

# Selective Cooperative Relaying over Time-Varying Channels

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**Abstract**—In selective cooperative relaying only a single relay out of the set of available relays is activated, hence the available power and bandwidth resources are efficiently utilized. However, implementing selective cooperative relaying in time-varying channels may cause frequent relay switchings that deteriorate the overall performance. In this paper, we study the rate at which a relay switching occurs in selective cooperative relaying applications in time-varying fading channels. In particular, we derive closed-form expressions for the relay switching rate (measured in Hz) for opportunistic relaying (OR) and distributed switch and stay combining (DSSC). Additionally, expressions for the average relay activation time for both of the considered schemes are also provided, reflecting the average time that a selected relay remains active until a switching occurs. Numerical results manifest that DSSC yields considerably lower relay switching rates than OR, along with larger average relay activation times, rendering it a better candidate for implementation of relay selection in fast fading environments.

**Index Terms**—Relay switching rate, selective cooperative relaying.

## I. INTRODUCTION

COOPERATIVE relaying has been recently proposed as a means of achieving the beneficial effects of diversity in wireless communications systems, without employing multiple antennas at neither the receiver nor the transmitter. Its operation is based upon the concept of employing wireless relaying terminals that assist the communication between a source and a destination terminal by receiving the message sent from the source, and then processing it appropriately and forwarding it to the destination. The relaying terminals may be either fixed, infrastructure-based terminals placed at selected spots in urban environments aiming at extending the coverage while avoiding the infrastructure cost that the deployment of base stations entails, or mobile, hand-held devices. In any of the above cases, apart from the apparent robustness against small-scale fading, cooperative relaying offers resilience against large attenuations due to path-loss, as well as shadowing. This, together with the advantages that cooperative relaying offers in the various levels of the open system interconnection

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(OSI) protocol stack, renders the cooperative concept a strong candidate for utilization in future wireless networks [1].

The most common cooperative relaying protocols were introduced in [2], where the term “cooperative diversity” was used so as to emphasize the diversity advantages of relay systems. In the same work, an outage analysis of the cooperative diversity concept was also conducted, showing remarkable performance benefits as compared to the case without relaying. Nonetheless, the analysis in [2] concerns the scenario where a single relay is available for cooperation. In cases where multiple relaying terminals are utilized, the error performance can be dramatically improved if the multiple relay transmissions occur in orthogonal channels and are combined by a maximal ratio combiner (MRC) at the destination. However, orthogonal channel utilization results in a reduced overall spectral efficiency. To this end, activating only a single relay out of the set of available relays has been shown to be an effective means of achieving cooperative diversity while limiting the negative effects of orthogonal relay transmissions, offering thus a good tradeoff between error performance and spectral efficiency.

Previous works on single relay selection in cooperative relaying scenarios include [3]–[6], where the relay selection was based on a maximum signal-to-noise-ratio (SNR) policy, thus attaining a diversity order equal to the number of available relays, i.e., the same diversity order as for the case where all the relays are activated, yet with considerably higher spectral efficiency. Due to the somewhat opportunistic usage of the available resources, the relay selection protocol based upon the maximum SNR rule is termed opportunistic relaying (OR) [4]. A simpler alternative to OR, for the case of two available relaying terminals, is the so-called distributed switch and stay combining (DSSC) protocol proposed in [7]. According to DSSC, a single relay remains active for as long as the corresponding SNR is greater than a predetermined threshold value; should this condition be violated, a relay switching occurs. The DSSC protocol hence requires only a single end-to-end channel estimation for each transmission, reducing thus the overall complexity while achieving the same outage performance as OR, albeit inferior error performance [7]. In the sequel, we use the term “selective cooperative relaying” to refer to the OR and DSSC protocols, i.e., to protocols where single relay selection takes place.

## A. Motivation

Despite the above benefits of selective cooperative relaying, however, a major issue that needs to be addressed is the rate

at which a switching of the active relaying terminal occurs in practical scenarios where the fading in each of the links involved is time-varying. In fact, this rate reflects the number of times per second the system has to switch from one relaying terminal to another, and corresponds to a complexity measure regarding the implementation of selective cooperative relaying in practice. More specifically, frequent relay switchings may cause synchronization problems due to the fact that the system needs to repeat the initialization process each time the active relay changes, in order to re-adapt to the channel conditions of the new branch. Apparently, frequent synchronization readjustments lead to increased implementation complexity, as well as potential delays and outages which may cause severe information loss with ultimate deteriorating effects on performance.

Particularly for the case of DSSC, where only one end-to-end branch is estimated in each transmission period, relay switchings have a negative impact on channel estimation along with synchronization. This is because each time a relay switching occurs, a previously idle relay needs to be “awakened”. Hence, a new training sequence needs to be initiated, which may not be long enough to provide accurate channel estimation, resulting in detection errors in addition to those owing to weak channel conditions.

Consequently, although there is a variety of works dealing with the performance of selective cooperative relaying in slow fading scenarios [4]-[5], [7], such analyses do not capture the effect of the time-varying nature of fading channels, which may be crucial especially under the fast fading assumption. To the best of the authors’ knowledge, the concept of relay switchings in time-varying fading environments has not been addressed in the literature before.

### B. Contribution

In this paper, we study the effect of the time-varying nature of fading channels in selective cooperative relaying applications. In particular, we provide closed-form expressions for the relay switching rate (measured in Hz) of OR and DSSC, as a function of the average channel gains and the maximum Doppler frequency of each of the source-relay and relay-destination links involved. It is emphasized that this rate strongly depends on the maximum Doppler frequency of the participating links, and differs from the probability of switching in slow fading scenarios where the Doppler frequency is practically zero (see, e.g., [8] for the Switch and Stay Combining (SSC) case where the antennas are co-located, i.e., no relaying takes place). The expressions obtained here consider the case of independent but not necessarily identically distributed (i.n.i.d.) Rayleigh fading channels, and account for both AF and decode-and-forward (DF) relaying. Particularly for OR, we derive the relay switching rate for arbitrary numbers of participating relaying terminals showing that this rate is an increasing function in the number of relays.

In addition to the switching rate, we obtain closed-form expressions for the average relay activation time, which is defined as the average time interval that the selected relay remains activated until the system switches to another relay. A set of numerical examples is provided showing that DSSC

results in a considerably lower relay switching rate, as well as a larger average relay activation time than OR. These results indicate that DSSC may be preferable in fast fading scenarios where the channel gains change rapidly making OR difficult to implement due to the frequent relay switchings, despite DSSC’s inferiority in terms of error performance [7].

### C. Outline

The rest of this paper is organized as follows. The mode of operation of OR and DSSC, together with some basic definitions are given in Section II. Section III provides expressions for the relay switching rate and the average relay activation time of OR, when operating over Rayleigh fading channels, while the corresponding expressions for DSSC are presented in Section IV. Some numerical examples are given in Section V. Finally, Section VI concludes the paper.

## II. BACKGROUND

We consider the cooperative relaying setup where a source terminal,  $S$ , communicates with a destination terminal,  $D$ , with the aid of  $L$  relaying terminals which are denoted here by  $R_i$ ,  $i \in \{1, \dots, L\}$ . The relaying terminals may operate in either the DF or the AF mode. In the former case, the relays fully decode the received signal and forward a noise-free symbol to the destination, while in the latter case the relaying terminals are used as simple analog repeaters which amplify the received signal and forward it to the destination without demodulating it. Additionally, the relays are assumed to be half-duplex, in the sense that they cannot receive and transmit simultaneously; instead, the source-relay and relay-destination transmissions are assumed to occur in time-orthogonal channels.

In this paper, we adopt the general notation,  $a_{AB}$ , to denote the channel gain of the link between terminals  $A$  and  $B$ , so that the channel gain of, e.g., the  $S$ - $R_i$  link, is represented by  $a_{SR_i}$ . The fading in each of the links involved is assumed to be Rayleigh distributed, with probability density function (PDF) given by

$$f_{a_{AB}}(x) = \frac{2x}{\Omega_{AB}} \exp\left(-\frac{x^2}{\Omega_{AB}}\right) \quad (1)$$

where  $\Omega_{AB}$  represents the average squared channel gain of the  $A$ - $B$  link, e.g.,  $\Omega_{SR_i} = E[a_{SR_i}^2]$  with  $E[\cdot]$  denoting expectation. We denote the maximum Doppler frequency of the  $A$ - $B$  link by  $\mathcal{F}_{AB}$ , e.g., the maximum Doppler frequency of the  $S$ - $R_i$  link is denoted by  $\mathcal{F}_{SR_i}$ . Moreover, all terminals are assumed to transmit with identical power, denoted by  $P_T$ , while the noise power in all of the links involved is identical and denoted by  $N_0$ .

