

An Efficient Approximation to the Correlated Nakagami- m Sums and its Application in Equal Gain Diversity Receivers

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Abstract—There are several cases in wireless communications theory where the statistics of the sum of independent or correlated Nakagami- m random variables (RVs) is necessary to be known. However, a closed-form solution to the distribution of this sum does not exist when the number of constituent RVs exceeds two, even for the special case of Rayleigh fading. In this paper, we present an efficient closed-form approximation for the distribution of the sum of arbitrary correlated Nakagami- m envelopes with identical and integer fading parameters. The distribution becomes exact for maximal correlation, while the tightness of the proposed approximation is validated statistically by using the Chi-square and the Kolmogorov-Smirnov goodness-of-fit tests. As an application, the approximation is used to study the performance of equal-gain combining (EGC) systems operating over arbitrary correlated Nakagami- m fading channels, by utilizing the available analytical results for the error-rate performance of an equivalent maximal-ratio combining (MRC) system.

Index Terms—Nakagami- m fading, arbitrary correlation, approximative statistics, equal gain combining (EGC), maximal ratio combining (MRC).

I. INTRODUCTION

THE analytical determination of the the probability distribution functions (PDF) and the cumulative distribution functions (CDF) of the sums of independent and correlated signals' envelopes is rather cumbersome, yielding difficulties in the theoretical performance analysis of some wireless communications systems [1]. A closed-form solution for the PDF and the CDF of the sum of Rayleigh random variables (RVs) has not been presented for more than 90 years, except when the number of RVs equals two. The famous Beaulieu series for computing PDF of a sum of independent RVs were proposed in [2]. Later, a finite range multifold integral for PDF of the sum of independent and identically distributed (i.i.d.) Nakagami- m RVs was proposed in [3]. A closed-form formula for the PDF of the sum of two i.i.d. Nakagami- m RVs was given in [4]-[6]. Exact infinite series representations for the sum of three and four i.i.d. Nakagami- m RVs was

presented in [7], although their usefulness is overshadowed by their computational complexity.

The most famous application, where these sums appear, deals with the analytical performance evaluation of equal gain combining (EGC) systems [8]-[13]. Only few papers address the performance of EGC receivers in correlated fading with arbitrary-order diversity. In [14], EGC was studied by approximating the moment generating function (MGF) of the output SNR, where the moments are determined exactly only for exponentially correlated Nakagami- m channels in terms of multi-fold infinite series. A completely novel approach for performance analysis of diversity combiners in equally correlated fading channels was proposed in [15], where the equally correlated Rayleigh fading channels are transformed into a set of conditionally independent Rician RVs. Based on this technique, the authors in [16] derived the moments of the EGC output signal-to-noise ratio (SNR) in equally correlated Nakagami- m channels in terms of the Appell hypergeometric function, and then used them to evaluate the EGC performance metrics, such as the outage probability and the error probability (using Gaussian quadrature with weights and abscissas computed by solving sets of nonlinear equations).

All of the above works yield to results that are not expressed in closed form due to the inherent intricacy of the exact sum statistics. This intricacy can be circumvented by searching for suitable highly accurate approximations for the PDF of a sum of arbitrary number of Nakagami- m RVs. Various simple and accurate approximations to the PDF of sum of independent Rayleigh, Rice and Nakagami- m RVs had been proposed in [17]-[21], which had been used for analytical EGC performance evaluation. Based on the ideas given in [1], the works [18]-[21] use various alternatives of the moment matching method to arrive at the required approximation.

In this paper, we present a highly accurate closed-form approximation for the PDF of the sum of non-identical arbitrarily correlated Nakagami- m RVs with identical (integer) fading parameters. By applying this approximation, we evaluate the performance of EGC systems in terms of the known performance of an equivalent maximal ratio combining (MRC) system [22], [24], thus avoiding many complex numerical evaluations inherent for the methods presented in the aforementioned previous works for the EGC performance analysis. Although approximate, the offered closed-form expressions allow to gain insight into system performance by considering, for example, operation in the low or high SNR region.

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II. AN ACCURATE APPROXIMATION TO THE SUM OF ARBITRARY CORRELATED NAKAGAMI- m ENVELOPES

Let Z be a sum of L non-identical correlated Nakagami- m envelopes, $\{Z_k\}_{k=1}^L$, defined as

$$Z = \sum_{k=1}^L Z_k. \quad (1)$$

The envelopes $\{Z_k\}_{k=1}^L$ are distributed according to the Nakagami- m distribution, whose PDF is given by [1]

$$f_{Z_k}(z) = \left(\frac{m_z}{\Omega_k}\right)^{m_z} \frac{2z^{2m_z-1}}{\Gamma(m_z)} \exp\left(-\frac{m_z}{\Omega_k}z^2\right), \quad z \geq 0, \quad (2)$$

with arbitrary average powers $E[Z_k^2] = \Omega_k$, $1 \leq k \leq L$, and the same (integer) fading parameter m_z . The power correlation coefficient between any given pair of envelopes (Z_i, Z_j) is defined as

$$\rho_{ij} = \frac{\text{cov}(Z_i^2, Z_j^2)}{\sqrt{\text{var}(Z_i^2) \text{var}(Z_j^2)}}, \quad i \neq j, \quad (3)$$

where $E[\cdot]$, $\text{cov}(\cdot, \cdot)$ and $\text{var}(\cdot)$ denote expectation, covariance and variance, respectively.

We propose the unknown PDF of Z be approximated by the PDF of R defined as

$$R = \sqrt{\sum_{k=1}^L R_k^2}, \quad (4)$$

where R_k , $1 \leq k \leq L$, denote a set of L correlated but identically distributed Nakagami- m envelopes with same average powers, $E[R_k^2] = \Omega_R$, and same fading parameters, m_R . The power correlation coefficients between any given pair (R_i, R_j) is assumed equal to that of the respective pair of the original envelopes (Z_i, Z_j) , ρ_{ij} .

The statistics of R^2 is easily seen to be equal to the statistics of the sum of correlated Gamma RVs. Thus, the MGF of R^2 is represented by [26, Eq. (11)]

$$M_{R^2}(s) = \det\left(\mathbf{I} - s \frac{\Omega_R}{m_R} \mathbf{\Lambda}\right)^{-m_R} = \prod_{k=1}^L \left(1 - s \frac{\Omega_R}{m_R} \lambda_k\right)^{-m_R} \quad (5)$$

where \mathbf{I} is the $L \times L$ identity matrix and $\mathbf{\Lambda}$ is the $L \times L$ positive definite matrix (denoted as the correlation matrix) whose elements are the square roots of the power correlation coefficients,

$$\mathbf{\Lambda} = \begin{bmatrix} 1 & \sqrt{\rho_{12}} & \cdots & \sqrt{\rho_{1L}} \\ \sqrt{\rho_{21}} & 1 & \cdots & \sqrt{\rho_{2L}} \\ \vdots & \vdots & \ddots & \vdots \\ \sqrt{\rho_{L1}} & \sqrt{\rho_{L2}} & \cdots & 1 \end{bmatrix}. \quad (6)$$

The L eigenvalues of the correlation matrix $\mathbf{\Lambda}$ are denoted by λ_k , $1 \leq k \leq L$.

