}/m_{SR_j})^k}\right)\right]\end{aligned} \tag{3}$$

where m_{AB} represents the Nakagami fading severity parameter of the $A \to B$ link; the parameters a and B are defined as $a_i = \sqrt{(B\bar{\gamma}_{SR_i}/m_{SR_i})/(1+B\bar{\gamma}_{SR_i}/m_{SR_i})}\cot(\pi/M)$ and $B = \sin^2(\pi/M)$, respectively, while T_{jk} is defined as

$$T_{jk} = \binom{2k}{k}\Bigg/\left[\binom{2(k-j)}{k-j}4^j(2(k-j)+1)\right]$$

The conditional error probability in Eq. (1), conditioned on the decoding set Θ, corresponds to the SEP of a multi-channel receiver with $|\Theta|+1$ independent input branches (i.e., the $S \to D$ branch plus the $|\Theta|$ relayed ones), where $|\Theta|$ denotes the cardinality of Θ. Therefore, for integer values of m, using [5, Eq. (18); Table 1] we may express $\Pr\{e|\Theta\}$ (and thereafter the overall SEP) in closed form for any of the aforementioned modulations, since $\Pr\{e|\Theta\}$ coincides with the probability that the sum of SNRs of the $|\Theta|+1$ active branches leads to an error. For the case of M-PSK modulation, for instance, denoting by $R_{\Theta j}$ the j-th relay that belongs to Θ, $j=\{1,\ldots,|\Theta|\}$, it holds

$$\Pr\{e|\Theta\} = A\sum_{j=1}^{|\Theta|+1}\sum_{k=1}^{m_{R_{\Theta j}D}}\left[\Xi_{|\Theta|+1}\left(j,k,\{m_{R_{\Theta q}D}\}_{q=1}^{|\Theta|+1},\left\{\frac{\bar{\gamma}_{R_{\Theta q}D}}{m_{R_{\Theta q}D}}\right\}_{q=1}^{|\Theta|+1},\{l_q\}_{q=1}^{|\Theta|-1}\right)\right.$$
$$\left.\times\frac{(2k-1)!!}{k!(2B)^k}\left(\frac{m_{R_{\Theta j}D}}{\bar{\gamma}_{R_{\Theta q}D}}\right)^k {}_2F_1\left(k,k+\frac{1}{2};k+1;-\frac{m_{R_{\Theta j}D}}{B\bar{\gamma}_{R_{\Theta q}D}}\right)\right] \tag{4}$$

where $A=1/2$ for BPSK and $A=1$ for higher PSK constellations; ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ denotes the Gauss hypergeometric function [6, Eq. (9.100)]; the notation $\Xi_L(\cdot,\cdot,\cdot,\cdot,\cdot)$ stands for a compact representation of the following multiple summation [5, Eq. (7)]:

$$\Xi_L(i,k,\{m_q\}_{q=1}^{L},\{\eta_q\}_{q=1}^{L},\{l_q\}_{q=1}^{L-2})$$
$$=\sum_{l_1=k}^{m_i}\sum_{l_2=k}^{l_1}\cdots\sum_{l_{L-2}=k}^{l_{L-3}}\left\{\frac{(-1)^{V_L-m_i}\eta_i^k}{\prod_{h=1}^{L}\eta_h^{m_h}}\frac{(m_i+m_{1+U(1-i)}-l_1-1)!}{(m_{1+U(1-i)}-1)!(m_i-l_1)!}\right.$$
$$\times\left(\frac{1}{\eta_i}-\frac{1}{\eta_{1+U(1-i)}}\right)^{l_1-m_i-m_{1+U(1-i)}}\frac{(l_{L-2}+m_{L-1+U(L-1-i)}-k-1)!}{(m_{L-1+U(L-1-i)}-1)!(l_{L-2}-k)!}$$
$$\times\left(\frac{1}{\eta_i}-\frac{1}{\eta_{L-1+U(L-1-i)}}\right)^{k-l_{L-2}-m_{L-1+U(L-1-i)}}\prod_{s=1}^{L-3}\left[\frac{(l_s+m_{s+1+U(s+1-i)}-l_{s+1}-1)!}{(m_{s+1+U(s+1-i)}-1)!(l_s-l_{s+1})!}\right.$$
$$\left.\left.\times\left(\frac{1}{\eta_i}-\frac{1}{\eta_{s+1+U(s+1-i)}}\right)^{l_{s+1}-l_s-m_{s+1+U(s+1-i)}}\right]\right\} \tag{5}$$