Throughout this work, two selective cooperative relaying protocols are considered: The opportunistic relaying (OR) protocol presented in [4], and a variant of the two-relay distributed switch and stay combining (DSSC) protocol proposed in [7], which is referred to as DSSC-B here; this notation is adopted since DSSC-B varies from DSSC in exactly the same way as SSC-B varies from SSC-A in [8]. The reasoning behind studying DSSC-B instead of the original DSSC protocol presented in [7] lies in the fact that DSSC-B yields less

frequent relay switchings than DSSC, similarly as SSC-B yields less frequent switchings than SSC-A [8].

The modes of operation of OR and DSSC-B are given in detail in the ensuing subsection. It is worth mentioning that in each case, only a single relay out of the set of available relays is activated and denoted by  $R_b$ , resulting in a somewhat distributed version of selection combining for OR; and a distributed version of SSC-B for DSSC-B. The selection is performed at the destination terminal, which collects the channel state information (CSI) of all the links involved. Then, after determining the “best” relay, the destination sends a feedback message to the relays indicating the activation of the selected relay and the deactivation of the previously selected relay. All other terminals remain inactive until they receive a proper activation message from the destination.

#### A. Mode of Operation of Schemes Under Consideration

1) *Opportunistic Relaying (OR)*: The OR protocol consists of selecting a single relay out of the set of  $L$  available relays, particularly the relay with the highest of some appropriately defined metric, which accounts for both the  $S$ - $R_i$  and  $R_i$ - $D$  links and corresponds to performance measures of the  $i$ th end-to-end path. Under the assumption of equal power transmitted by the potential relaying terminals, such metrics may be the min equivalent defined as

$$a_i = \min(a_{SR_i}, a_{R_iD}) \quad (2)$$

or the “half harmonic mean” equivalent defined as

$$a_i = \sqrt{\frac{a_{SR_i}^2 a_{R_iD}^2}{a_{SR_i}^2 + a_{R_iD}^2}}. \quad (3)$$

The min equivalent metric accounts for determining the end-to-end path based on the weakest intermediate link. It is appropriate for DF relaying because it corresponds to an outage-equivalent of the end-to-end DF channel, since the outage probability of DF relaying equals the cumulative distribution function (CDF) of  $a_i$  evaluated at the outage threshold SNR. The half harmonic mean equivalent is appropriate for AF relaying since it corresponds to a tight approximation of the end-to-end SNR of the  $S$ - $R_i$ - $D$  link [9]. It should be noted, however, that both criteria lead to approximately the same results, since the min equivalent represents a tight upper upper bound of the half harmonic mean equivalent, particularly when the SNRs of the source-relay and relay-destination links are very different, e.g., if  $a_{SR_i} \gg a_{R_iD}$  or  $a_{R_iD} \gg a_{SR_i}$  [10], [11]. For this reason, in the sequel we adopt the max-min criterion for both DF and AF relaying for determining the selected relay,  $R_b$ , so that the relay associated with the maximum “bottleneck” of the source-relay and relay-destination links is selected, i.e.,

$$b = \arg \max_{i \in \{1, \dots, L\}} \min(a_{SR_i}, a_{R_iD}). \quad (4)$$

Additionally, it is assumed that the decision on the selected relay is made at the destination terminal, after collecting the CSI of all the intermediate links involved through some signalling process.

2) *DSSC-B*: The DSSC protocol [7] applies to the case where there are two relays available for cooperation. Its simplicity over OR lies in the fact that in each training period, only a single end-to-end channel has to be estimated. This estimation is used so as to check whether the active branch is of sufficient quality. The system thus switches from one relay to another only if the equivalent SNR of the active branch lies below a predefined switching threshold,  $T$ . Using the min equivalent metric defined above, a relay switching occurs when  $a_b < \sqrt{T/\Gamma}$ , where  $\Gamma = P_T/N_0$  denotes the ratio of the power transmitted by the source and each relay divided by the noise power, i.e., the common SNR without fading.

Nevertheless, there may exist transmission periods where both the available end-to-end channels are not strong enough; in such cases the system switches continuously from one relay to another, though without reaching the desired SNR level. Therefore, in order to avoid these excessive, as well as unavailing switchings, throughout this paper we study a variant of the DSSC protocol denoted here by DSSC-B. According to DSSC-B, a switching occurs whenever the SNR of the active branch down-crosses the switching threshold,  $T$ ; the system then stays connected with the new branch, regardless of whether the SNR of the new branch is greater or lower than  $T$ , until this SNR down-crosses  $T$ . Mathematically speaking, denoting by  $a_i^j$  the min equivalent of the  $i$ th branch for a transmission period  $j$ , the active relay,  $R_b^{j+1}$ , for the ensuing transmission period is determined by

$$\text{If } R_b^j = R_1, \quad (5a)$$

$$R_b^{j+1} = \begin{cases} R_1 & \text{if } \left[ a_1^{j-1} < \sqrt{T/\Gamma} \text{ or } a_1^j > \sqrt{T/\Gamma} \right] \\ R_2 & \text{if } \left[ a_1^{j-1} > \sqrt{T/\Gamma} \text{ and } a_1^j < \sqrt{T/\Gamma} \right] \end{cases}$$

$$\text{If } R_b^j = R_2, \quad (5b)$$

$$R_b^{j+1} = \begin{cases} R_2 & \text{if } \left[ a_2^{j-1} < \sqrt{T/\Gamma} \text{ or } a_2^j > \sqrt{T/\Gamma} \right] \\ R_1 & \text{if } \left[ a_2^{j-1} > \sqrt{T/\Gamma} \text{ and } a_2^j < \sqrt{T/\Gamma} \right] \end{cases}$$

It is interesting to note that, under the min equivalent criterion, the  $i$ th end-to-end path between source and destination in both OR and DSSC-B is treated as a *virtual* channel with channel gain  $a_i = \min(a_{SR_i}, a_{R_iD})$ . The OR scheme can be thus regarded as a virtual selection diversity scheme, where the instantaneous channel gain of the  $i$ th input branch is  $a_i$ ; the DSSC-B scheme can be interpreted as a virtual SSC-B scheme with channel gains  $a_1$  and  $a_2$ . We note, however, that although this virtual gain represents a useful tool for studying the effect of time-varying fading in selective cooperative schemes, it may not be associated to the second-order statistics of the  $i$ th inherent end-to-end channel between source and destination.

#### B. Basic Definitions

The rest of the paper focuses on deriving expressions, as well as providing numerical examples, for the following performance measures for both OR and DSSC-B:

- The *relay switching rate*, defined as the number of times per second that a switching of the active relay takes place; that is, the destination stops receiving from a certain relaying terminal and connects with another one. We

emphasize that this switching rate strongly depends on the Doppler effect in the fading channels involved, and that it is completely different from the probability of switching in slow fading, investigated in [8] for the SSC case, as has been pointed out also in [12].

- The *average relay activation time*, defined as the average time duration that the selected relay remains activated, starting from the time it receives an activation message until the system switches to another relaying terminal.

### III. RELAY SWITCHING RATES AND AVERAGE RELAY ACTIVATION TIME OF OR

#### A. Two Relays, i.n.i.d. Fading

Let us first consider the two-relay OR scenario, where the fading in all the intermediate links involved is i.n.i.d.

