Two Hop Amplify-and-Forward Transmission in Mixed Rayleigh and Rician Fading Channels

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***Abstract*—The performance of a two hop amplify-and-forward relay system, where the source-relay and the relay-destination channels experience Rayleigh and Rician fading respectively, is investigated. We derive exact and lower bound expressions for the outage probability and average bit error probability, where the bounds become tight at high signal-to-noise ratios (SNR). Our results are verified through comparison with Monte Carlo simulations, where we also illustrate the positive impact of the Rician factor on the system performance.**

***Index Terms*—Relay, amplify-and-forward, Rician fading.**

I. Introduction

RELAY technologies have become a major topic in the research community because they enable high speed information exchange over challenging (possibly shadowed) wireless environments. In many practical scenarios, different links in relay networks can experience asymmetric fading conditions. For example, a base station-relay link may have a Rician link because of a strong line-of-sight (LoS) component, while the relay-mobile link may only experience Rayleigh fading [1]. A recently released WINNER II project deliverable [2] also documents mixed Rayleigh/Rician propagation characteristics in micro/macro cellular multi-hop transmissions.

To date, several works have studied the performance of decode-and-forward (DF) and amplify-and-forward (AF) relay systems using performance measures such as the outage probability [3] and the average bit error probability (ABEP) [4]–[8]. In [4], the ABEP of two hop systems with AF relaying over Rayleigh fading channels has been studied. In [6] Karagiannidis presented performance bounds for AF multihop systems over Rician, Nakagami-m and Nakagami-q (Hoyt) generalized fading channels. In [7] lower bounds for the performance of a two hop channel state information (CSI) assisted AF relay system over non identically distributed generalized gamma fading channels have been presented. The generalized gamma distribution includes Rayleigh, Nakagami-m, and Weibull distributions as special cases.

In addition to the widely used Rayleigh and Nakagami-m fading assumptions, Rician fading is often used in the technical literature to model wireless propagation comprising a LoS component and a scattered component. Despite the importance of the Rician model, only a few works have analyzed the performance of relays under LoS fading conditions [1], [3], [8]. Ribeiro *et al.* [8] derived asymptotic expressions for the symbol error probability for a variety of cooperative diversity network schemes such as single relay, multibranch, multihop multibranch. In [9], the performance of a single fixed gain AF relay in Rayleigh/Rician fading has been investigated.

In this paper, we examine the outage probability and ABEP of a two hop AF relay system in an asymmetric fading environment. In particular, we derive exact expressions in terms of an infinite series, which we show numerically converges quickly for a finite number of terms. We also derive lower bounds which become tight for high signal-to-noise ratios (SNRs). Simulation results are presented to verify the theoretical analysis.

II. System and Channel Model

We consider a two hop system where the communication from the source S to the destination D via a relay R takes place in two time slots. In the first time slot, S sends its signal to R. In the second time slot, R amplifies the received signal by a gain factor $\mathcal{G}$ and forwards the resultant signal to D. The source is located in a deeply shadowed environment, so it is assumed that there is no direct path between S and D. The instantaneous end-to-end SNR, γ_{eq}, at the destination is

$$\gamma_{\text{eq}} = \frac{(P_1/N_0)|h_{SR}|^2 (P_2/N_0)|h_{RD}|^2}{(P_2/N_0)|h_{RD}|^2 + (1/\mathcal{G}^2 N_0)} \qquad (1)$$

where P_1 and P_2 are the transmit power at S and R respectively, $|h_{SR}|$ and $|h_{RD}|$ are the fading amplitudes of the wireless channels in the $S-R$ and $R-D$ links respectively and N_0 is the power of the additive white Gaussian noise (AWGN) component. If $\mathcal{G}$ is selected according to the instantaneous CSI assisted relay gain [4, Eq. (4)], then γ_{eq} can be re-expressed as

$$\gamma_{\text{eq}} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + c} \qquad (2)$$

where $\gamma_1 = |h_{SR}|^2 P_1/N_0$ and $\gamma_2 = |h_{RD}|^2 P_2/N_0$ are the per hop SNRs. In addition, exact γ_{eq} is given by substituting $c = 1$, and well approximated at high SNR by substituting $c = 0$.