Throughout literature, the PDF of R^2 is determined by using several different approaches that result in alternative closed-form solutions, two of which are given by [23, Eq. (29)] and [24, Eq. (10)]. After a simple RV transformation, these two

alternatives for the PDF of R are expressed as

$$f_R(r) = \frac{2r}{\pi} \int_0^\infty \frac{\cos\left[m_R \sum_{k=0}^{L-1} \arctan\left(t \frac{\Omega_R \lambda_k}{m_R}\right) - tr^2\right]}{\prod_{k=0}^{L-1} \left[1 + \left(t \frac{\Omega_R \lambda_k}{m_R}\right)^2\right]^{m_R/2}} dt \quad (7)$$

$$= \frac{2r^{2Lm_R-1}}{\Gamma(Lm_R)} \left(\frac{m_R}{\Omega_R}\right)^{Lm_R} \left(\frac{1}{\det(\mathbf{\Lambda})}\right)^{m_R} \times \Phi_2^{(L)}\left(m_R, m_R, \dots, m_R; Lm_R; -\frac{m_R}{\Omega_R} \frac{r^2}{\lambda_1}, \dots, -\frac{m_R}{\Omega_R} \frac{r^2}{\lambda_L}\right), \quad (8)$$

where $\Phi_2^{(L)}(\cdot)$ is the confluent Lauricella hypergeometric function of L variables, defined in [33] and [24, Eqs. (9)-(10)]. Note that (8) is here presented to demonstrate existence of an exact closed-form solution, whereas (7) is much more convenient for accurate and efficient numerical integration. For example, the PDF may be obtained using the Gauss-Legendre quadrature rule [34, Eq. (25.4.29)] over (7) [23].

Next, we apply the moment matching method to determine the parameters Ω_R and m_R of the proposed approximation (7)-(8) to the PDF of Z . In wireless communications, moment matching methods are most typically applied to approximate distributions of the sum of log-normal RVs [29]. Most recently, a variant of moment matching, matching of the normalized first and second moments, had been applied to arrive at an improved approximation to the sum of independent Nakagami- m RVs via the α - μ distribution [20]-[21].

We arrive at required approximation by matching the first and the second moments of the powers of Z and R , i.e., the second and fourth moments of the envelopes Z and R ,

$$E[Z^2] = E[R^2], \quad E[Z^4] = E[R^4]. \quad (9)$$

Matching the first and the second moments of the powers aids the analytical tractability of the proposed approximation due to the availability of the MGF of R^2 in closed form, given by (5). The second and the fourth moments of R are determined straightforwardly by applying the moment theorem over (5), yielding

$$E[R^2] = \left. \frac{dM_{R^2}(s)}{ds} \right|_{s=0} = \Omega_R \sum_{l=1}^L \lambda_l = \Omega_R L, \quad (10)$$

$$E[R^4] = \left. \frac{d^2 M_{R^2}(s)}{ds^2} \right|_{s=0} = \frac{\Omega_R^2}{m_R} \left[\sum_{l=1}^L \lambda_l^2 + m_R L^2 \right]. \quad (11)$$

Introducing (10) and (11) into (9), one obtains the unknown parameters for the statistics of R

$$\Omega_R = \frac{E[Z^2]}{L}, \quad m_R = \frac{\sum_{l=1}^L \lambda_l^2}{L^2} \frac{(E[Z^2])^2}{E[Z^4] - (E[Z^2])^2}. \quad (12)$$

Using the multinomial theorem and [1, Eq. (137)], the second and the fourth moments of Z are determined as

$$E[Z^2] = \sum_{k=1}^L \Omega_k + \frac{2\Gamma^2(m_z + 1/2)}{m_z \Gamma^2(m_z)} \times \sum_{i=1}^L \sum_{j=i+1}^L \sqrt{\Omega_i \Omega_j} {}_2F_1(-1/2, -1/2; m_z; \rho_{ij}), \quad (13)$$

$$\begin{aligned}
E[Z^4] &= \frac{m_z + 1}{m_z} \sum_{m=1}^L \Omega_m^2 + \frac{6\Gamma^2(m_z + 1)}{m_z^2 \Gamma^2(m_z)} \sum_{i=1}^L \sum_{j=i+1}^L \Omega_i \Omega_j \\
&\quad \times {}_2F_1(-1, -1; m_z; \rho_{ij}) + \frac{4\Gamma(m_z + 3/2)\Gamma(m_z + 1/2)}{m_z^2 \Gamma^2(m_z)} \\
&\quad \times \sum_{i=1}^L \sum_{j=i+1}^L (\Omega_i^{3/2} \Omega_j^{1/2} + \Omega_i^{1/2} \Omega_j^{3/2}) {}_2F_1\left(-\frac{3}{2}, -\frac{1}{2}; m_z; \rho_{ij}\right) \\
&\quad + 12 \sum_{m=1}^L \sum_{i=m+1}^L \sum_{j=i+1}^L \Omega_m \sqrt{\Omega_i \Omega_j} E[Z_m^2 Z_i Z_j] \\
&\quad + 12 \sum_{m=1}^L \sum_{i=m+1}^L \sum_{j=i+1}^L \sqrt{\Omega_m} \Omega_i \sqrt{\Omega_j} E[Z_m Z_i^2 Z_j] \\
&\quad + 12 \sum_{m=1}^L \sum_{i=m+1}^L \sum_{j=i+1}^L \sqrt{\Omega_m \Omega_i} \Omega_j E[Z_m Z_i Z_j^2] \\
&\quad + 24 \sum_{m=1}^L \sum_{n=m+1}^L \sum_{i=n+1}^L \sum_{j=i+1}^L \sqrt{\Omega_m \Omega_n \Omega_i \Omega_j} E[Z_m Z_n Z_i Z_j], \tag{14}
\end{aligned}$$

where ${}_2F_1(\cdot)$ is the Gauss hypergeometric function [31]. The joint moments $E[Z_m^2 Z_i Z_j]$, $E[Z_m Z_i^2 Z_j]$, $E[Z_m Z_i Z_j^2]$ and $E[Z_m Z_n Z_i Z_j]$ are not known in closed-form for arbitrary branch correlation. Exact closed-form expressions are available only for some particular correlation models, such as the exponential and the equal correlation models. For the case of arbitrary correlation, we utilize the method presented in [27], where an arbitrary correlation matrix $\mathbf{\Lambda}$ is approximated by its respective Green's matrix, followed by the application of the available joint moments of the exponential correlation model.