where in (5) $V_L=\Sigma_{j=1}^{L}m_j$, and $U(.)$ stands for the unitary step function defined as $U(x)=1$ if $x\geqslant 0$ and zero otherwise. The reader may note that for compactness of exposition, the $S\to D$ channel is also included in Eq. (4), hence the terms $m_{R_{\Theta(|\Theta|+1)}}$and $\bar{\gamma}_{R_{\Theta(|\Theta|+1)}}$ refer to m_{SD} and $\bar{\gamma}_{SD}$, respectively.

4. Numerical results

In this section, some numerical examples that demonstrate the SEP performance of cooperative diversity with multiple DF relays under various fading realizations are presented.

The SEP for BPSK modulation versus the average SNR is depicted in Fig. 2. The four curves shown in this figure correspond to four different realizations regarding the number of available relays and the relative strength of the channels involved. Specifically, realizations 1 and 2 refer both to the case where $L=2$; in the former, the average SNRs and Nakagami-m parameters are as follows: $\bar{\gamma}_{SR_i}=\bar{\gamma}_{R_iD}=\text{SNR}$, $i=\{1,2\}$, $(m_{SR_1},m_{R_1D})=(2,1)$ and $(m_{SR_2},m_{R_2D})=(1,2)$; in the latter, the same parameters are as follows: $\bar{\gamma}_{SR_1}=0.5\bar{\gamma}_{R_1D}=\text{SNR}$, $\bar{\gamma}_{SR_2}=2\bar{\gamma}_{R_2D}=2\text{SNR}$, $(m_{SR_1},m_{R_1D})=(2,1)$ and $(m_{SR_2},m_{R_2D})=(1,2)$. Realizations 3 and 4 refer both to the $L=4$ case; in the former, $\bar{\gamma}_{SR_i}=\bar{\gamma}_{R_iD}=\text{SNR}$, $i=\{1,\ldots,4\}$, $m_{SR_i}=1$, $i=\{1,\ldots,4\}$ and $(m_{R_1D},m_{R_2D},m_{R_3D},m_{R_4D})=(1,2,3,4)$; in the latter, $\bar{\gamma}_{SR_i}=\text{SNR}$, $i=\{1,\ldots,4\}$, $\bar{\gamma}_{R_iD}=(2/2^{i-1})\text{SNR}$, $i=\{1,\ldots,4\}$, $m_{SR_i}=1$, $i=$