*Theorem 1:* The relay switching rate of OR with two available relays and i.n.i.d. fading is given by

$$SR_{OR} = \frac{\pi\sqrt{2\Omega_1\Omega_2}}{(\Omega_1 + \Omega_2)^{3/2} (\Omega_{SR_1} + \Omega_{R_1D}) (\Omega_{SR_2} + \Omega_{R_2D})} \times \left[ \Omega_{SR_1}\Omega_{SR_2}\sqrt{\Omega_{R_1D}\mathcal{F}_{R_1D}^2 + \Omega_{R_2D}\mathcal{F}_{R_2D}^2} + \Omega_{R_1D}\Omega_{SR_2}\sqrt{\Omega_{SR_1}\mathcal{F}_{SR_1}^2 + \Omega_{R_2D}\mathcal{F}_{R_2D}^2} + \Omega_{R_2D}\Omega_{SR_1}\sqrt{\Omega_{SR_2}\mathcal{F}_{SR_2}^2 + \Omega_{R_1D}\mathcal{F}_{R_1D}^2} + \Omega_{R_1D}\Omega_{R_2D}\sqrt{\Omega_{SR_1}\mathcal{F}_{SR_1}^2 + \Omega_{SR_2}\mathcal{F}_{SR_2}^2} \right] \quad (6)$$

where  $\Omega_i = E[a_i^2] = \Omega_{SR_i}\Omega_{R_iD}/(\Omega_{SR_i} + \Omega_{R_iD})$  represents the average squared value of  $a_i$ ,  $i \in \{1, 2\}$ .

*Proof:* Let us consider the random processes

$$a_1(t) = \min(a_{SR_1}(t), a_{R_1D}(t)) \quad (7)$$

$$a_2(t) = \min(a_{SR_2}(t), a_{R_2D}(t)) \quad (8)$$

which correspond to the fading processes of the virtual end-to-end channels  $S$ - $R_1$ - $D$  and  $S$ - $R_2$ - $D$ , respectively. Additionally, let us define the random process  $Z(t)$  as

$$Z(t) = a_1(t) - a_2(t) \quad (9)$$

so that the active relay in each time instance  $t$  is determined by the signum of  $Z(t)$  in this time instance, i.e.,  $R_1$  is active at time  $t$  if  $Z(t) > 0$ ;  $R_2$  is active at time  $t$  if  $Z(t) < 0$ . Consequently, in order to derive the average relay switching rate it suffices to evaluate the average number of times the process  $Z(t)$  crosses zero. This is equivalent to obtaining the level crossing rate (LCR) of  $Z(t)$ , evaluated at zero. Then, the positive-going LCR of  $Z(t)$  corresponds to the average number of times the system switches from  $R_2$  to  $R_1$ , while the negative-going LCR of  $Z(t)$  accounts for the average number of times the system switches from  $R_1$  to  $R_2$ .

The relay switching rate equals the sum of positive-going and negative-going zero-crossing rates of  $Z(t)$ , which can be expressed as [13]

$$SR_{OR} = \int_{-\infty}^0 \left| \dot{z} \right| f(0, \dot{z}) d\dot{z} + \int_0^{\infty} \dot{z} f(0, \dot{z}) d\dot{z} \quad (10)$$

where  $f(z, \dot{z})$  denotes the joint PDF of  $Z(t)$  and the time-derivative of  $Z(t)$ ,  $\dot{Z}(t)$ . Because of the independence of the

fading process and its time derivative in each of the intermediate links involved, owing to the Rayleigh fading assumption, the processes  $Z(t)$  and  $\dot{Z}(t)$  are independent [12]. Therefore,  $f(z, \dot{z})$  can be expressed as  $f(z, \dot{z}) = f_Z(z) f_{\dot{Z}}(\dot{z})$ , with  $f_Z(\cdot)$  and  $f_{\dot{Z}}(\cdot)$  denoting the PDFs of  $Z(t)$  and  $\dot{Z}(t)$ , respectively. Equation (10) thus yields

$$SR_{OR} = f_Z(0) \left[ \int_{-\infty}^0 \left| \dot{z} \right| f_{\dot{Z}}(\dot{z}) d\dot{z} + \int_0^{\infty} \dot{z} f_{\dot{Z}}(\dot{z}) d\dot{z} \right] \quad (11)$$

$$= 2f_Z(0) \int_0^{\infty} \dot{z} f_{\dot{Z}}(\dot{z}) d\dot{z} \quad (12)$$

where we used the fact that the two integrals in (11) are equal to each other since, apparently, the number of times the system switches from  $R_1$  to  $R_2$  equals that of switching from  $R_2$  to  $R_1$ , in the long run. The PDF of  $Z(t)$  evaluated at the origin is derived as (see Appendix A)

$$f_Z(0) = \frac{\sqrt{\pi}\sqrt{\Omega_1\Omega_2}}{(\Omega_1 + \Omega_2)^{3/2}}. \quad (13)$$

Moreover, the second term in (12) is derived as (see Appendix B)

$$\int_0^{\infty} \dot{z} f_{\dot{Z}}(\dot{z}) d\dot{z} = \frac{\sqrt{\pi}}{\sqrt{2}(\Omega_{SR_1} + \Omega_{R_1D})(\Omega_{SR_2} + \Omega_{R_2D})} \times \left[ \Omega_{SR_1}\Omega_{SR_2}\sqrt{\Omega_{R_1D}\mathcal{F}_{R_1D}^2 + \Omega_{R_2D}\mathcal{F}_{R_2D}^2} + \Omega_{R_1D}\Omega_{SR_2}\sqrt{\Omega_{SR_1}\mathcal{F}_{SR_1}^2 + \Omega_{R_2D}\mathcal{F}_{R_2D}^2} + \Omega_{R_2D}\Omega_{SR_1}\sqrt{\Omega_{SR_2}\mathcal{F}_{SR_2}^2 + \Omega_{R_1D}\mathcal{F}_{R_1D}^2} + \Omega_{R_1D}\Omega_{R_2D}\sqrt{\Omega_{SR_1}\mathcal{F}_{SR_1}^2 + \Omega_{SR_2}\mathcal{F}_{SR_2}^2} \right]. \quad (14)$$

The relay switching rate of OR for the case of two relays with i.n.i.d. fading channels is obtained by substituting (13) and (14) into (12), completing the proof. ■

*Corollary 1:* The average relay activation time for OR with two available relays and i.n.i.d. fading is given by

$$AT_{i,OR} = \frac{2\Omega_i}{SR_{OR}(\Omega_1 + \Omega_2)}, \quad i \in \{1, 2\} \quad (15)$$

where  $SR_{OR}$  is given in (6).

*Proof:* Let us denote by  $\rho_i^{OR}$  the steady-state probability of selecting relay  $R_i$  in the OR setup, i.e.,  $\rho_1^{OR} = \Pr\{a_1 > a_2\}$ ;  $\rho_2^{OR} = \Pr\{a_1 < a_2\}$ . It holds

$$\rho_i^{OR} = \frac{SR_{OR}}{2} AT_{i,OR} \quad (16)$$

where we used the fact that the average switching rate from  $R_1$  to  $R_2$  equals the average switching rate from  $R_2$  to  $R_1$ . The average relay activation time is then derived as

$$AT_{i,OR} = \frac{2\rho_i^{OR}}{SR_{OR}}. \quad (17)$$

Using the fact that  $\rho_i^{OR} = \Omega_i/(\Omega_1 + \Omega_2)$  (see Eqs. (52), (53)), the proof is complete. ■

### B. Two Relays, Independent and Identically Distributed (i.i.d.) Fading

*Corollary 2:* The relay switching rate of OR with two available relays and i.i.d. fading is given by

$$SR_{OR}^{iid} = \frac{\pi \left[ \sqrt{\mathcal{F}_{SR_1}^2 + \mathcal{F}_{SR_2}^2} + \sqrt{\mathcal{F}_{SR_1}^2 + \mathcal{F}_{R_2D}^2} \right]}{4\sqrt{2}} + \frac{\pi \left[ \sqrt{\mathcal{F}_{R_1D}^2 + \mathcal{F}_{SR_2}^2} + \sqrt{\mathcal{F}_{R_1D}^2 + \mathcal{F}_{R_2D}^2} \right]}{4\sqrt{2}}. \quad (18)$$

Particularly for the case where the maximum Doppler frequencies are also identical (i.e.,  $\mathcal{F}_{R_i} = \mathcal{F}_{R_iD} = \mathcal{F}$ ,  $i \in \{1, 2\}$ ), the relay switching rate is given by

$$SR_{OR}^{iid} = \pi\mathcal{F}. \quad (19)$$

*Proof:* The proof follows directly from (6) after simple algebraic manipulations. ■

One may note that the relay switching rate in the i.i.d. case is independent of the channel amplitude and is determined only by the maximum Doppler frequency in each of the links involved. Moreover, it is interesting to note that (19) yields a relay switching rate which is identical to that of conventional selection diversity (i.e., where no relaying takes place) with identical Rayleigh fading, as given in [12], except for a factor of  $\sqrt{2}$ . This implies that the OR setup can be considered as a distributed selection diversity scheme with two *virtual* Rayleigh channels, where the maximum Doppler frequency of each virtual channel equals  $\sqrt{2}$ -times the maximum Doppler frequency of the intermediate links. This is due to the fact that in our case we deal with four time-varying links, whereas in conventional selection diversity there are only two time-varying links involved.