A. Equal correlation model

Equal correlation typically corresponds to the scenario of multichannel reception from closely spaced diversity antennas (e.g., three antennas placed on an equilateral triangle). This model may be employed as a worst case correlation scenario, since the impact of correlation on system performance for other correlation models typically will be less severe [22], [30].

For this correlation model, the power correlation coefficients are all equal,

$$\rho_{ij} = \rho, \quad i \neq j, \quad 0 \leq \rho \leq 1. \tag{15}$$

When m_z is assumed to be integer, the unknown joint moments in (14) can be expressed in closed-form as [16, Eq. (43)]

$$\begin{aligned}
E[Z_m^2 Z_i Z_j] &= E[Z_m Z_i^2 Z_j] = E[Z_m Z_i Z_j^2] \\
&= \left(\frac{1 - \sqrt{\rho}}{m_z}\right)^2 W(2, 1, 1), \tag{16}
\end{aligned}$$

$$E[Z_m Z_n Z_i Z_j] = \left(\frac{1 - \sqrt{\rho}}{m_z}\right)^2 W(1, 1, 1, 1), \tag{17}$$

where the coefficients $W(\dots)$ are determined as

$$\begin{aligned}
W(k_1, \dots, k_N) &= \left(\prod_{j=1}^N \frac{\Gamma(m_z + k_j/2)}{\Gamma(m_z)}\right) \left(\frac{1 - \sqrt{\rho}}{1 + (N-1)\sqrt{\rho}}\right)^{m_z} \\
&\quad \times F_A^{(N)}\left(m_z; m_z + \frac{k_1}{2}, \dots, m_z + \frac{k_N}{2}; m_z, \dots, m_z; \right. \\
&\quad \left. \frac{\sqrt{\rho}}{1 + (N-1)\sqrt{\rho}}, \dots, \frac{\sqrt{\rho}}{1 + (N-1)\sqrt{\rho}}\right), \tag{18}
\end{aligned}$$

with $F_A^{(N)}(\cdot)$ denoting the Lauricella F_A hypergeometric function of N variables, defined by [31, Eq. (9.19)] and [25, Eqs. (11)-(13)].

Note that the coefficient $W(2, 1, 1)$ needs to be evaluated when $L \geq 3$, whereas the coefficient $W(1, 1, 1, 1)$ needs to be evaluated when $L \geq 4$. In Appendix A, $W(2, 1, 1)$ is reduced to the more familiar hypergeometric functions, attaining the form given by (A.2). $W(1, 1, 1, 1)$ requires numerical evaluation of the Lauricella F_A function of 4 variables, which can be computed with desired accuracy by using one of the two numerical methods presented in [25, Section IV.A].

The assumption of equal average powers, $\Omega_k = \Omega_Z$, $1 \leq k \leq L$, yields independence of m_R from Ω_Z . For this case, Table I gives the values of m_R for several combinations of ρ , L and m_z . The use of Table I aids the practical applicability of our approach for the case of equal average powers.

For the equal correlation model, the eigenvalues of $\mathbf{\Lambda}$ are exactly found as $\lambda_1 = 1 + (L-1)\sqrt{\rho}$ and $\lambda_k = (1 - \sqrt{\rho})$ for $2 \leq k \leq L$, so the statistics of R^2 is identical to that of the sum of a pair of independent Nakagami RVs. Thus, the MGF of R^2 is given by [30, Eq. (9.213)], whereas the PDF of R is given by [30, Eq. (9.208)]

$$\begin{aligned}
f_R(r) &= \left(\frac{m_R}{\Omega_R}\right)^{m_R L} \\
&\quad \times \frac{2r^{2m_R L - 1} \exp(-m_R r^2 / ((1 - \sqrt{\rho})\Omega_R))}{\Gamma(m_R L) (1 - \sqrt{\rho})^{m_R(L-1)} (1 + (L-1)\sqrt{\rho})^{m_R}} \\
&\quad \times {}_1F_1\left(m_R; m_R L; \frac{m_R L \sqrt{\rho}}{(1 - \sqrt{\rho})(1 + (L-1)\sqrt{\rho})\Omega_R} r^2\right), \tag{19}
\end{aligned}$$

where ${}_1F_1(\cdot)$ is the Kummer confluent hypergeometric function [31, Eq. (9.210)].

B. Exponential correlation model

Exponential correlation typically corresponds to the scenario of multichannel reception from equispaced diversity antennas in which the correlation between the pairs of combined signals decays as the spacing between the antennas increases [22], [30].

For this correlation model, the power correlation coefficients are determined as

$$\rho_{ij} = \rho^{|i-j|}, \quad 0 \leq \rho \leq 1. \tag{20}$$

The unknown joint moments in (14), $E[Z_m^2 Z_i Z_j]$, $E[Z_m Z_i^2 Z_j]$, $E[Z_m Z_i Z_j^2]$ and $E[Z_m Z_n Z_i Z_j]$, can be calculated from [14, Eqs. (11) and (12)]. The Appendix B derives simpler alternatives to [14, Eqs. (11) and (12)], which involve a single infinite sum and a familiar hypergeometric

TABLE I
 FADING PARAMETER m_R FOR SOME FEASIBLE SCENARIOS WITH EQUAL CORRELATION

ρ	$m_z = 1$			$m_z = 2$			$m_z = 3$		
	$L = 2$	$L = 3$	$L = 4$	$L = 2$	$L = 3$	$L = 4$	$L = 2$	$L = 3$	$L = 4$
	m_R	m_R	m_R	m_R	m_R	m_R	m_R	m_R	m_R
0	0.9552	0.9411	0.9343	1.947	1.93	1.9217	2.943	2.9258	2.9168
0.2	0.9195	0.8884	0.8709	1.9102	1.876	1.8569	2.9068	2.8715	2.8518
0.4	0.9156	0.8841	0.8672	1.907	1.8722	1.8535	2.9039	2.868	2.8487
0.6	0.9304	0.9056	0.8929	1.9242	1.8971	1.8831	2.9222	2.8944	2.8799
0.8	0.9587	0.9445	0.9374	1.956	1.9409	1.9333	2.9553	2.9399	2.9321