We consider an asymmetric scenario for the fading distributions of the $S-R$ and $R-D$ links, namely: *The $S-R$ link is subject to Rayleigh fading and the $R-D$ link is subject to Rician fading*. The proposed model can represent either an up or down link in a mobile communication network. In the first case a mobile station acts as S, another mobile station as R and a base station as D. Note that due to the symmetry of γ_{eq}

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w.r.t to γ_1 and γ_2, the statistics of γ_{eq} will be identical in the down link where the $S-R$ link is subject to Rician fading and the $R-D$ link is subject to Rayleigh fading.

If a link experiences Rayleigh fading, γ_1 is exponentially distributed with probability density function (PDF) given by

$$p_{\gamma_1}(\gamma) = \frac{1}{\bar{\gamma}_1} e^{-\gamma/\bar{\gamma}_1} \tag{3}$$

where $\bar{\gamma}_1 = \mathcal{E}\{|h_{SR}|^2\}P_1/N_0$ is the average per hop SNR of the $S-R$ channel and $\mathcal{E}\{\cdot\}$ is the statistical expectation.

If a link experiences Rician fading, γ_2 is distributed according to a noncentral-χ^2 distribution given by

$$p_{\gamma_2}(\gamma) = \frac{(K+1)e^{-K}}{\bar{\gamma}_2} e^{-\frac{(K+1)\gamma}{\bar{\gamma}_2}} I_0\left(2\sqrt{\frac{K(K+1)\gamma}{\bar{\gamma}_2}}\right) \tag{4}$$

where K is the Rician K-factor defined as the ratio of the powers of the LoS component to the scattered components, $\bar{\gamma}_2 = \mathcal{E}\{|h_{RD}|^2\}P_2/N_0$ is the average per hop SNR of the $R-D$ and $I_0(\cdot)$ is the zeroth order modified Bessel function of the first kind [10, p. 916].

III. Performance Analysis

A. Outage Probability

Outage probability is an important performance measure that is commonly used to characterize a wireless communication system. It is defined as the probability that the instantaneous end-to-end SNR falls below a threshold γ_{th}. Therefore mathematically, the outage probability is

$$P_{\text{out}} = F_{\gamma_{\text{eq}}}(\gamma_{\text{th}}) = \Pr\left[\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+c} < \gamma_{\text{th}}\right] \tag{5}$$

where $F_{\gamma_{\text{eq}}}(\gamma)$ is the CDF of the end-to-end SNR. After applying some algebraic manipulations $F_{\gamma_{\text{eq}}}(\gamma_{\text{th}})$ can be re-expressed as

$$F_{\gamma_{\text{eq}}}(\gamma_{\text{th}}) = 1 - \int_0^\infty C_{\gamma_1}\left(\gamma_{\text{th}} + \frac{\gamma_{\text{th}}^2 + c\gamma_{\text{th}}}{w}\right) p_{\gamma_2}(\gamma_{\text{th}}+w)dw \tag{6}$$

where $C_{\gamma_i}(\cdot) = 1 - F_{\gamma_i}(\cdot)$ is the complementary CDF of γ_i. Therefore, we can express $F_{\gamma_{\text{eq}}}(\gamma_{\text{th}})$ as

$$F_{\gamma_{\text{eq}}}(z) = 1 - \frac{K+1}{\bar{\gamma}_2} e^{-K-(1/\bar{\gamma}_1+(K+1)/\bar{\gamma}_2)z} \times \int_0^\infty e^{-\frac{z^2+cz}{\bar{\gamma}_1 w} - \frac{(K+1)w}{\bar{\gamma}_2}} I_0\left(2\sqrt{\frac{K(K+1)(z+w)}{\bar{\gamma}_2}}\right) dw. \tag{7}$$

We are unaware of a closed-form analytical solution to this integral. Nevertheless, using the infinite-series representation of $I_0(x)$ [10, Eq. (8.447.1)] and the integral result of [10, Eq. (3.471.9)] (7) can be solved. Therefore, P_{out} is given by (8). In (8) $K_\ell(\cdot)$ is the ℓ-order modified Bessel function of the second kind [10, Eq. (8.446)] and $\lambda = (\bar{\gamma}_2 + (K+1)\bar{\gamma}_1)/\bar{\gamma}_1\bar{\gamma}_2$. As we see later, (8) converges quickly for a finite number of summation terms, and for arbitrary SNRs.

