Semi-blind amplify-and-forward with partial relay selection

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The performance of the dual-hop semi-blind amplify-and-forward relay channel, where the intermediate relay node is selected depending on the instantaneous and partial channel knowledge, is investigated. Using closed-form expressions for the cumulative distribution function of the end-to-end signal-to-noise ratio (SNR), outage probability, SNR moments and average bit error rate are derived.

Introduction: There has been much research interest in cooperative schemes, where one or more relay nodes are selected according to some network parameters [1–5]. More specifically, we refer interested readers to [2] for a detailed exposition of the relay selection schemes proposed so far. In [4, 5], amplify-and-forward (AF) relay selection based on the best instantaneous signal-to-noise ratio (SNR) composed of SNR across both source-relay and relay-destination hops has been addressed. However, in some applications such as ad hoc and sensor networks, monitoring the global connectivity among different links can drain precious battery power as well as introduce additional network delays. Such challenges have prompted some authors to propose sub-optimal relay selection schemes which require only local (single hop) information. For example, in [1] a partial relay selection policy based only on the average SNR and instantaneous channel state information (CSI) of the source-relay link for CSI assisted AF relays has been proposed. In this Letter, we analyse the performance of an AF semi-blind relaying system with partial relay selection providing closed-form expressions for the outage probability, end-to-end (e2e) SNR moments and the average bit error rate (BER).

Overview of partial relay selection: Consider a relay system where a source communicates with a destination, D, using K relays denoted by R_i, $i = 1, \ldots, K$. According to the partial relay selection policy described in [1], the source selects a single ith relay among the K relays having the highest $S - R_i$ hop instantaneous SNR, γ_{SR_i}. The relays are assumed to be operative in the half-duplex AF mode. Therefore, each transmission period between S and D via the selected R_i is divided into two sub-slots.

The instantaneous e2e SNR of the virtual $S - R_i - D$ channel is given by

$$\gamma_i = \frac{\gamma_{SR_i}\gamma_{R_iD}}{C + \gamma_{R_iD}} \tag{1}$$

where γ_{R_iD} represents the instantaneous SNR of the $R_i - D$ link; C is a constant. All links ($S - R_i$, $R_i - D$) in the network experience flat and slow Rayleigh fading, i.e. the random variables γ_{SRi} and γ_{R_iD} are exponentially distributed. We assume that the relays use the semi-blind fixed gain [6]. Hence, C is expressed as

$$C = \frac{\bar{\gamma}_{SR_i}\exp(-1/\bar{\gamma}_{SR_i})}{E_1(1/\bar{\gamma}_{SR_i})} \tag{2}$$

where $E_1(x) = \int_x^\infty e^{-t}/t dt$ is the first-order exponential integral function and the overbar $(\bar{\bullet})$ denotes averaging. In the sequel, $\bar{\gamma}_{SR_i}$, $\bar{\gamma}_{R_iD}$ are denoted for each i by $\bar{\gamma}_1$ and $\bar{\gamma}_2$, respectively. Note that with semi-blind mode of operation, the relays do not need to calculate the instantaneous channel gain of the $S - R_i$ channel as in the case of CSI assisted relays. At the beginning of each coherent time interval of the $S - R$ channel, the relays can periodically transmit pilots. Therefore, using the channel reciprocity property, S can estimate CSI for the forward $S - R_i$ channel.

Outage probability and SNR moments: The outage probability is defined as the probability that the instantaneous e2e SNR, γ_{eq}, falls below a threshold γ_T. Let $\gamma_1 = \max_i\{SR_i\}$ and $\gamma_2 = \gamma_{RD_i}$. Therefore, $P_o(\gamma_T) = \Pr[\gamma_{eq} < \gamma_T] = F_{\gamma_eq}(\gamma_T)$ is given by

$$\begin{aligned} P_o(\gamma_T) &= \Pr\left[\frac{\gamma_1\gamma_2}{C + \gamma_2} < \gamma_T\right] \\ &= \int_0^\infty \Pr\left[\gamma_1 < \frac{\gamma_T}{\gamma_2}(C + \gamma_2)|\gamma_2\right] p_{\gamma_2}(\gamma)d\gamma \end{aligned} \tag{3}$$

The corresponding cumulative distribution function (CDF) of γ_1, $F_{\gamma_1}(\gamma)$, is given by

$$F_{\gamma_1}(\gamma) = K\sum_{l=0}^{K-1}\frac{(-1)^l}{1+l}\binom{K-1}{l}(1 - e^{-(1+l)\gamma/\bar{\gamma}_1}) \tag{4}$$