*Corollary 3:* The average relay activation time of two-relay OR for the i.i.d. scenario with identical maximum Doppler frequencies is given by

$$AT_{OR}^{iid} = \frac{1}{\pi\mathcal{F}}. \quad (20)$$

*Proof:* The proof follows from (15), in conjunction with (19). ■

### C. L Relays, i.n.i.d. Fading

Similarly to the two-relay scenario, the relay switching rate for the versatile case of OR with  $L$  available relays is derived as follows. Let us define the rate at which the system switches to  $R_i$  from any of the remaining relays as  $SR_{OR,i}$ ,  $i \in \{1, \dots, L\}$ . The overall relay switching rate equals the sum of the individual switching rates, i.e.,

$$SR_{OR} = SR_{OR,1} + \dots + SR_{OR,L}. \quad (21)$$

Each of the individual switching rates in (21) is obtained by utilizing an auxiliary random process  $Z_i(t)$ ,  $i \in \{1, \dots, L\}$ , similarly as the  $L = 2$  case. In particular, the rate  $SR_{OR,1}$  equals the number of times the random variable

$$Z_1(t) = a_1(t) - \max(a_2(t), \dots, a_L(t)) \quad (22)$$

up-crosses zero. This rate is obtained as

$$SR_{OR,1} = f_{Z_1}(0) \int_0^\infty z_1 f_{Z_1}(z) dz_1. \quad (23)$$

Let us define

$$\alpha_i(t) = \max_{j \in \{1, \dots, L\}, j \neq i} a_j(t). \quad (24)$$

That is,  $\alpha_i(t)$  represents the maximum virtual channel gain of all the end-to-end channels involved (as defined in (2)), except  $a_i(t)$ . The value of  $f_{Z_1}(0)$  evaluated at the origin is expressed as

$$f_{Z_1}(0) = \int_0^\infty f_{a_1}(x) f_{\alpha_1}(x) dx. \quad (25)$$

A closed-form expression for  $f_{Z_1}(0)$  is derived through a process similar to that in Appendix A. For instance, when  $L = 3$  (25) yields after algebraic manipulations

$$f_{Z_1}(0) = \sqrt{\pi}\Omega_1^{5/2} \left[ \frac{\sqrt{\Omega_2}}{(\Omega_1 + \Omega_2)^{3/2}} - \frac{\sqrt{\Omega_3}}{(\Omega_1 + \Omega_3)^{3/2}} - \frac{\sqrt{\Omega_2\Omega_3}(\Omega_2 + \Omega_3)}{(\Omega_1 + \Omega_2 + \Omega_3)^{3/2}} \right]. \quad (26)$$

Working similarly as in Appendix B, a closed-form expression for the second term of (23) is derived. We note, however, that the resulting expression is quite cumbersome even for  $L = 3$ , therefore it is omitted. The individual switching rate  $SR_{OR,1}$  is then obtained from (23); the expressions for  $SR_{OR,2}, \dots, SR_{OR,L}$  are derived likewise. The overall relay switching rate is derived by plugging these expressions into (21).

Having then an expression for  $SR_{OR,i}$ , we infer the average activation time of the  $R_i$  relay as

$$AT_{i,OR} = \frac{\rho_i^{OR}}{SR_{OR,i}} \quad (27)$$

where  $\rho_i^{OR}$  is obtained through the following formula

$$\rho_i^{OR} = \Pr\{a_i > \alpha_i\} = \int_0^\infty f_{a_i}(x) F_{\alpha_i}(x) dx. \quad (28)$$

For the case of  $L = 3$ , for example,  $\rho_1^{OR}, \rho_2^{OR}, \rho_3^{OR}$  are derived as

$$\rho_1^{OR} = \frac{\Omega_1}{\Omega_1 + \Omega_3} - \frac{\Omega_1\Omega_2^2}{(\Omega_1 + \Omega_2)(\Omega_1\Omega_2 + \Omega_1\Omega_3 + \Omega_2\Omega_3)} \quad (29)$$

$$\rho_2^{OR} = \frac{\Omega_2}{\Omega_2 + \Omega_3} - \frac{\Omega_1^2\Omega_2}{(\Omega_1 + \Omega_2)(\Omega_1\Omega_2 + \Omega_1\Omega_3 + \Omega_2\Omega_3)} \quad (30)$$

$$\rho_3^{OR} = \frac{\Omega_3}{\Omega_2 + \Omega_3} - \frac{\Omega_1^2\Omega_3}{(\Omega_1 + \Omega_3)(\Omega_1\Omega_2 + \Omega_1\Omega_3 + \Omega_2\Omega_3)} \quad (31)$$

### D. L Relays, i.i.d. Fading

Let us now consider the case of  $L$  relays where all the  $S$ - $R_i$  and  $R_i$ - $D$  links experience i.i.d. fading, as well as identical maximum Doppler frequency. In this case, the rate at which the system switches from  $R_i$  to any other relay is not affected

by the index  $i$ , because of symmetry. This key assumption allows for much simpler results than the i.n.i.d. case, as shown below.

*Theorem 2:* The relay switching rate of OR with  $L$  available relays, i.i.d. Rayleigh fading and identical maximum Doppler frequency  $\mathcal{F}$  in each of the intermediate channels involved, is given by

$$SR_{OR}^{iid} = \sqrt{2}L(L-1)\pi\mathcal{F} \sum_{l=0}^{L-2} (-1)^l \binom{L-2}{l} \left(\frac{1}{l+2}\right)^{\frac{3}{2}} \quad (32)$$

*Proof:* The switching rate for the  $L$ -branch case is evaluated through the use of the process

$$Z_i(t) = a_i(t) - \alpha_i(t). \quad (33)$$

It is important to note that, due to symmetry, the statistics of  $Z_i(t)$  are the same for each  $i \in \{1, \dots, L\}$ . Moreover, the i.i.d. assumption yields

$$SR_{OR,1} = \dots = SR_{OR,L} = SR_{OR}^{iid}/L. \quad (34)$$

Consequently, the relay switching rate for this case is obtained as

$$SR_{OR}^{iid} = Lf_Z(0) \int_0^\infty \dot{z} f_{\dot{Z}}(\dot{z}) d\dot{z}. \quad (35)$$

Using trivial integrations and the expressions for  $f_Z(0)$  and  $f_{\dot{Z}}(\dot{z})$  given in Appendix C, in (61) and (64), respectively, (35) yields (32); the proof is thus complete. As a cross-check, one may notice that for  $L = 2$ , (32) reduces to (19). ■

It is interesting to note from (32) that the relay switching rate is an increasing function of  $L$ , implying that, as expected, the larger the number of relays the more frequently relay switchings occurs.

*Corollary 4:* The average relay activation time for the i.i.d. scenario with arbitrary number of available relays is derived as

$$AT_{OR}^{iid} = \frac{1}{SR_{OR}^{iid}}. \quad (36)$$

*Proof:* The proof follows directly from (27) and (34) by considering that, due to symmetry,  $\rho_1^{OR} = \dots = \rho_L^{OR} = 1/L$ . ■

#### IV. RELAY SWITCHING RATES AND AVERAGE RELAY ACTIVATION TIME OF DSSC-B

*Theorem 3:* The relay switching rate for the two-relay DSSC-B setup is derived as in (37), shown at the top of the next page.