Several recent papers have approximated γ_{eq} using an upper bound γ_b given by [7]

$$\gamma_b = \min(\gamma_1, \gamma_2). \tag{9}$$

Therefore, a closed-form lower bound to (8) is given by

$$\begin{aligned} \mathcal{P}_{\text{out}} &= \Pr\left[\min(\gamma_1,\gamma_2) < \gamma_{\text{th}}\right] \\ &= 1 - C_{\gamma_1}(\gamma_{\text{th}})C_{\gamma_2}(\gamma_{\text{th}}) \\ &= 1 - e^{-\gamma_{\text{th}}/\bar{\gamma}_1} Q\left(\sqrt{2K}, \sqrt{\frac{2(1+K)\gamma_{\text{th}}}{\bar{\gamma}_2}}\right) \end{aligned} \tag{10}$$

where $Q(\cdot,\cdot)$ is the first-order Marcum Q-function [3].

B. Average Bit Error Probability

The ABEP is a useful measure of evaluating the performance of wireless communication applications. Traditionally, the ABEP is computed by determining the PDF of γ_{eq} and then averaging the conditional BEP in AWGN, $P_b(e|\gamma)$, over this PDF. Therefore,

$$P_b(e) = \int_0^\infty P_b(e|\gamma)p_{\gamma_{\text{eq}}}(\gamma)d\gamma. \tag{11}$$

Assume $P_b(e|\gamma) = \mathcal{Q}\left(\sqrt{\beta\gamma}\right)$, where β is a constant and $\mathcal{Q}(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-t^2/2}dt$ is the Gaussian Q-function. It is trivial to extend the following error performance derivations to well-known modulation schemes employed in communication systems such as BPSK ($\beta = 2$), QPSK ($\beta = 1$) and square/rectangular M-QAM.

In order to compute the ABEP of (11), we identify that the integral needs to be computed is of the form: $\mathcal{J}_1 = \int_0^\infty \mathcal{Q}(\sqrt{\beta\gamma})p_{\gamma_{\text{eq}}}(\gamma)d\gamma$. After integration by parts, $\mathcal{J}_1$ can be rewritten as

$$\mathcal{J}_1 = \frac{1}{\sqrt{2\pi}}\int_0^\infty F_{\gamma_{\text{eq}}}\left(\frac{t^2}{\beta}\right) e^{-\frac{t^2}{2}}dt. \tag{12}$$

Now employing (8) with $c = 0$ which is analytically more tractable, $\mathcal{J}_1$ can be expressed as (13). Using [10, Eq. (6.621.3)] (13) can be solved as shown in (14) where $\varphi = m+n$, $\psi = 2e^{-K}\bar{\gamma}_1\bar{\gamma}_2\sqrt{2\beta\bar{\gamma}_1\bar{\gamma}_2}$, $q = (2\lambda+\beta)\bar{\gamma}_1\bar{\gamma}_2/2 + 2\sqrt{(K+1)\bar{\gamma}_1\bar{\gamma}_2}$, $p = q - 4\sqrt{(K+1)\bar{\gamma}_1\bar{\gamma}_2}$, ${}_2F_1(a,b;c;x)$ is the Gauss hypergeometric function [10, Sec. 9.1] and $\Gamma(x) = \int_0^\infty t^{z-1}e^{-t}dt$ is the gamma function.

If (10) is employed in (12), the ABEP can be expressed as

$$\mathcal{J}_2 = \frac{1}{2} - \frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-\frac{2+\beta\bar{\gamma}_1}{2\beta\bar{\gamma}_1}t^2} Q\left(\sqrt{2K}, \sqrt{\frac{2(1+K)}{\beta\bar{\gamma}_2}}t\right)dt. \tag{15}$$

Using [11, Eq. (62)] $\mathcal{J}_2$ can be expressed in closed-form and is given by (16), where $\nu = 1/\sqrt{1+2/(\beta\bar{\gamma}_1)}$ and $s = \frac{(1+K)\bar{\gamma}_1}{(1+\beta\bar{\gamma}_1/2)\bar{\gamma}_2}$.