 TABLE II
 FADING PARAMETER m_R FOR SOME FEASIBLE SCENARIOS WITH EXPONENTIAL CORRELATION

ρ	$m_z = 1$			$m_z = 2$			$m_z = 3$		
	$L = 2$	$L = 3$	$L = 4$	$L = 2$	$L = 3$	$L = 4$	$L = 2$	$L = 3$	$L = 4$
	m_R	m_R	m_R	m_R	m_R	m_R	m_R	m_R	m_R
0	0.9552	0.9411	0.9343	1.947	1.93	1.9217	2.943	2.9258	2.9168
0.2	0.9195	0.9033	0.9015	1.9102	1.892	1.8897	2.9068	2.8878	2.8852
0.4	0.9156	0.8887	0.88	1.907	1.877	1.8675	2.9039	2.8728	2.8629
0.6	0.9304	0.8988	0.8817	1.9242	1.889	1.87	2.9222	2.8858	2.866
0.8	0.9587	0.934	0.9162	1.956	1.9291	1.9093	2.9553	2.9277	2.9072

function,

$$\begin{aligned}
 E[Z_m^{n_1} Z_i^{n_2} Z_j^{n_3}] &= \frac{|\Delta|^{m_z}}{\delta_{11}^{m_z+n_1/2} \delta_{22}^{m_z+n_2/2} \delta_{33}^{m_z+n_3/2}} \\
 &\times \frac{\Gamma(m_z + n_3/2)}{\Gamma^2(m_z) m_z^{(n_1+n_2+n_3)/2}} \sum_{k=0}^{\infty} \left(\frac{\delta_{12}^2}{\delta_{11} \delta_{22}} \right)^k \\
 &\times \frac{\Gamma(m_z + k + n_1/2) \Gamma(m_z + k + n_2/2)}{\Gamma(m_z + k) k!} \\
 &\times {}_2F_1 \left(m_z + k + \frac{n_2}{2}, m_z + \frac{n_3}{2}, m_z, \frac{\delta_{23}^2}{\delta_{22} \delta_{33}} \right), \quad (21)
 \end{aligned}$$

$$\begin{aligned}
 E[Z_m Z_n Z_i Z_j] &= \frac{|\Psi|^{m_z}}{(\psi_{11} \psi_{22} \psi_{33} \psi_{44})^{m_z+1/2}} \frac{\Gamma^2(m_z + 1/2)}{\Gamma^3(m_z) m_z^2} \\
 &\times \sum_{k=0}^{\infty} \frac{\Gamma^2(m_z + k + 1/2)}{k! \Gamma(m_z + k)} \left(\frac{\psi_{23}^2}{\psi_{22} \psi_{33}} \right)^k \\
 &\times {}_2F_1 \left(m_z + \frac{1}{2}, k + m_z + \frac{1}{2}, m_z, \frac{\psi_{12}^2}{\psi_{11} \psi_{22}} \right) \\
 &\times {}_2F_1 \left(m_z + \frac{1}{2}, k + m_z + \frac{1}{2}, m_z, \frac{\psi_{34}^2}{\psi_{33} \psi_{44}} \right). \quad (22)
 \end{aligned}$$

In (21), $(n_1, n_2, n_3) = (2, 1, 1)$ for the calculation of $E[Z_m^2 Z_i Z_j]$, $(n_1, n_2, n_3) = (1, 2, 1)$ for the calculation of $E[Z_m Z_i^2 Z_j]$ and $(n_1, n_2, n_3) = (1, 1, 2)$ for the calculation of $E[Z_m Z_i Z_j^2]$. The matrix $\Delta = [\delta_{i,j}]$ is the inverse of Λ 's principal submatrix composed of the m -th, i -th and j -th rows and columns of Λ , whereas the matrix $\Psi = [\psi_{i,j}]$ is the inverse of Λ 's principal submatrix composed of the m -th, n -th, i -th and j -th rows and columns of Λ ,

$$\begin{aligned}
 \Delta &= \begin{bmatrix} 1 & \sqrt{\rho_{mi}} & \sqrt{\rho_{mj}} \\ \sqrt{\rho_{im}} & 1 & \sqrt{\rho_{ij}} \\ \sqrt{\rho_{jm}} & \sqrt{\rho_{ji}} & 1 \end{bmatrix}^{-1}, \\
 \Psi &= \begin{bmatrix} 1 & \sqrt{\rho_{mn}} & \sqrt{\rho_{mi}} & \sqrt{\rho_{mj}} \\ \sqrt{\rho_{nm}} & 1 & \sqrt{\rho_{ni}} & \sqrt{\rho_{nj}} \\ \sqrt{\rho_{im}} & \sqrt{\rho_{in}} & 1 & \sqrt{\rho_{ij}} \\ \sqrt{\rho_{jm}} & \sqrt{\rho_{jm}} & \sqrt{\rho_{ji}} & 1 \end{bmatrix}^{-1}. \quad (23)
 \end{aligned}$$

The exactness of (21)-(22) arise from the fact that both matrices Δ and Ψ are tridiagonal matrices due to (20) [27, Section IV]. Introducing (21)-(22) into (14), one obtains the closed-form expression for $E[Z^4]$, which is omitted here for brevity. Combining (13)-(14) into (12), one obtains the unknown parameters Ω_R and m_R for the statistics of R .

The assumption of equal average powers, $\Omega_k = \Omega_Z$, $1 \leq k \leq L$, again renders independence of m_R from Ω_Z for the exponential correlation model. Under such assumptions, Table II displays the values of m_R for several illustrative combinations of ρ , L and m_z .

C. Arbitrary correlation model

In the general case of arbitrary branch correlations, the correlation matrix Λ is approximated by its appropriate Green's matrix, \mathbf{C} , utilizing the method presented in [27, Section IV]. Since principal submatrices of Green's matrices are also Green's matrices, the matrices Δ and Ψ , defined by (23), are determined to be tridiagonal, yielding direct applicability of the results presented in Section II.B to determine the unknown parameters Ω_R and m_R for the statistics of R . Thus, the statistics of Z are approximated by the statistics of R , whose arbitrary correlation matrix Λ is approximated by the Green's matrix \mathbf{C} . In the following subsection, we illustrate the highly accurate approximation to the PDF of Z facilitated by this approach.

D. Validation via statistical goodness-of-fit tests

We now statistically validate the proposed PDF approximations for equal, exponential and arbitrary branch correlation by using two different goodness-of-fit tests. The Chi-square (C-S) and Kolmogorov-Smirnov (K-S) tests provide two different statistical metrics, χ_n^2 and D_n , which describe the discrepancy between the observed samples of Z and the samples expected under the analytical distribution (7)-(8).