Substituting (4) in (3), and using the identity $\int_0^\infty x^{\nu-1}e^{-\beta/x-\gamma x}dx = 2(\beta/\gamma)^{\nu/2}K_\nu(2\sqrt{\beta\gamma})$ for the calculation of the resulting integral, we obtain the outage probability as

$$\begin{aligned} P_0(\gamma_T) = 1 - 2K\sum_{l=0}^{K-1}\frac{(-1)^l}{1+l}\binom{K-1}{l}\sqrt{\frac{C(1+l)\gamma_T}{\bar{\gamma}_1\bar{\gamma}_2}} \\ \times e^{-(1+l)\gamma_T/\bar{\gamma}_1}K_1\left(2\sqrt{\frac{C(1+l)\gamma_T}{\bar{\gamma}_1\bar{\gamma}_2}}\right) \end{aligned} \tag{5}$$

where $K_\nu(\bullet)$ is the νth-order modified Bessel function of the second kind. Using (5), the probability density function (PDF) of γ_{eq}, can be derived by taking the derivative with respect to γ_T:

$$\begin{aligned} P_{\gamma_{eq}}(\gamma) = \frac{2K}{\bar{\gamma}_1}\sum_{l=0}^{K-1}(-1)^l\binom{K-1}{l}e^{-(1+l)\gamma/\bar{\gamma}_1} \\ \times\left[\sqrt{\frac{C(1+l)\gamma}{\bar{\gamma}_1\bar{\gamma}_2}}K_1\left(2\sqrt{\frac{C(1+l)\gamma}{\bar{\gamma}_1\bar{\gamma}_2}}\right)\right. \\ \left.+\frac{C}{\bar{\gamma}_2}K_0\left(2\sqrt{\frac{C(1+l)\gamma}{\bar{\gamma}_1\bar{\gamma}_2}}\right)\right] \end{aligned} \tag{6}$$

where $dK_1(x)/dx = -K_0(x) - K_1(x)/x$ is applied.

We now characterise the moments of the e2e SNR. The SNR moments are important performance measures of the average output SNR, as well as its variance. The moments of γ_{eq} are given by

$$\mathrm{E}\langle\gamma_{eq}^n\rangle = \int_0^\infty \gamma^n p_{\gamma_{eq}}(\gamma)d\gamma = n\int_0^\infty \gamma^{n-1}(1 - F_{\gamma_{eq}}(\gamma))d\gamma \tag{7}$$

where $\mathrm{E}\langle\bullet\rangle$ denotes the statistical expectation operator. Let $t^2 = \gamma$, and using (5) and (7) the moments of e2e SNR can be expressed as

$$\begin{aligned} \mathrm{E}\langle\gamma_{eq}^n\rangle = 4nK\sqrt{\frac{C}{\bar{\gamma}_1\bar{\gamma}_2}}\sum_{l=0}^{K-1}\frac{(-1)^l}{\sqrt{(1+l)}}\binom{K-1}{l} \\ \times\int_0^\infty t^{2n}e^{-(1+l)t^2/\bar{\gamma}_1}K_1\left(2t\sqrt{\frac{C(1+l)}{\bar{\gamma}_1\bar{\gamma}_2}}\right)dt \end{aligned} \tag{8}$$

The integral in (8) can be expressed in closed-form using [7, eqn. 2.16.8.4]. Therefore, $\mathrm{E}\langle\gamma_{eq}^n\rangle$ is given by

$$\begin{aligned} \mathrm{E}\langle\gamma_{eq}^n\rangle = K(n!)^2(\bar{\gamma}_1)^n\exp\left(\frac{C}{2\bar{\gamma}_2}\right)W_{-n,1/2}\left(\frac{C}{\bar{\gamma}_2}\right) \\ \times\left(\sum_{l=0}^{K-1}\frac{(-1)^l}{(1+l)^{n+1}}\binom{K-1}{l}\right) \end{aligned} \tag{9}$$

where $W_{\alpha,\beta}(\bullet)$ is the Whittaker function [7, p. 748].

Average bit error rate: The straightforward approach to obtain the BER, $P_b(e)$, is to average the conditional BER, $P_{be}(\gamma)$, over the PDF $P_{\gamma_{eq}}(\gamma)$, i.e.

$$P_b(e) = \int_0^\infty P_{be}(\gamma)p_{\gamma_{eq}}(\gamma)d\gamma \tag{10}$$

In the cases of BPSK, M-ary phase-shift keying (MPSK), M-ary quadrature amplitude modulation (M-QAM) $P_{be}(\gamma)$ can be expressed in the form of $aQ(\sqrt{b\gamma})$ where $(a,\ b)$ are constants depending on the modulation scheme and $Q(x) = 1/\sqrt{2\pi}\int_x^\infty e^{-t^2/2}dt$. To evaluate the BER we employ integration by parts. Therefore, the average BER, given by $I = a\int_0^\infty Q(\sqrt{b\gamma})p_{\gamma_{eq}}(\gamma)d\gamma$, can be evaluated as

$$I = \frac{a}{\sqrt{2\pi}}\int_0^\infty F_{\gamma_{eq}}\left(\frac{y^2}{b}\right)\exp\left(-\frac{y^2}{2}\right)dy \tag{11}$$