IV. Numerical Results

In this section, we present some numerical results to verify our analysis. Fig. 1 shows the outage probability for two different K factors. The analytical curves are from (8) using only 20 summations ($n = 0, \ldots, 20$), and we clearly see an exact match with the Monte Carlo simulated curves. Furthermore, we see that the lower bound analytical curves from (10) converge at high SNR to the exact outage probability curves, as expected. Fig. 2 shows the ABEP for various K factors using QPSK modulation. We have also plotted the

$$P_{\text{out}} = 1 - e^{-K-\lambda\gamma_{\text{th}}} \sum_{n=0}^{\infty} \frac{2K^n}{(n!)^2} \sum_{m=0}^{n} \binom{n}{m} \gamma_{\text{th}}^{n-m} \left(\frac{K+1}{\bar{\gamma}_2}\right)^{\frac{2n-m+1}{2}} \left(\frac{\gamma_{\text{th}}^2 + c\gamma_{\text{th}}}{\bar{\gamma}_1}\right)^{\frac{m+1}{2}} K_{m+1}\left(2\sqrt{\frac{(K+1)(\gamma_{\text{th}}^2 + c\gamma_{\text{th}})}{\bar{\gamma}_1\bar{\gamma}_2}}\right) \quad (8)$$

$$\mathcal{J}_1 = \frac{1}{2} - \frac{e^{-K}}{\sqrt{2\pi}\beta} \sum_{n=0}^{\infty} \frac{(K/\beta)^n}{(n!)^2} \sum_{m=0}^{n} \binom{n}{m} \left(\frac{1}{\bar{\gamma}_1}\right)^{\frac{m+1}{2}} \left(\frac{K+1}{\bar{\gamma}_2}\right)^{\frac{2n-m+1}{2}} \int_0^{\infty} t^{n+\frac{1}{2}} e^{-\left(\frac{2\lambda+\beta}{2\beta}\right)t} K_{m+1}\left(\frac{2t}{\beta}\sqrt{\frac{K+1}{\bar{\gamma}_1\bar{\gamma}_2}}\right) dt. \quad (13)$$

$$\mathcal{J}_1 = \frac{1}{2} - \psi \sum_{n=0}^{\infty} \frac{(K\bar{\gamma}_1)^n (K+1)^{n+1}}{(n!)^2} \sum_{m=0}^{n} \binom{n}{m} \frac{(4\bar{\gamma}_2)^m \Gamma(\varphi+\frac{5}{2})\Gamma(n-m+\frac{1}{2})}{\Gamma(n+2)\, q^{\varphi+2.5}} \, {}_2F_1\left(\varphi+\frac{5}{2}, m+\frac{3}{2}; n+2; \frac{p}{q}\right) \quad (14)$$

$$\mathcal{J}_2 = \frac{1}{2} - \frac{\nu}{2}\left[1 - 2Q\left(\sqrt{\frac{K}{2}}\left(1-\frac{1}{\sqrt{1+s}}\right), \sqrt{\frac{K}{2}}\left(1+\frac{1}{\sqrt{1+s}}\right)\right) + \left(1+\frac{1}{\sqrt{1+s}}\right) e^{-K\frac{2+s}{2+2s}} I_0\left(\frac{Ks}{2+2s}\right)\right] \quad (16)$$

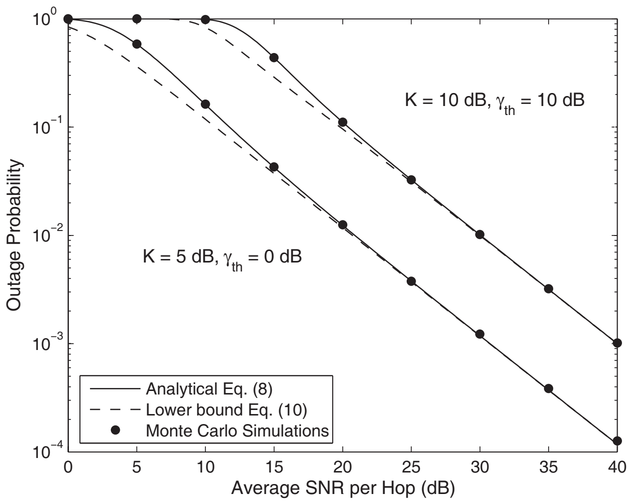

Fig. 1. Outage probability for various K factors with $\bar{\gamma}_2 = \bar{\gamma}_1$.