Each metric is averaged over 100 statistical samples, where each statistical sample comprises of 10000 independent random samples of Z . The random samples of Z are generated by computer simulations of correlated Nakagami- m RVs based on the method proposed in [28, Section VII].

TABLE III
SIGNIFICANCE LEVELS OF C-S AND K-S TESTS FOR GOODNESS
OF FIT BETWEEN THE EXACT AND THE APPROXIMATIVE DISTRIBUTIONS OF FIG. 1

ρ	$m_z = 1$				$m_z = 3$			
	$L = 2$		$L = 5$		$L = 2$		$L = 5$	
	α_{CS}	α_{KS}	α_{CS}	α_{KS}	α_{CS}	α_{KS}	α_{CS}	α_{KS}
0.2	0.004	0.02	0.17	0.06	< 0.001	< 0.001	< 0.001	< 0.001
0.7	0.04	0.03	0.2	0.18	< 0.001	< 0.001	< 0.001	< 0.001

TABLE IV
SIGNIFICANCE LEVELS OF C-S AND K-S TESTS FOR GOODNESS
OF FIT BETWEEN THE EXACT AND THE APPROXIMATIVE DISTRIBUTIONS OF FIG. 2

ρ	$m_z = 1$				$m_z = 3$			
	$L = 2$		$L = 5$		$L = 2$		$L = 5$	
	α_{CS}	α_{KS}	α_{CS}	α_{KS}	α_{CS}	α_{KS}	α_{CS}	α_{KS}
0.2	< 0.001	0.02	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
0.7	0.08	0.04	0.02	0.17	< 0.001	< 0.001	< 0.001	< 0.001

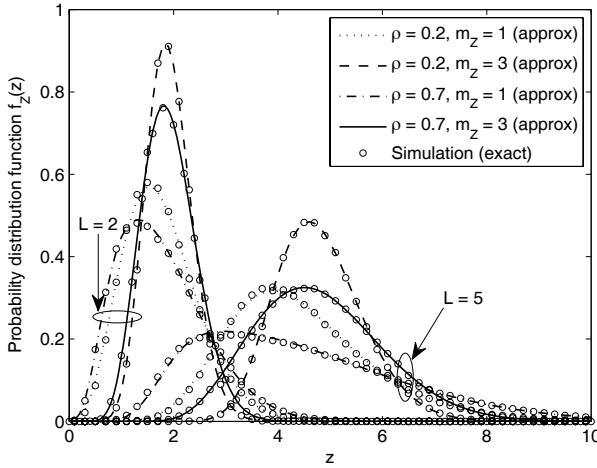


Fig. 1. Exact (obtained by simulation) and the approximative analytical PDFs to the sum of equally correlated Nakagami- m RVs with equal average powers, when $\Omega_Z = 1$

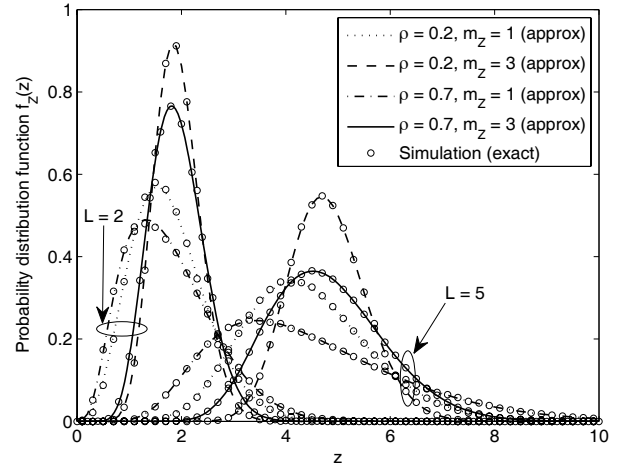


Fig. 2. Exact (obtained by simulation) and the approximative analytical PDFs to the sum of exponentially correlated Nakagami- m RVs with equal average powers, when $\Omega_Z = 1$

For each metric, we calculate the significance level α from the C-S and K-S distributions, respectively denoted as α_{CS} and α_{KS} . The significance level α represents the probability of rejecting the tested null hypothesis (H_0 : "the random samples of Z , obtained from (1), belong to the distribution given by (7)-(8)", when it is actually true. The small values of α indicate a good fit.

Note that, significance levels α less than 0.2 still indicate a good fit, due to the rigourousness of both C-S and K-S tests in accepting the null hypothesis H_0 .

1) *Equal and exponential correlation*: For the equal and exponential correlation models, the goodness-of-fit testing is conducted for combinations of the followings input parameters: $L = 2$ and 5 , $m_Z = 1$ and 3 , $\rho = 0.2$ and 0.7 , whereas the average powers of Z_k are assumed equal to unity ($\Omega_Z = 1$). The needed fading parameter m_R of distribution (7)-(8) is obtained directly from Tables I and II, whereas the average power Ω_R is calculated from (12).

Figs. 1 and 2 depict the excellent (visual) match between the histogram obtained from generated samples of Z and the proposed approximation (7)-(8), for the cases of equal and exponential correlation models, respectively. Tables III and IV complement Figs. 1 and 2, by presenting the significance levels α for the corresponding input parameters' combinations. The Table

III and the Table IV entries reveal the very low significance levels α for all input parameters' combinations, thus proving an excellent goodness of fit in statistical sense.

2) *Arbitrary correlation*: For illustrative purposes, we use same two example correlation matrices from [27, Sections V.B and V.D], Σ_{3_lin} and Σ_{4_circ} , here denoted as Λ_1 and Λ_2 , respectively. They are approximated by their Green's matrices C_{3_lin} and C_{4_circ} , here denoted as C_1 and C_2 , respectively.

Using C_1 and C_2 , one obtains the needed tridiagonal matrices Δ and Ψ from their definitions given by (23). The required joint moments are then calculated from (21) and (22), which are then substituted into (13) and (14) to calculate $E[Z^2]$ and $E[Z^4]$, and then (12) is used to describe the statistics of R .

Fig. 3 depicts the excellent (visual) match between the histogram obtained from generated samples of Z and the proposed approximation (7)-(8), for the two example correlation matrices Λ_1 and Λ_2 . Table V complements Figs. 3, by revealing the very low significance levels α_s , thus again proving an excellent goodness of fit.