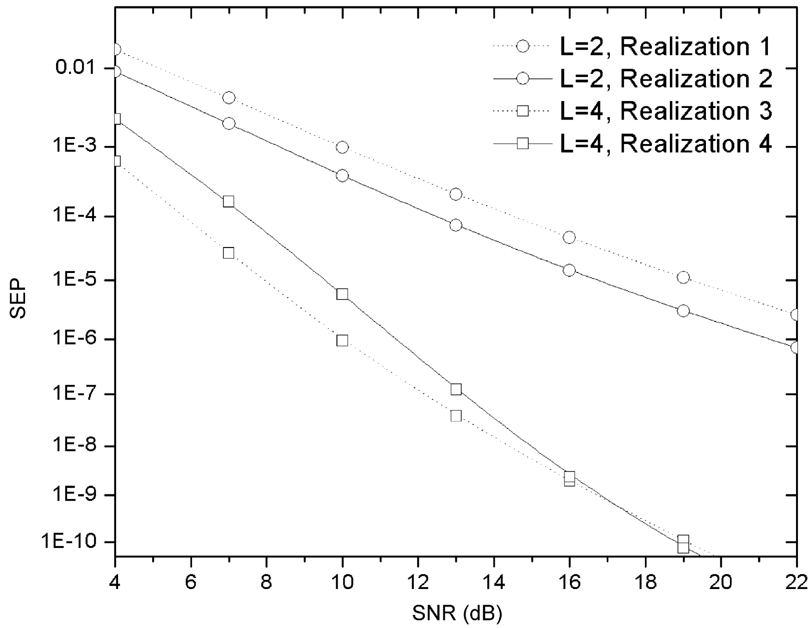

Fig. 2. SEP of DF cooperative diversity versus the average SNR for the case of two and four available relays, assuming BPSK modulation and two different channel realizations per case.

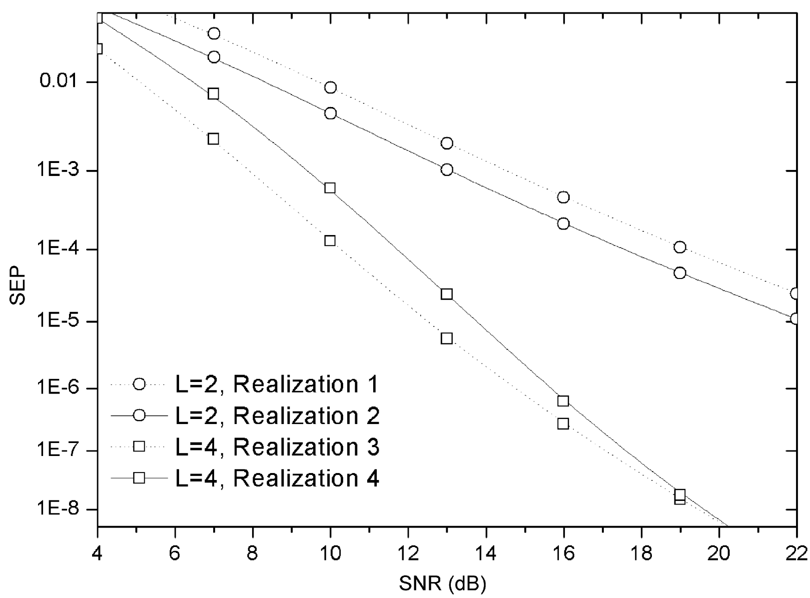

Fig. 3. SEP of DF cooperative diversity versus the average SNR for the case of two and four available relays, assuming QPSK modulation and two different channel realizations per case.

$\{1, \ldots, 4\}$ and $(m_{R_1D}, m_{R_2D}, m_{R_3D}, m_{R_4D}) = (1, 2, 3, 4)$. The direct ($S \rightarrow D$) channel in all the aforementioned scenarios is supposed shadowed enough to be considered negligible. As it shown in Fig. 2, for $L = 2$ case realization, 2 corresponds to generally better error performance; for $L = 4$ case, realization 3 is associated with a lower SEP in the low and medium SNR regions, however a cross point exists, hence for high SNRs the relative performances of realizations 3 and 4 are inverted.

Similar observations regarding the QPSK error performance of cooperative diversity with multiple DF relays are extracted from Fig. 3, under the same fading realizations as previous. One may also notice in these two figures that the slope of the curves in the high-SNR regime implies that the diversity gain of the scheme under investigation is, under the weak $S \rightarrow D$ channel assumption, on the order of the number of available relays.

5. Conclusions

In this paper, the error performance of cooperative diversity with multiple DF relays under Nakagami-*m* distributed fading was investigated, for the cases of *M*-ary QAM and *M*-ary PSK modulations. Numerical results reveal that, as expected, the diversity gain that such systems offer is, under the weak source–destination channel assumption, on the order of the number of available relays.

Acknowledgments

This work was performed within the framework of PENED'03, co-financed by the National and Community Funds (25% from the Greek Ministry of Development-General Secretariat of Research and Technology and 75% from E.U.-European Social Fund).

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