*Proof:* In the two-relay DSSC-B scheme, a relay switching occurs whenever the negative-going slope of the output SNR crosses the switching threshold,  $T$ ; equivalently, a relay switching occurs whenever the process  $a_i(t)$ ,  $i \in \{1, 2\}$ , crosses  $\sqrt{T/\Gamma}$  in a negative-going direction; recall that  $\Gamma$  denotes the common SNR without fading. The switching rate for relay  $R_i$ ,  $i \in \{1, 2\}$ , is given by

$$N_i(T) = \int_{-\infty}^0 |x| f_{a_i, \dot{a}_i}(\sqrt{T/\Gamma}, x) dx \quad (39)$$

where  $f_{a_i, \dot{a}_i}(\cdot, \cdot)$  denotes the joint PDF of  $a(t)$  and  $\dot{a}_i(t)$ . Using (46), (54) and the fact that  $a(t)$  and  $\dot{a}_i(t)$  are independent processes, (39) yields

$$N_i(T) = \sqrt{2\pi} \left( \sqrt{\frac{T}{\Gamma\Omega_{SR_i}}} \mathcal{F}_{SR_i} + \sqrt{\frac{T}{\Gamma\Omega_{R_iD}}} \mathcal{F}_{R_iD} \right) \times \exp\left(-\frac{\Omega_{SR_i} + \Omega_{R_iD} T}{\Omega_{SR_i} \Omega_{R_iD} \Gamma}\right). \quad (40)$$

The steady-state probabilities of activating  $R_1$  and  $R_2$  in the DSSC-B scheme are derived in Appendix D as shown in (3), given at the top of the next page, where

$$F_{a_i^2}(x) = 1 - \exp(-x/\Omega_i) \quad (41)$$

denotes the CDF of  $a_i^2(t)$ ,  $i \in \{1, 2\}$ . Therefore, the relay switching rate can be expressed as

$$SR_{DSSC}(T) = \rho_1^{DSSC}(T) N_1(T) + \rho_2^{DSSC}(T) N_2(T). \quad (42)$$

Substituting (40)-(41) into (42) and after some algebraic manipulations, we arrive at (37). ■

*Corollary 5:* The average relay activation time for DSSC-B is given by

$$AT_{i,DSSC}(T) = \frac{2\rho_i^{DSSC}(T)}{SR_{DSSC}(T)} \quad (43)$$

*Proof:* The proof is identical to that of Corollary 1. We note that the factor 2 in the numerator is present because the system switches to  $R_i$  from the other available relay at a rate that equals  $SR_{DSSC}(T)/2$  since, apparently, the number of times the system switches from  $R_2$  to  $R_1$  equals the number of switches from  $R_1$  to  $R_2$ , in the long run. ■

#### V. NUMERICAL EXAMPLES AND DISCUSSION

##### A. Implementation Issues

In classical diversity communication systems (e.g. conventional selection combining or switch and stay combining, where no relaying takes place), the switching between the diversity branches causes several problems, such as ‘‘an internal outage’’ due to the corruption of the receiver filters and data signal chains, as well as phase estimation failures [12]. In cooperative relaying systems, however, where signals have to be exchanged between spatially distributed nodes, an additional important issue has to be addressed; such issue is time synchronization, which requires the local clocks of the relay nodes to be synchronized, requiring various degrees of precision [14].

It should be pointed out that in classical switched diversity systems, keeping time synchronization between the transmitter and the receiver after switching to a different diversity branch is not a serious problem, since the relative time delays of the different branches are practically identical, because the link distances are practically identical. On the contrary, in cooperative relaying systems, especially in those employing mobile relays, the time delay between the transmitter and the destination may change dramatically as soon as a relay switching occurs. Time synchronization becomes particularly challenging in cooperative networks since the network dynamics such as propagation time or physical channel access

$$\begin{aligned}
SR_{DSSC}(T) = & \frac{\sqrt{2\pi}e^{-\frac{T(\Omega_1+\Omega_2)}{\Gamma\Omega_1\Omega_2}} \left( e^{\frac{2T}{\Gamma\Omega_2}} - e^{\frac{T}{\Gamma\Omega_2}} \right) \left( \sqrt{\frac{T}{\Gamma\Omega_{SR_1}}} \mathcal{F}_{SR_1} + \sqrt{\frac{T}{\Gamma\Omega_{R_1D}}} \mathcal{F}_{R_1D} \right)}{2e^{\frac{T}{\Gamma\Omega_1}} - e^{\frac{2T}{\Gamma\Omega_1}} + 2e^{\frac{T}{\Gamma\Omega_2}} - e^{\frac{2T}{\Gamma\Omega_2}} - 2e^{\frac{T(\Omega_1+\Omega_2)}{\Gamma\Omega_1\Omega_2}} + e^{\frac{T(\Omega_1+2\Omega_2)}{\Gamma\Omega_1\Omega_2}} + e^{\frac{T(2\Omega_1+\Omega_2)}{\Gamma\Omega_1\Omega_2}} - 2} \\
& + \frac{\sqrt{2\pi}e^{-\frac{T(\Omega_1+\Omega_2)}{\Gamma\Omega_1\Omega_2}} \left( e^{\frac{2T}{\Gamma\Omega_1}} - e^{\frac{T}{\Gamma\Omega_1}} \right) \left( \sqrt{\frac{T}{\Gamma\Omega_{SR_2}}} \mathcal{F}_{SR_2} + \sqrt{\frac{T}{\Gamma\Omega_{R_2D}}} \mathcal{F}_{R_2D} \right)}{2e^{\frac{T}{\Gamma\Omega_1}} - e^{\frac{2T}{\Gamma\Omega_1}} + 2e^{\frac{T}{\Gamma\Omega_2}} - e^{\frac{2T}{\Gamma\Omega_2}} - 2e^{\frac{T(\Omega_1+\Omega_2)}{\Gamma\Omega_1\Omega_2}} + e^{\frac{T(\Omega_1+2\Omega_2)}{\Gamma\Omega_1\Omega_2}} + e^{\frac{T(2\Omega_1+\Omega_2)}{\Gamma\Omega_1\Omega_2}} - 2} \\
& + \frac{\sqrt{2\pi} \left( \sqrt{\frac{T}{\Gamma\Omega_{SR_1}}} \mathcal{F}_{SR_1} + \sqrt{\frac{T}{\Gamma\Omega_{R_1D}}} \mathcal{F}_{R_1D} + \sqrt{\frac{T}{\Gamma\Omega_{SR_2}}} \mathcal{F}_{SR_2} + \sqrt{\frac{T}{\Gamma\Omega_{R_2D}}} \mathcal{F}_{R_2D} \right) \left( e^{\frac{T}{\Gamma\Omega_1}} - 1 \right) \left( e^{\frac{T}{\Gamma\Omega_2}} - 1 \right)}{2e^{\frac{T}{\Gamma\Omega_1}} - e^{\frac{2T}{\Gamma\Omega_1}} + 2e^{\frac{T}{\Gamma\Omega_2}} - e^{\frac{2T}{\Gamma\Omega_2}} - 2e^{\frac{T(\Omega_1+\Omega_2)}{\Gamma\Omega_1\Omega_2}} + e^{\frac{T(\Omega_1+2\Omega_2)}{\Gamma\Omega_1\Omega_2}} + e^{\frac{T(2\Omega_1+\Omega_2)}{\Gamma\Omega_1\Omega_2}} - 2}
\end{aligned} \quad (37)$$

$$\begin{aligned}
\rho_1^{DSSC}(T) &= \frac{F_{a_2^2}(T/\Gamma) \left[ F_{a_2^2}(T/\Gamma) - 1 \right] \left[ 1 - F_{a_1^2}(T/\Gamma) + F_{a_1^2}^2(T/\Gamma) \right]}{F_{a_1^2}(T/\Gamma) \left[ F_{a_1^2}(T/\Gamma) - 1 \right] \left[ 1 - 2F_{a_2^2}(T/\Gamma) + 2F_{a_2^2}^2(T/\Gamma) \right] + F_{a_2^2}(T/\Gamma) \left[ F_{a_2^2}(T/\Gamma) - 1 \right]} \\
\rho_2^{DSSC}(T) &= \frac{F_{a_1^2}(T/\Gamma) \left[ F_{a_1^2}(T/\Gamma) - 1 \right] \left[ 1 - F_{a_2^2}(T/\Gamma) + F_{a_2^2}^2(T/\Gamma) \right]}{F_{a_2^2}(T/\Gamma) \left[ F_{a_2^2}(T/\Gamma) - 1 \right] \left[ 1 - 2F_{a_1^2}(T/\Gamma) + 2F_{a_1^2}^2(T/\Gamma) \right] + F_{a_1^2}(T/\Gamma) \left[ F_{a_1^2}(T/\Gamma) - 1 \right]}
\end{aligned} \quad (38)$$

time are in general non-deterministic, because of the relative distances between source, relays, and destination. In other words, switching to another relay would provoke a time synchronization readjustment between the transmitting relay and the destination, as well as the source. Therefore, it is evident that frequent relay switchings are not desirable as they significantly increase the implementation complexity of relay selection protocols, rendering them difficult to implement in fast fading scenarios.