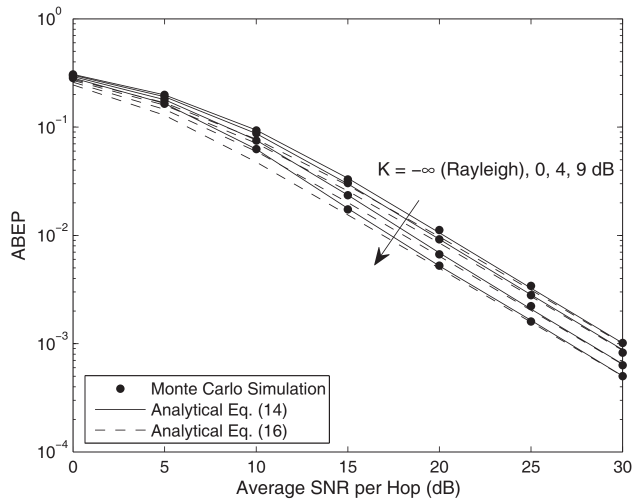

Fig. 2. ABEP using QPSK modulation with $\bar{\gamma}_2 = \bar{\gamma}_1$.

curve for Rayleigh/Rayleigh fading for comparison purposes. The analytical curves are from (14) using 20 summations, and we clearly see an exact match with Monte Carlo simulated curves. Again ABEPs calculated from (16) only match the simulations in the medium to high SNR regime, i.e. SNR > 20 dB. The diversity order of the system is one, but, a large K-factor provides SNR gains. As seen from Fig. 2, the offered gain becomes marginal for $K > 7$ dB.

V. Conclusions

We have derived infinite-series representations and tight lower bounds for the outage probability and ABEP of a two hop communication system equipped with a single CSI assisted AF relay in Rayleigh/Rician fading environments. This analysis is useful to the system design engineer for performance evaluation purposes in LoS/non LoS conditions.

References

[1] Y. Kim and H. Liu, "Infrastructure relay transmission with cooperative MIMO," *IEEE Trans. Veh. Technol.*, vol. 57, pp. 2180-2188, July 2008.

[2] P. Kyösti, J. Meinilä, L. Hentilä *et al.*, WINNER II Interim Channel Models (IST-4-027756 WINNER II D1.1.1 V1.1) [Online]. Available: http://www.ist-winner.org/WINNER2-Deliverables/D1.1.1.pdf.

[3] Y. Zhu, Y. Xin, and P.-Y. Kam, "Outage probability of Rician fading relay channels," *IEEE Trans. Veh. Technol.*, vol. 57, pp. 2648-2652, July 2008.

[4] M. O. Hasna and M.-S. Alouini, "End-to-end performance of transmission systems with relays over Rayleigh-fading channels," *IEEE Trans. Wireless Commun.*, vol. 2, pp. 1126-1131, Nov. 2003.

[5] J. Adeane, M. R. D. Rodriques, and I. J. Wassell, "Characterization of the performance of cooperative networks in Ricean fading channels," in *Proc. ICT 2005*, Cape Town, South Africa, May 2005.

[6] G. K. Karagiannidis, "Performance bounds of multihop wireless communications with blind relays over generalized fading channels," *IEEE Trans. Wireless Commun.*, vol. 5, pp. 498-503, Mar. 2006.

[7] S. Ikki and M. H. Ahmed, "Performance analysis of dual-hop relaying communications over generalized Gamma fading channels," in *Proc. IEEE Global Communications Conf. (Globecom 2007)*, pp. 3888-3893.

[8] A. Ribeiro, X. Cai, and G. B. Giannakis, "Symbol error probabilities for general cooperative links," *IEEE Trans. Wireless Commun.*, vol. 4, pp. 1264-1273, May 2005.

[9] H. A. Suraweera, G. K. Karagiannidis, and P. J. Smith, "Performance analysis of the dual-hop asymmetric fading channel," *IEEE Trans. Wireless Commun.*, accepted for publication.

[10] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*. San Diego, CA: Academic Press, 2000.

[11] A. H. Nuttall, "Some integrals involving the Q-function," Naval Underwater Systems Center (NUSC), Technical Report 4297, Apr. 1972.