Plugging (5) into (11), thereafter using [7, eqn. 2.16.8.7] and after algebraic manipulation we obtain

$$I = \frac{a}{2}\left[1 - \frac{KC}{b\bar{\gamma}_1\bar{\gamma}_2}\sum_{l=0}^{K-1}(-1)^l\binom{K-1}{l}\left(\frac{b\bar{\gamma}_1}{2(1+l)+b\bar{\gamma}_1}\right)^{3/2} \times \exp\left(\frac{C(1+l)}{2(1+l)+b\bar{\gamma}_1)\bar{\gamma}_2}\right)\Phi\left(\frac{C(1+l)}{(2(1+l)+b\bar{\gamma}_1)\bar{\gamma}_2}\right)\right] \quad (12)$$

where the auxiliary function, $\Phi(x) = K_1(x) - K_0(x)$.

To illustrate the usefulness of the above analytical results, Figs 1 and 2 show the outage and QPSK-modulated average BER performances against the average SNR of the $S-R$ link of partial relay selection, respectively. In Fig. 2, three available relays are assumed, i.e. $K = 3$. Three cases regarding the relative strength of the $S-R$ and $R-D$ channels are considered, namely $\bar{\gamma}_1 = \bar{\gamma}_2$, $\bar{\gamma}_1 = 5\bar{\gamma}_2$ and $\bar{\gamma}_1 = 0.2\bar{\gamma}_2$. As can be observed from Figs 1 and 2, partial relay selection does not result in favourable diversity gain compared to the case where the relays are selected based on the e2e channel [4, 5], since in partial relay selection the $R-D$ channel is ignored at the selection process. Nevertheless, in some applications where obtaining end-to-end channel information is practically cumbersome due to, e.g., complexity constraints, partial relay selection is the only applicable relay selection method.

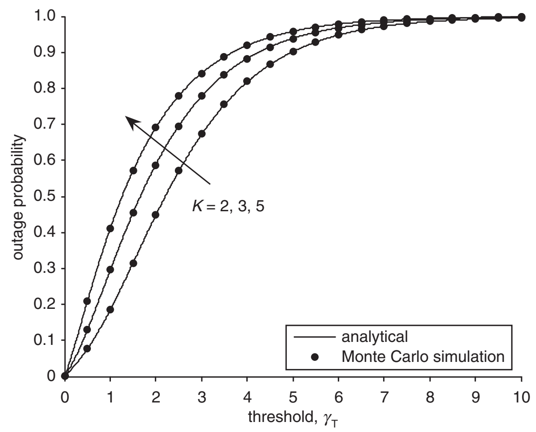

Fig. 1 *Outage probability of partial relay selection for different number of available relays*

$\bar{\gamma}_1 = 3$ dB, $\bar{\gamma}_2 = 7$ dB

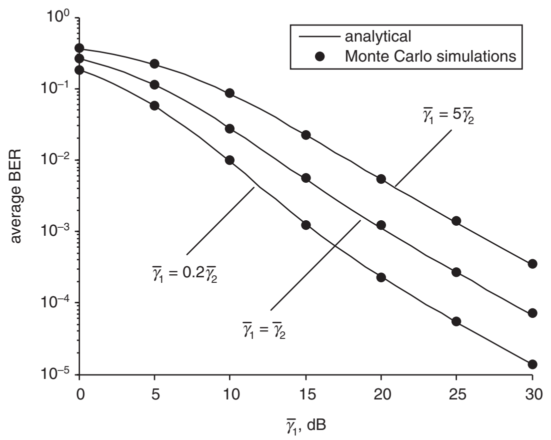

Fig. 2 *Average BER of partial relay selection for, and several assumptions regarding, the relative strength of S-R and R-D channel, for QPSK modulation*

Conclusions: The usefulness of relay selection based on single hop information for some practical ad hoc networks has been highlighted in recent research. In the work reported in this Letter, we developed exact expressions for the outage probability, moments of the end-to-end signal-to-noise ratio and the average bit error rate in order to evaluate the performance of dual hop semi-blind amplify-and-forward when partial relay selection is applied. These results are confirmed through comparison with Monte Carlo simulations. To the best of our knowledge, such exact expressions considering a semi-blind relay gain have not been reported elsewhere in the technical literature.

29 October 2008
doi: 10.1049/el.2009.3089

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