### B. Numerical Results

In this subsection, we illustrate some numerical results, regarding the relaying switching rate and average relaying activation time of both OR and DSSC-B. Recall that the relay selection criterion for both DF and AF relaying is identical and given in (4), hence the numerical results presented in this section apply to both DF and AF relaying. All the results were also confirmed by simulations, which were made according to the auto-regressive model for simulating time-varying channels [15].

As already mentioned, the relay switching rate and the average relaying activation time constitute a vital part in the design and implementation of selective cooperative relaying, due to the complexity issues that each relay switching entails. Fig. 1 depicts the normalized switching rates of both OR and DSSC-B versus the ratio  $\mathcal{F}_{SR_i}/\mathcal{F}_{R_iD}$  for  $L = 2$  relays, when the fading powers in the  $S$ - $R$  and  $R$ - $D$  links are unbalanced. Specifically, the average channel gains of the  $S$ - $R_1$  and  $S$ - $R_2$  links are assumed equal to each other, and the same is true for the  $R_1$ - $D$  and  $R_2$ - $D$  links; the difference in the corresponding average SNRs is 10dB, so that  $\Gamma\Omega_{SR_i} = \Gamma\Omega_{R_iD} \pm 10\text{dB}$ ,  $i \in \{1, 2\}$ . As observed from Fig. 1, the switching rate of DSSC-B is always smaller than that of OR, with the latter to be under specific conditions even ten times greater than the former. This verifies the intuition that DSSC-B leads to less frequent relay switchings, accounting thus for simpler practical implementations. It should be noted that in Fig. 1

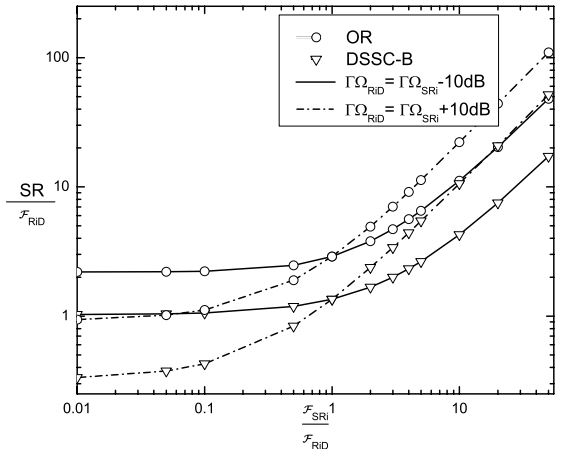


Fig. 1. Relay switching rates of OR and DSSC.

the switching threshold,  $T$ , used is that threshold which leads to the maximum possible switching rate of DSSC-B, which is determined through numerical optimization methods. In other words, the DSSC-B curves depict the worst case in terms of switching rate, demonstrating that even in this scenario the DSSC-B switching rate is much less than that of OR.

The effect of the number of available relays on the switching rate of OR for the i.i.d. case with equal maximum Doppler frequencies in all of the links involved, is plotted in Fig. 2. As expected, the switching rate continuously increases as the number of the relays increases, yet in a non-linear manner. The main result extracted from Fig. 2 is that in cases where the number of available relays is large, leaving some of them out of the selection set may be preferable in practical applications with high Doppler spread, despite the apparent performance cost.

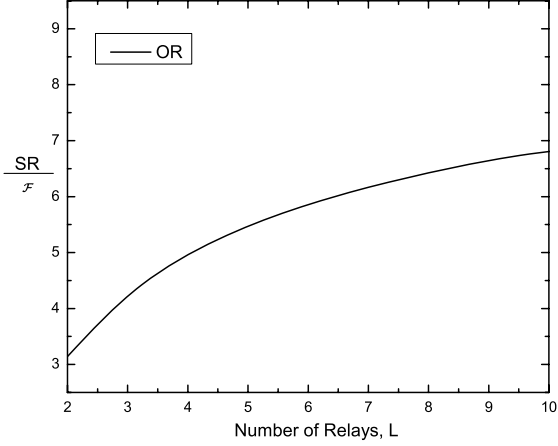


Fig. 2. Relay switching rate of OR versus the number of available relays.

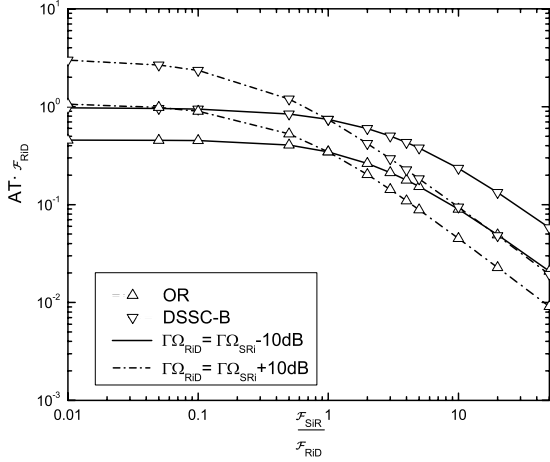


Fig. 3. Average activation time of OR and DSSC.

The normalized average relay activation time of the relays for the same conditions as in Fig. 1 is plotted in Fig. 3. Again the average activation time in DSSC-B is the minimum possible, i.e., the parameter  $T$  is properly selected so as to account for the worst case. Fig. 3 manifests that the average relay activation time of DSSC-B is considerably larger than that of OR, regardless of the assumptions on the relative  $S$ - $R$  and  $R$ - $D$  channel strengths. The effect of the number of relays on the activation time for OR is shown in Fig. 4. One may observe that the activation time reduces in a non-linear fashion as the number of relays reduces.

Finally, in Fig. 5, the switching rates of OR and DSSC-B are plotted for  $L = 2$  relays and several values of the switching threshold,  $T$ , assuming that the channel gains in the  $S$ - $R$  and  $R$ - $D$  links are unbalanced, as well as that  $\mathcal{F}_{SR_i} = 2\mathcal{F}_{R_iD}$ . For each of the scenarios shown here, the switching rate of DSSC-B is considerably lower than that of OR. The ratio

of switching rates is, however, crucially dependent on the switching threshold of DSSC-B, implying that the difference of the switching rates of OR and DSSC-B shown in Fig. 1 is significantly expanded for  $T$  values different from the worst (in terms of relay switching rate) case for DSSC-B.

## VI. CONCLUSION

We conducted a study of selective cooperative relaying in time-varying fading channels. In particular, we derived the relay switching rate for selective cooperative relaying, which reflects the concept of how frequently a cooperative scheme switches from one relay to another. Together with the relay switching rate, a closed-form expression for the average relay activation time was also derived, which corresponds to the average time which a relay remains activated until a switching occurs. Numerical results indicated that for the case where there are two relays available, selecting the active relay in a max SNR-based fashion (a.k.a. opportunistic relaying - OR) results in a considerably higher switching rate than selecting the relay in a DSSC-B fashion. Therefore, considering the complexity issues associated with frequent relay switching, DSSC-B leads to a simpler implementation of selective cooperative relaying than OR, and may be preferable in fast fading scenarios, despite being inferior in terms of error performance.