E. Validation in case of maximal correlation

We now consider the case of maximal correlation coefficient between any pair of Nakagami- m envelopes Z_i and Z_j , i.e.,

TABLE V
SIGNIFICANCE LEVELS OF C-S AND K-S TESTS FOR GOODNESS OF FIT BETWEEN EXACT AND APPROXIMATIVE DISTRIBUTIONS OF FIG. 3

	$m_z = 1$		$m_z = 3$	
	α_{CS}	α_{KS}	α_{CS}	α_{KS}
Λ_1	0.19	0.18	< 0.001	< 0.001
Λ_2	0.12	0.11	< 0.001	< 0.001

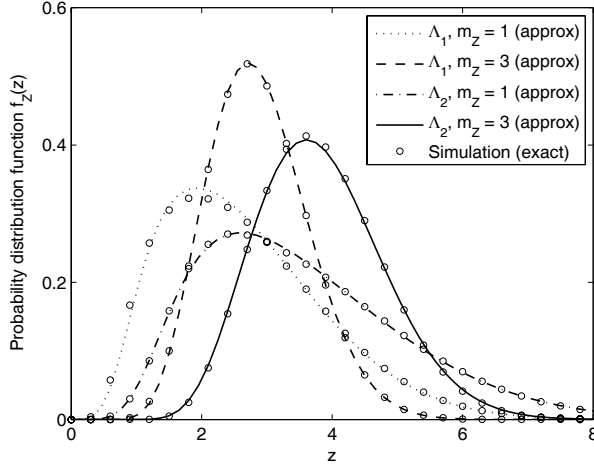


Fig. 3. Exact (obtained by simulation) and the approximative analytical PDFs to the sum of correlated Nakagami- m RVs with equal average powers, when $\Omega_Z = 1$, and correlation matrices Λ_1 and Λ_2

$\rho_{ij} = 1$. It indicates a perfect linear relationship between these pairs, which, after applying the model from [16, Eq. (37)], can be defined as $Z_i = \sqrt{\Omega_i} Z_0$ for $1 \leq i \leq L$, where Z_0 is an auxiliary Nakagami- m RV with unity average power and same fading parameter m_z . After replacing the latter expression into (1), Z is transformed into a Nakagami- m RV with fading parameter m_z and average power of

$$E[Z^2] = \left(\sum_{i=1}^L \sqrt{\Omega_i} \right)^2 = \sum_{i=1}^L \sum_{j=1}^L \sqrt{\Omega_i \Omega_j}, \quad (24)$$

which agrees with (13) when $\rho_{ij} \rightarrow 1$.

Replacing $\rho_{ij} = 1$ into (6), the $L - 1$ eigenvalues of the matrix Λ turn up equal to 0, except $\lambda_1 = L$. After plugging these eigenvalues into (5), R is transformed into a Nakagami- m RV with fading parameter m_R and average power $L\Omega_R$. After the moment matching, Ω_R and m_R can be obtained from (12), as

$$\Omega_R = \frac{1}{L} \sum_{i=1}^L \sum_{j=1}^L \sqrt{\Omega_i \Omega_j}, \quad m_R = \frac{(E[Z^2])^2}{E[Z^4] - (E[Z^2])^2} = m_z, \quad (25)$$

respectively, where the latter equality is attributed to the definition of the Nakagami- m fading parameter, given by [1, Eq. (4)].

Thus, maximal correlation yields (7)-(8) as an accurate distribution of Z , when our moment matching approach is applied. This conclusion further validates our approach.

III. APPLICATION TO THE PERFORMANCE ANALYSIS OF EGC RECEIVERS

We now consider a typical L -branch EGC diversity receiver exposed to slow and flat Nakagami- m fading. The envelopes of

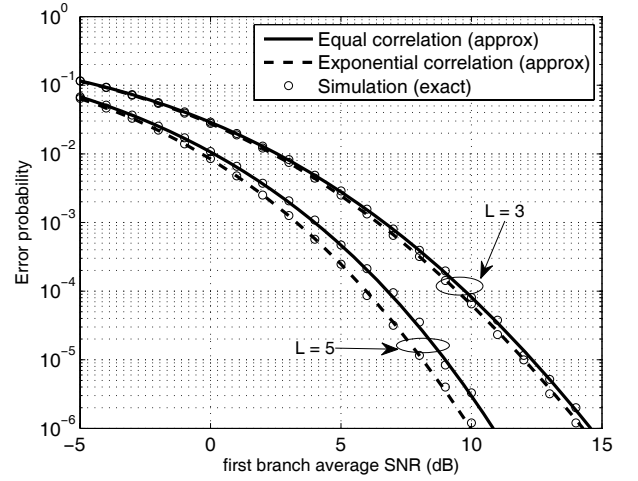


Fig. 4. Exact and approximate error probabilities of an EGC receiver with correlated Nakagami- m branches, when $m_z = 2$, $\mu = 0$ and $\rho = 0.7$

the branch signals Z_k are non-identical correlated Nakagami- m random processes with PDFs given by (2), whereas their respective phases are i.i.d. uniform random processes. Each branch is also corrupted by additive white Gaussian noise (AWGN) with power spectral density $N_0/2$, which is added to the useful branch signal. In the EGC receiver, the random phases of the branch signals are compensated (co-phased), equally weighted and then summed together to produce the decision variable. The envelope of the composite useful signal, denoted by Z , is given by (1), whereas the composite noise power is given by $\sigma_{EGC}^2 = LN_0/2$, resulting in the instantaneous output SNR given by

$$\gamma_{EGC} = \frac{Z^2}{2\sigma_{EGC}^2} = \frac{1}{LN_0} \left(\sum_{k=1}^L Z_k \right)^2 = \left(\sum_{k=1}^L G_k \right)^2 \quad (26)$$

where RVs $G_k = Z_k/\sqrt{LN_0}$, $1 \leq k \leq L$, form a set of L non-identical equally correlated Nakagami- m RVs with $E[G_k^2] = \bar{\gamma}_k/L$, same fading parameters m_z and correlation coefficient ρ_{ij} between branch pair (i, j) . Note that $\bar{\gamma}_k = \Omega_k/N_0$ denotes the average SNR in k -th branch.

Using the results from Section II, the MGF and the PDF of (26) can be approximated using (5) and (7)-(8), respectively, when Ω_R is replaced by $\bar{\gamma}_R = \Omega_R/(LN_0)$. These approximations are then used to determine the outage probability $F_{\gamma_{EGC}}$ and the error probability \bar{P}_{EGC} of an L -branch EGC systems in correlated Nakagami- m fading with high accuracy.