## APPENDIX A

### EVALUATION OF $f_Z(0)$

The PDF of  $Z(t)$  evaluated at zero reflects the probability that the absolute difference of the virtual fading gains of the  $S$ - $R_1$ - $D$  and  $S$ - $R_2$ - $D$  channels lies in the infinitesimal interval  $[0, dz]$ . Therefore,  $f_Z(0)$  can be expressed as

$$f_Z(0) = \int_0^\infty f_{a_1}(x) f_{a_2}(x) dx \quad (44)$$

where  $f_{a_1}(\cdot)$  and  $f_{a_2}(\cdot)$  denote the PDFs of the processes  $a_1(t)$  and  $a_2(t)$ , respectively. Using (1) and (2), the CDF of  $a_i(t)$ ,  $i \in \{1, 2\}$ , is expressed as

$$\begin{aligned} F_{a_i}(x) &= 1 - \int_x^\infty f_{SR_i}(\omega) d\omega \int_x^\infty f_{R_iD}(\omega) d\omega \\ &= 1 - \exp\left(-\frac{\Omega_{SR_i} + \Omega_{R_iD}}{\Omega_{SR_i}\Omega_{R_iD}}x^2\right) \end{aligned} \quad (45)$$

while the PDF of  $a_i(t)$  is derived by differentiating (45), yielding

$$f_{a_i}(x) = 2x \frac{\Omega_{SR_i} + \Omega_{R_iD}}{\Omega_{SR_i}\Omega_{R_iD}} \exp\left(-\frac{\Omega_{SR_i} + \Omega_{R_iD}}{\Omega_{SR_i}\Omega_{R_iD}}x^2\right). \quad (46)$$

Note that the process  $a_i(t)$  is also Rayleigh distributed; its average squared value is denoted by  $\Omega_i$  and is given by

$$\Omega_i = \frac{\Omega_{SR_i}\Omega_{R_iD}}{\Omega_{SR_i} + \Omega_{R_iD}}. \quad (47)$$

Then,  $f_Z(0)$  is derived as the integral of the product of two Rayleigh distributions, yielding

$$f_Z(0) = \frac{4}{\Omega_1\Omega_2} \int_0^\infty x^2 \exp\left(-\frac{\Omega_1 + \Omega_2}{\Omega_1\Omega_2}x^2\right) dx. \quad (48)$$

Using [16, Eq. (3.321.5)], (48) yields (13).



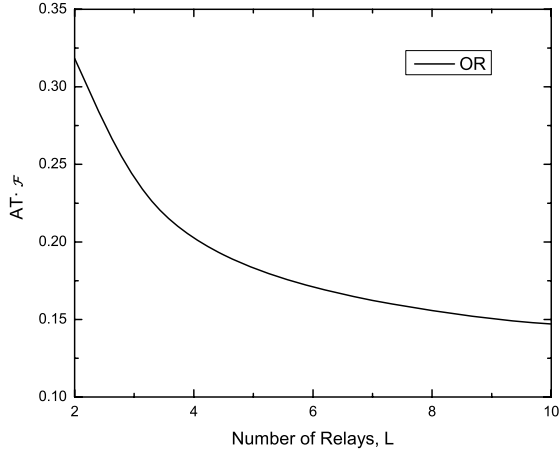


Fig. 4. Average activation time of OR versus the number of available relays.

#### APPENDIX B DERIVATION OF $f_{\dot{Z}}(\cdot)$

The time derivative of  $Z(t)$ ,  $\dot{Z}(t)$ , equals the difference of the time derivatives  $\dot{a}_1(t)$  and  $\dot{a}_2(t)$ . Before proceeding in deriving  $f_{\dot{Z}}(\dot{z})$ , we first obtain the PDF of  $\dot{a}_i(t)$ ,  $i \in \{1, 2\}$ , as

$$f_{\dot{a}_i}(x) = \Pr\{a_{SR_i} \leq a_{R_iD}\} f_{\dot{a}_{SR_i}}(x) + \Pr\{a_{SR_i} > a_{R_iD}\} f_{\dot{a}_{R_iD}}(x) \quad (49)$$

where  $f_{\dot{a}_{SR_i}}(\cdot)$  and  $f_{\dot{a}_{R_iD}}(\cdot)$  denote the PDF of the time derivatives of  $a_{SR_i}$  and  $a_{R_iD}$ , respectively. Given that  $a_{SR_i}$  and  $a_{R_iD}$  are Rayleigh distributed,  $f_{\dot{a}_{SR_i}}(\cdot)$  and  $f_{\dot{a}_{R_iD}}(\cdot)$  can be expressed as [13]

$$f_{\dot{a}_{SR_i}}(x) = \frac{1}{\sqrt{2\pi}\sigma_{SR_i}} \exp\left(-\frac{x^2}{2\sigma_{SR_i}^2}\right) \quad (50)$$

$$f_{\dot{a}_{R_iD}}(x) = \frac{1}{\sqrt{2\pi}\sigma_{R_iD}} \exp\left(-\frac{x^2}{2\sigma_{R_iD}^2}\right) \quad (51)$$

i.e.,  $\dot{a}_{SR_i}$  and  $\dot{a}_{R_iD}$  are zero-mean Gaussian random variables (RVs) with standard deviations  $\sigma_{SR_i} = \pi\mathcal{F}_{SR_i}\sqrt{\Omega_{SR_i}}$  and  $\sigma_{R_iD} = \pi\mathcal{F}_{R_iD}\sqrt{\Omega_{R_iD}}$ , respectively. The steady-state probabilities  $\Pr\{a_{SR_i} \leq a_{R_iD}\}$  and  $\Pr\{a_{SR_i} > a_{R_iD}\}$  are given by

$$\begin{aligned} \Pr\{a_{SR_i} \leq a_{R_iD}\} &= \int_0^\infty f_{\dot{a}_{SR_i}}(x) [1 - F_{a_{R_iD}}(x)] dx \\ &= \frac{\Omega_{R_iD}}{\Omega_{SR_i} + \Omega_{R_iD}} \end{aligned} \quad (52)$$

$$\Pr\{a_{SR_i} > a_{R_iD}\} = \frac{\Omega_{SR_i}}{\Omega_{SR_i} + \Omega_{R_iD}}. \quad (53)$$

The PDF of  $\dot{a}_i(t)$  is therefore derived as

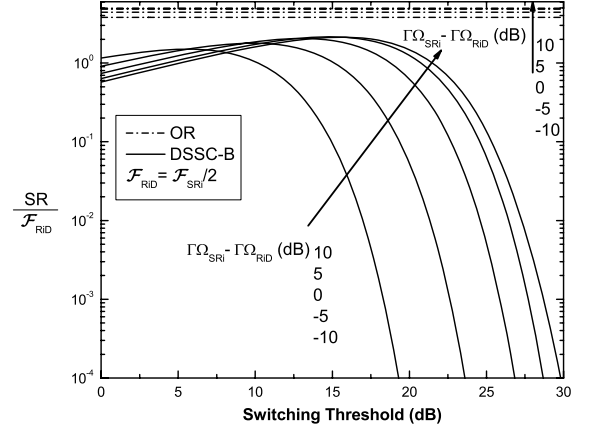


Fig. 5. Relay switching rates of OR and DSSC versus the normalized switching threshold.

$$\begin{aligned} f_{\dot{a}_i}(x) &= \frac{1}{\sqrt{2\pi}^{3/2} (\Omega_{SR_i} + \Omega_{R_iD})} \\ &\times \left[ \frac{\Omega_{R_iD} \exp\left(-\frac{x^2}{2\pi^2 \mathcal{F}_{SR_i}^2 \Omega_{SR_i}}\right)}{\sqrt{\Omega_{SR_i} \mathcal{F}_{SR_i}}} \right. \\ &\left. + \frac{\Omega_{SR_i} \exp\left(-\frac{x^2}{2\pi^2 \mathcal{F}_{R_iD}^2 \Omega_{R_iD}}\right)}{\sqrt{\Omega_{R_iD} \mathcal{F}_{R_iD}}} \right] \end{aligned} \quad (54)$$

where  $i \in \{1, 2\}$ . Consequently, it follows from (9) that the PDF of the process  $\dot{Z}(t)$  is expressed as [17, ch. 6]

$$f_{\dot{Z}}(x) = \int_{-\infty}^{\infty} f_{\dot{a}_1}(y+x) f_{\dot{a}_2}(y) dy. \quad (55)$$