A. Outage probability

The outage probability of the EGC system with arbitrary correlated Nakagami- m fading branches, whose output SNR drops below threshold t , is approximated by the known outage probability expressions of an equivalent MRC system [23, Eq. (28)], [24, Eq. (13)],

$$\begin{aligned} F_{\gamma_{EGC}}(t) &\approx F_{\gamma_{MRC}}(t) \\ &= \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{\sin \left[m_R \sum_{k=0}^{L-1} \arctan \left(x \frac{\Omega_R \lambda_k}{m_R} \right) - xt \right] dx}{\prod_{k=0}^{L-1} \left[1 + \left(x \frac{\Omega_R \lambda_k}{m_R} \right)^2 \right]^{m_R/2}} \frac{dx}{x} \end{aligned}$$

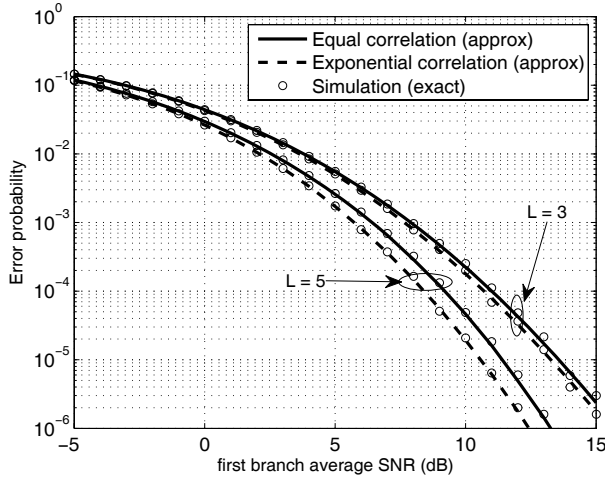


Fig. 5. Exact and approximate error performance of an EGC receiver with correlated Nakagami- m branches, when $m_z = 2$, $\mu = 0.3$ and $\rho = 0.7$

$$= \frac{1}{\Gamma(1 + Lm_R)} \left(\frac{m_R}{\Omega_R} t \right)^{Lm_R} \frac{1}{\det(\Lambda)} \times \Phi_2^{(L)} \left(m_R, m_R, \dots, m_R; 1 + Lm_R; -\frac{m_R}{\Omega_R \lambda_1} t, -\frac{m_R}{\Omega_R \lambda_2} t, \dots, -\frac{m_R}{\Omega_R \lambda_L} t \right). \quad (27)$$

For the equal correlation model, (27) can be simplified using [31, Eq. (2.1.3(1))].

B. Average error probability

The average error probability of the correlated Nakagami- m EGC system with BPSK modulation / coherent demodulation is approximated using the available expressions for the average error probability of the equivalent MRC systems. Based on [30, Eq. (9.11)] and [24, Eq. (17)], the error performance of this EGC system is alternatively approximated as

$$\begin{aligned} \bar{P}_{EGC-BPSK} &\approx \bar{P}_{MRC-BPSK} = \frac{1}{\pi} \int_0^{\pi/2} M_{R^2} \left(\frac{-1}{\sin^2 \theta} \right) d\theta \\ &= \frac{\Gamma(Lm_R + 1/2)}{2\sqrt{\pi}\Gamma(Lm_R + 1)} \left(\frac{m_R}{\Omega_R} \right)^{Lm_R} \left(\frac{1}{\det(\Lambda)} \right)^{m_R} \\ &\quad \times F_D^{(L)} \left(Lm_R + 1/2, m_R, \dots, m_R; Lm_R + 1; -\frac{m_R}{\Omega_R \lambda_1}, -\frac{m_R}{\Omega_R \lambda_2}, \dots, -\frac{m_R}{\Omega_R \lambda_L} \right). \quad (29) \end{aligned}$$

In (28), $M_{R^2}(\cdot)$ is replaced with the MGF given by (5). In (29), $F_D^{(L)}(\cdot)$ denotes the Lauricella F_D hypergeometric function of L variables, defined in [33] and [24, Eq. (18)]. For the equal correlation model, the average error probability can be calculated using [22, Eq. (32)], which is a special case of (29).

Note that (29) is here presented to demonstrate existence of an exact closed-form solution, whereas (28) is much more convenient for accurate and efficient numerical integration. For example, the average error probability may be obtained by applying the Gauss-Chebyshev quadrature rule [34, Eq. (25.4.38)] over (28). In the case of the balanced diversity

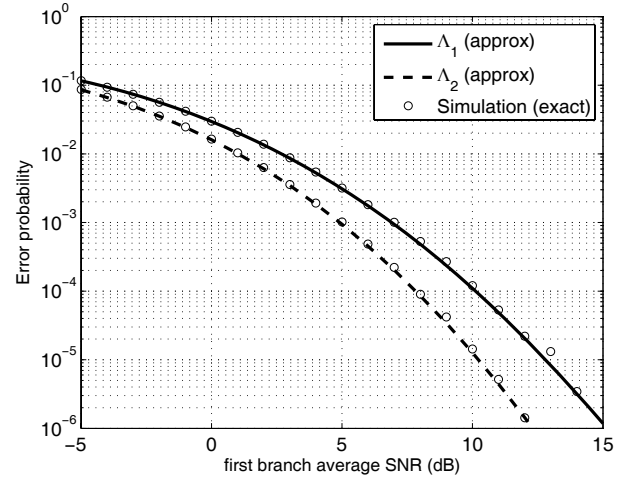


Fig. 6. Exact and approximate error performance of an EGC receiver with correlated Nakagami- m branches, when correlation is described by correlation matrices Λ_1 and Λ_2 , $m_z = 2$ and $\mu = 0$

branches with equal or exponential correlation, the combination of this quadrature rule with Tables I and II allows efficient and extremely accurate evaluation of the EGC performance.

The average error probability of correlated Nakagami- m EGC system with BPSK modulation / non-coherent demodulation is approximated by known expression of the equivalent MRC system [26, Eq. (16)], $\bar{P}_{EGC-BFSK} \approx \bar{P}_{MRC-BFSK} = \frac{1}{2} M_{R^2}(-\frac{1}{2})$, where $M_{R^2}(\cdot)$ is given by (5).

C. Validation via Monte-Carlo simulations

Next, we illustrate the tightness of the error performance of an correlated Nakagami- m EGC system with BPSK modulation / coherent demodulation to that of the equivalent MRC system. The results for the actual EGC system are obtained by Monte-Carlo simulations, whereas those of the equivalent MRC system are obtained using (28).

1) *Equal and exponential branch correlation:* Figs. 4 and 5 displays the comparative error performance of the actual EGC and the equivalent MRC systems, for several combinations of (ρ, L, m_z, Ω_k) . In order to accommodate unequal average branch powers (thus, unequal average branch SNRs), we used the exponentially decaying profile, modelled as

$$\Omega_k = \Omega_1 \exp(-\mu(k-1)), \quad 1 \leq k \leq L, \quad (30)$$

where Ω_1 is the average power of branch 1 and μ is the decaying exponent, with $\mu = 0$ denoting the case of branches with equal power (i.e., the balanced branches).

2) *Arbitrary branch correlation:* Fig. 6 depicts the comparative error performances of the EGC with same correlation matrices from Section II.D, Λ_1 and Λ_2 , and the equivalent MRC system with respective Green's matrices C_1 and C_2 . The high accuracy of our approach is maintained for arbitrary branch correlations.

IV. CONCLUSIONS

A tight closed-form approximation to the distribution of the sum of correlated Nakagami- m RVs was introduced for the case of identical and integer fading parameters. The proposed

method approximates this distribution by using the statistics of the square-root of the sum of statistically independent Gamma RVs. Examples indicate that the new approximation is highly accurate over the entire range of abscissas. To demonstrate this more rigorously, the proposed distribution is tested against the computer generated data by the use of the Chi-square and the Kologorov-Smirnov goodness-of-fit tests. In case of maximal correlation, the proposed distribution becomes the exact distribution.

The presented approach allowed to successfully tackle the famous problem of analytical performance evaluation of an EGC system with arbitrarily correlated and unbalanced Nakagami- m branches. The significance of the presented results is underpinned by the existence of a large body of literature dealing with MRC performance analysis, which permits highly accurate and efficient EGC performance evaluation.

APPENDIX A

Using [31, Eqs. (9.212 (1)) and (7.622 (1))], one has the following identity

$$\begin{aligned} J(m, a, p, q) &= \frac{1}{\Gamma(m)} \\ &\times \int_0^\infty u^{m-1} e^{-u} {}_1F_1\left(-\frac{p}{2}; m; -au\right) {}_1F_1\left(-\frac{q}{2}; m; -au\right) du \\ &= (1+a)^{\frac{p}{2}} \left(\frac{1+2a}{1+a}\right)^{\frac{q}{2}} {}_2F_1\left(m + \frac{p}{2}, -\frac{q}{2}; m; -\frac{a^2}{1+2a}\right) \end{aligned} \quad (\text{A.1})$$

Using [31, Eq. (9.212 (3)), pp. 1023] with some simple algebraic manipulations, the general form (18) of the coefficient $W(2, 1, 1)$ can be simplified as

$$\begin{aligned} W(2, 1, 1) &= m_z \left(\frac{\Gamma(m_z + 1/2)}{\Gamma(m_z)}\right)^2 \left(\frac{1 - \sqrt{\rho}}{1 + (N-1)\sqrt{\rho}}\right)^{m_z} \\ &\times \left[J(m_z, a, 1, 1) + \frac{a(m_z + 1/2)^2}{m_z^2} \right. \\ &\quad \times J(m_z + 1, a, 1, 1) + \frac{a}{4m_z^2} J(m_z + 1, a, -1, -1) \\ &\quad \left. - \frac{a(m_z + 1/2)^2}{m_z^2} J(m_z + 1, a, -1, 1) \right] \end{aligned} \quad (\text{A.2})$$

where $a = \sqrt{\rho}/(1 + (N-1)\sqrt{\rho})$

APPENDIX B

The unknown joint moments in (14), $E[Z_m^2 Z_i Z_j]$, $E[Z_m Z_i^2 Z_j]$, $E[Z_m Z_i Z_j^2]$ and $E[Z_m Z_n Z_i Z_j]$ can be calculated from [14, Eqs. (11) and (12)]. Here we derive their simpler and computationally more efficient alternatives. The alternative to [14, Eq. (12)] is derived directly from the definition of the joint moment $E[Z_1 Z_2 Z_3 Z_4]$,

$$\begin{aligned} E[Z_m Z_n Z_i Z_j] &= \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty z_m z_n z_i z_j \\ &\quad \times f_{Z_m Z_n Z_i Z_j}(z_m, z_n, z_i, z_j) dz_m dz_n dz_i dz_j, \end{aligned} \quad (\text{B.1})$$

where joint pdf of four exponentially correlated Nakagami- m RVs is expressed as [14, Eq. (9)]

$$\begin{aligned} f_{Z_m Z_n Z_i Z_j}(z_m, z_n, z_i, z_j) &= \frac{2^4 m_z^{m_z+3} |\Psi|^{m_z} z_m^{m_z} z_n z_i z_j^{m_z}}{\Gamma(m_z) |\psi_{12} \psi_{23} \psi_{34}|^{m_z-1}} \\ &\times I_{m_z-1}(2m_z |\psi_{12}| z_m z_n) I_{m_z-1}(2m_z |\psi_{23}| z_n z_i) \\ &\times I_{m_z-1}(2m_z |\psi_{34}| z_i z_j) \exp\left(-m_z (\psi_{11} z_m^2 \right. \\ &\quad \left. + \psi_{22} z_n^2 + \psi_{33} z_i^2 + \psi_{44} z_j^2)\right), \end{aligned} \quad (\text{B.2})$$

where $\Psi = [\psi_{i,j}]$ is defined by (23). Now, we integrate [14, Eq. (11)] over z_m and z_j , respectively obtaining

$$\begin{aligned} \int_0^\infty z_m^{m_z+1} \exp(-m_z \psi_{11} z_m^2) I_{m_z-1}(2m_z \psi_{12} z_m z_n) \\ = \frac{1}{2} (m_z \psi_{12} z_n)^{m_z-1} (m_z \psi_{11})^{-(m_z+1/2)} \\ \times \frac{\Gamma(m_z + 1/2)}{\Gamma(m_z)} {}_1F_1\left(m_z + \frac{1}{2}; m_z; m_z \frac{\psi_{12}^2}{\psi_{11}} z_n^2\right), \end{aligned} \quad (\text{B.3})$$

and

$$\begin{aligned} \int_0^\infty z_j^{m_z+1} \exp(-m_z \psi_{44} z_j^2) I_{m_z-1}(2m_z \psi_{34} z_i z_j) \\ = \frac{1}{2} (m_z \psi_{34} z_i)^{m_z-1} (m_z \psi_{44})^{-(m_z+1/2)} \\ \times \frac{\Gamma(m_z + 1/2)}{\Gamma(m_z)} {}_1F_1\left(m_z + \frac{1}{2}; m_z; m_z \frac{\psi_{34}^2}{\psi_{44}} z_i^2\right). \end{aligned} \quad (\text{B.4})$$

We then use the series expansion of the modified Bessel function of first kind [31, Eq. (8.445)] that allows to separate the integrations per variables z_n and z_i , yielding (22). A similar procedure yields to an alternative of [14, Eq. (11)] given by (21).

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cooperative diversity.



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