Substituting (54) in (55) and making use of the integral [16, Eq. (3.323.2)]

$$\begin{aligned} &\int_{-\infty}^{\infty} \exp\left(-\frac{(y+x)^2}{\alpha_1}\right) \exp\left(-\frac{y^2}{\alpha_2}\right) dy \\ &= \sqrt{\pi \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2}} \exp\left(-\frac{x^2}{\alpha_1 + \alpha_2}\right) \end{aligned} \quad (56)$$

we derive the PDF of  $\dot{Z}(t)$ ,  $f_{\dot{Z}}(\cdot)$ , as

$$\begin{aligned} f_{\dot{Z}}(x) &= \frac{1}{\sqrt{2\pi}^{3/2} (\Omega_{SR_1} + \Omega_{R_1D}) (\Omega_{SR_2} + \Omega_{R_2D})} \\ &\times \left[ \frac{\Omega_{SR_1} \Omega_{SR_2} \exp\left(-\frac{x^2}{2\pi^2 (\Omega_{R_1D} \mathcal{F}_{R_1D}^2 + \Omega_{R_2D} \mathcal{F}_{R_2D}^2)}\right)}{\sqrt{\Omega_{R_1D} \mathcal{F}_{R_1D}^2 + \Omega_{R_2D} \mathcal{F}_{R_2D}^2}} \right. \\ &\left. + \frac{\Omega_{R_1D} \Omega_{SR_2} \exp\left(-\frac{x^2}{2\pi^2 (\Omega_{SR_1} \mathcal{F}_{SR_1}^2 + \Omega_{R_2D} \mathcal{F}_{R_2D}^2)}\right)}{\sqrt{\Omega_{SR_1} \mathcal{F}_{SR_1}^2 + \Omega_{R_2D} \mathcal{F}_{R_2D}^2}} \right] \end{aligned}$$

$$\left. \begin{aligned} & + \frac{\Omega_{R_2 D} \Omega_{S R_1} \exp\left(-\frac{x^2}{2\pi^2(\Omega_{S R_2} \mathcal{F}_{S R_2}^2 + \Omega_{R_1 D} \mathcal{F}_{R_1 D}^2)}\right)}{\sqrt{\Omega_{S R_2} \mathcal{F}_{S R_2}^2 + \Omega_{R_1 D} \mathcal{F}_{R_1 D}^2}} \\ & + \frac{\Omega_{R_1 D} \Omega_{R_2 D} \exp\left(-\frac{x^2}{2\pi^2(\Omega_{S R_1} \mathcal{F}_{S R_1}^2 + \Omega_{S R_2} \mathcal{F}_{S R_2}^2)}\right)}{\sqrt{\Omega_{S R_1} \mathcal{F}_{S R_1}^2 + \Omega_{S R_2} \mathcal{F}_{S R_2}^2}} \end{aligned} \right] . \quad (57)$$

Having an expression for  $f_{\dot{z}}(\cdot)$ , the last term in (12) is derived using [16, Eq. (3.321.4)] as shown in (14).

### APPENDIX C

#### RELAY SWITCHING RATE OF OR WITH $L$ AVAILABLE RELAYS

*Evaluation of  $f_Z(0)$ :* Let us first derive the PDF of  $a_k(t)$  as the PDF of the maximum of  $L-1$  i.i.d. Rayleigh RVs, yielding

$$\begin{aligned} f_{a_k}(x) &= \sum_{l=1}^{L-1} f_{a_l}(x) \prod_{\substack{j=1 \\ j \neq l}}^{L-1} F_{a_j}(x) \\ &= (L-1) f_a(x) [F_a(x)]^{L-2} \end{aligned} \quad (58)$$

with  $f_a(x) = (2x/\Omega) \exp(-x^2/\Omega)$  and  $F_a(x) = 1 - \exp(-x^2/\Omega)$ , where  $\Omega = \Omega_{S R_1}/2 = \Omega_{R_1 D}/2 = \dots = \Omega_{S R_L}/2 = \Omega_{R_L D}/2$  denotes the average squared channel gain in each of the virtual end-to-end channels involved. Then,  $f_Z(0)$  is derived as

$$\begin{aligned} f_Z(0) &= \int_0^\infty f_a(x) f_{a_k}(x) dx \\ &= (L-1) \int_0^\infty f_a^2(x) [F_a(x)]^{L-2} dx. \end{aligned} \quad (59)$$

Using the product expansion

$$\left[1 - \exp\left(-\frac{x^2}{\Omega}\right)\right]^{L-2} = 1 + \sum_{l=1}^{L-2} (-1)^l \binom{L-2}{l} \exp\left(-l\frac{x^2}{\Omega}\right) \quad (60)$$

(59) yields

$$f_Z(0) = \frac{(L-1)\sqrt{\pi}}{\Omega^2} \sum_{l=0}^{L-2} (-1)^l \binom{L-2}{l} \left(\frac{\Omega}{l+2}\right)^{\frac{3}{2}}. \quad (61)$$

*Derivation of  $f_{\dot{z}}(\cdot)$ :* Because of the i.i.d. assumption, the PDF of the time-derivative  $\dot{a}_i(t)$  is derived from (54) as

$$f_{\dot{a}_i}(x) = \frac{1}{2\pi^{3/2}\mathcal{F}\sqrt{\Omega}} \exp\left(-\frac{x^2}{4\pi^2\mathcal{F}^2\Omega}\right) \quad (62)$$

that is,  $\dot{a}_i$  is a zero-mean Gaussian RV with standard deviation  $\hat{\sigma}_{a_i} = \pi\mathcal{F}\sqrt{\Omega}$ . The PDF of the time-derivative of  $a_k(t)$ ,  $\dot{a}_k(t)$ , is derived as

$$f_{\dot{a}_k}(x) = \sum_{j=1}^{L-1} \rho_j^{OR} f_{\dot{a}_j}(x) = f_{\dot{a}_i}(x) \quad (63)$$

which implies that, due to symmetry, the PDF of the time-derivative of the maximum of  $L-1$  i.i.d. RVs, equals the PDF

of each of the  $L-1$  RVs. Using (55), the PDF of  $\dot{Z}(t) = \dot{a}_i(t) - \dot{a}_k(t)$  is derived as

$$f_{\dot{Z}}(x) = \frac{1}{(2\pi)^{3/2}\mathcal{F}\sqrt{\Omega}} \exp\left(-\frac{x^2}{8\pi^2\mathcal{F}^2\Omega}\right). \quad (64)$$

### APPENDIX D

#### STEADY-STATE RELAY ACTIVATION PROBABILITIES OF DSSC-B

Considering that relay switchings in DSSC-B are determined in exactly the same way as branch switchings in SSC-B, the steady-state relay activation probabilities for DSSC-B are derived through the Markov states of SSC-B given in [8]. Specifically, the Markov chain of DSSC-B yields six states, as follows. State 1 corresponds to the case where “ $R_1$  is active and the overall SNR down-crosses  $T$ ”; state 2 corresponds to “ $R_1$  is active and the overall SNR is below  $T$ ”; state 3 corresponds to “ $R_1$  is active and the overall SNR is greater than  $T$ ”; states 4, 5, 6 refer to the case where  $R_2$  is active, and are defined analogous to 1, 2, 3, respectively. The stationary probabilities,  $\pi_j^{DSSC-B}$ ,  $j \in \{1, \dots, 6\}$ , of the above Markov states are taken from [8, Eq. (21)], yielding

$$\pi_1^{DSSC-B} = \frac{[1-q_1]q_1[1-q_2]q_2}{[q_1+q_2][1+2q_1q_2] - [q_1+q_2]^2 - 2q_1^2q_2^2} \quad (65)$$

$$\pi_2^{DSSC-B} = \frac{q_1^2q_2[1-q_2]}{[q_1+q_2][1+2q_1q_2] - [q_1+q_2]^2 - 2q_1^2q_2^2} \quad (66)$$

$$\pi_3^{DSSC-B} = \frac{[1-q_1]^2[1-q_2]q_2}{[q_1+q_2][1+2q_1q_2] - [q_1+q_2]^2 - 2q_1^2q_2^2} \quad (67)$$

$$\pi_4^{DSSC-B} = \frac{[1-q_1]q_1[1-q_2]q_2^2}{[q_1+q_2][1+2q_1q_2] - [q_1+q_2]^2 - 2q_1^2q_2^2} \quad (68)$$

$$\pi_5^{DSSC-B} = \frac{q_1^2q_1[1-q_1]}{[q_1+q_2][1+2q_1q_2] - [q_1+q_2]^2 - 2q_1^2q_2^2} \quad (69)$$

$$\pi_6^{DSSC-B} = \frac{[1-q_1]q_1[1-q_2]q_2}{[q_1+q_2][1+2q_1q_2] - [q_1+q_2]^2 - 2q_1^2q_2^2} \quad (70)$$

where  $q_1 = F_{a_1^2}(T/\Gamma)$ ;  $q_2 = F_{a_2^2}(T/\Gamma)$ . Considering that  $R_1$  is active in the states 1, 2, 3, while  $R_2$  in states 4, 5, 6, the steady-state relay activation probabilities of DSSC-B are derived as  $\rho_1^{DSSC} = \sum_{j=1}^3 \pi_j^{DSSC-B}$ ;  $\rho_2^{DSSC} = \sum_{j=4}^6 \pi_j^{DSSC-B}$ , yielding (38).

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